

DISPERSION- AND DIFFRACTION-INDUCED EVOLUTION OF LONGITUDINAL POLARIZATION FORCES IN ULTRASHORT LASER PULSES

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Abstract. In this work, we solve the (3+1)D paraxial equation, including second-order dispersion and diffraction. The dynamics of the longitudinal polarization forces generated by ultrashort laser pulses are analyzed. In this way, analytical evolution laws for the pulse envelope, force density, and potential are derived. In our previous work, the polarization force was investigated under the first-order dispersion approximation and in the absence of diffraction. Stationary solutions for the force density and potential in 3D space were obtained. The main aim of the present work is to uncover the temporal evolution of the longitudinal polarization force and potential, depending on the diffraction and dispersion of a femtosecond laser pulse. This research provides valuable guidance for developing and designing laser accelerators, with potential applications in laser-driven neutral particle technologies.

Keywords: Attractive longitudinal optical force, neutral particles, femtosecond laser pulses, (3+1)D paraxial equation, diffraction, dispersion

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1. Introduction

Ashkin first introduced optical trapping of neutral matter using transverse (gradient) forces in focused laser beams [1, 2]. Later, [3] showed that, in the pulse-propagation regime, an additional longitudinal polarization force arises, associated with the time derivative of the Poynting vector. These forces have been widely studied and applied to manipulate individual spherical nanoparticles in so-called optical tweezers. Researchers have used various methods to address this problem, including Rayleigh scattering [1], Maxwell's stress tensor [4], and the dipole approximation [5]. Reference [6] derived an analytical model that captures both linear and nonlinear contributions to the optical force on a Rayleigh particle. In applications where medium-induced effects must be minimized, nanoparticles may be manipulated in a vacuum chamber to reduce the surrounding medium's influence.

By contrast, the optical forces discussed by the authors in [1,3] act collectively on neutral, polarizable species within the pulse volume in a gaseous medium. The radiation-force density generated by a laser pulse in an ensemble of neutral atoms and molecules was first considered in [3]. In [7], it was estimated that this force density drives particles toward

the center of the optical pulse, and that the attractive potential is proportional to the negative of the intensity profile.

The results, presented in [7, 8], were obtained in the near-field approximation, accounting for first-order dispersion while neglecting diffraction for propagation distances much shorter than the diffraction length. Under these conditions, stationary three-dimensional profiles of the optical force density and potential were obtained in the co-moving frame. Beyond the near field, however, the pulse intensity profile is strongly modified by the combined effects of diffraction and dispersion. Therefore, the spatiotemporal shape of the trapping potential and the longitudinal optical force, both of which depend on the pulse-intensity profile and its first time derivative, also change during propagation.

To quantify this evolution, we first perform an analytical investigation of the corresponding linear (3+1)D paraxial equation for the laser-pulse envelope. This treatment also yields an analytical expression for the spatiotemporal evolution of the longitudinal polarization force density and its associated potential density. However, for practical implementations, the density is not a directly measurable quantity. Therefore, from the force density, we obtain the real force evolution acting on the particles within the optical pulse at the e^{-1} level of its maximum. A rigorous analytical treatment that includes both diffraction and second-order dispersion of the optical force density and the averaged force, for laser pulses propagating over several diffraction lengths, has not been reported to date. In this work, we derive closed-form solutions describing the resulting evolution of the longitudinal force density and the averaged real force at several diffraction and dispersion lengths in dispersive media.

2. The connection between optical-force dynamics and pulse envelope deformation

In the laboratory frame, the nonlinear polarization-force density can be written as [7]:

$$\vec{f}^P(x, y, z, t) = \frac{4\pi}{c^2} \frac{d}{dt} \left[\left(\chi^{(1)} + \chi^{(3)} |\vec{A}|^2 \right) \vec{S} \right], \quad (1)$$

where c is the speed of light, $\chi^{(1)}$ and $\chi^{(3)}$ denote the linear and third-order nonlinear susceptibilities of the medium, respectively, $\vec{A}$ is the electric-field envelope. The vector $\vec{S}$ denotes the Poynting vector, while the superscript P in $\vec{f}^P$ denotes the polarization contribution to the force density. All electromagnetic quantities in Eq. (1) are expressed in Gaussian units. Under the conditions considered here, the nonlinear contribution, proportional to $\chi^{(3)} |\vec{A}|^2$ is much smaller than the linear contribution proportional to $\chi^{(1)}$. Consequently, we neglect this nonlinear contribution in the subsequent analytical treatment and focus on the linear propagation regime with laser-pulse power below the self-focusing threshold. In Gaussian units, the Poynting vector is defined as $\vec{S} = c/4\pi (\vec{E} \times \vec{H})$. For the nonmagnetic gaseous medium considered here, $\mu = 1$, so that $\vec{H} = \vec{B}$.

Both the Poynting vector and the corresponding force density depend on spatial coordinates and time. To describe the force density in a frame moving with the pulse envelope, we introduce a “local time” coordinate system, moving with group velocity v_{gr} .

The laboratory and co-moving variables are related by:

$$\xi = t - \frac{z}{v_{gr}}, \quad z' = z$$

With this convention, Eq. (1) transforms to:

$$\vec{f}^P(x, y, \xi, z) = \frac{4\pi\chi^{(1)}}{c^2} \frac{d\vec{S}(x, y, \xi, t)}{d\xi}. \quad (2)$$

For a linearly polarized pulse propagating along the longitudinal direction, the Poynting vector can be expressed in terms of the optical intensity as $\vec{S} = I\hat{z}$, where I is the optical intensity and $\hat{z}$ is the unit vector along the propagation direction. The relation between the intensity and the envelope amplitude is well known: $I = n_0 c |A_x|^2 / 2\pi$, where n_0 denotes the refractive index of the medium, and A_x is the amplitude of the linearly polarized laser pulse, which gives $\vec{S} = S_z \vec{z} = I\vec{z} = \left[n_0 c |A_x|^2 / 2\pi \right] \vec{z}$ where n_0 denotes the refractive index of the medium and A_x is the amplitude of the linearly polarized laser pulse. Using the relation between the optical intensity and the squared modulus of the complex field envelope, Eq. (2) can then be expressed in terms of $|A_x|^2$ as:

$$\vec{f}^P(x, y, \xi, z) = f_z(x, y, \xi, z) \vec{z} = \frac{2n_0\chi^{(1)}}{c} \frac{d|A_x|^2}{d\xi} \vec{z}. \quad (3)$$

In linear propagation through a transparent medium, the spatiotemporal profile of an ultrashort pulse evolves primarily due to diffraction and group-velocity dispersion. Because the longitudinal polarization-force density in Eq. (3) depends directly on the pulse envelope, these propagation effects also govern the evolution of the optical force. Therefore, we model the pulse envelope using the linear (3+1)D paraxial equation, which accounts for both Fresnel diffraction and second-order dispersion.

3. Finding analytical solutions of the amplitude equation

The propagation of a linearly polarized laser pulse with amplitude $A = A_x$ in an isotropic medium, under the influence of dispersion and diffraction, is governed by the paraxial equation for the amplitude function [9]:

$$2ik_0 \frac{\partial A}{\partial z} = \Delta_{\perp} A - \beta \frac{\partial^2 A}{\partial \xi^2}, \quad (4)$$

where $A(x, y, \xi, z)$ denotes the complex slowly varying envelope of the linearly polarized electric field and the transverse Laplacian is defined as $\Delta_{\perp} = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$. The dispersion constant β can be written as $\beta = k_0 k''$, where k'' is the group-velocity dispersion coefficient of the medium ($\beta > 0$ for normal dispersion and $\beta < 0$ for anomalous dispersion), and k_0 is the carrier wavenumber. The first term of the r.h.s in Eq. (4) describes transverse Fresnel diffraction, whereas the second term accounts for group-velocity dispersion.

We assume that the initial pulse envelope at $z=0$ has a Gaussian profile:

$$A(x, y, \xi, z=0) = A_0 \exp\left(-\frac{x^2 + y^2}{2d_0^2} - \frac{\xi^2}{2\xi_0^2}\right), \quad (5)$$

where A_0 is the initial peak amplitude, d_0 is the transverse Gaussian spot radius, and ξ_0 characterizes the pulse's initial longitudinal duration. We solve Eq. (4) analytically by applying a three-dimensional Fourier transform defined as:

$$A(x, y, \xi, z) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{A}(k_x, k_y, \omega_{\xi}, z) \exp(ik_x x) \exp(ik_y y) \exp(i\omega_{\xi} \xi) dk_x dk_y d\omega_{\xi}. \quad (6)$$

Substituting Eq. (6) into Eq. (4) yields the following ordinary differential equation for the Fourier-transformed envelope $\hat{A}(z, k_x, k_y, k_\xi)$:

$$2ik_0 \frac{\partial \hat{A}}{\partial z} = -\left(k_x^2 + k_y^2 - \beta\omega_\xi^2\right)\hat{A}. \quad (7)$$

The corresponding solution to Eq. (7) is given by:

$$\hat{A}(z, k_x, k_y, k_\xi) = \hat{A}(0, k_x, k_y, k_\xi) \exp\left(i \frac{k_x^2 + k_y^2 - \beta\omega_\xi^2}{2k_0} z\right), \quad (8)$$

where the Fourier transform of the initial pulse profile is expressed as:

$$\hat{A}(0, k_x, k_y, k_\xi) = d_0^2 \xi_0 A_0 \exp\left(-\frac{(k_x^2 + k_y^2)d_0^2 + \omega_\xi^2 \xi_0^2}{2}\right). \quad (9)$$

Substituting Eq. (9) into Eq. (8), we obtain:

$$\hat{A}(z, k_x, k_y, k_\xi) = d_0^2 \xi_0 A_0 \exp\left(-C_1^2(k_x^2 + k_y^2)/2 - C_2^2\omega_\xi^2/2\right), \quad (10)$$

where $C_1^2 = d_0^2(1 + iz/k_0 d_0^2)$ is the complex propagation parameter governing the transverse (k_x, k_y) dependence, and $C_2^2 = \xi_0^2(1 - i\beta z/k_0 \xi_0^2)$ is the corresponding parameter, related to the longitudinal ω_ξ dependence.

The analytical solution to Eq. (4) subject to the initial condition in Eq. (5) is obtained by applying the inverse Fourier transform to Eq. (10). Since the solution in Fourier space is separable with respect to k_x, k_y , and ω_ξ , the inverse transform can be written as the product of three one-dimensional Fourier integrals:

$$\begin{aligned} A(t, x, y, \xi) &= d_0^2 \xi_0 A_0 \int_{-\infty}^{\infty} \exp(-C_1^2 k_x^2 / 2) \exp(ik_x x) dk_x \\ &\quad \times \int_{-\infty}^{\infty} \exp(-C_1^2 k_y^2 / 2) \exp(ik_y y) dk_y \\ &\quad \times \int_{-\infty}^{\infty} \exp(-C_2^2 \omega_\xi^2 / 2) \exp(i\xi \omega_\xi) d\omega_\xi \end{aligned} \quad (11)$$

The Fourier integrals in Eq. (11) can be evaluated analytically. For the Gaussian initial condition given by Eq. (5), the resulting solution to Eq. (4) is:

$$\begin{aligned} A(z, x, y, \xi) &= \frac{d_0^2 \xi_0 A_0}{C_1^2 C_2} \exp\left(-\frac{(x^2 + y^2)}{2C_1^2} - \frac{\xi^2}{2C_2^2}\right) \\ &= \frac{A_0}{(1 + iz/z_{diff}) \sqrt{1 - iz/z_{disp}}} \\ &= \exp\left(-\frac{x^2 + y^2}{2d_0^2(1 + iz/z_{diff})} - \frac{\xi^2}{2\xi_0^2(1 - iz/z_{disp})}\right), \end{aligned} \quad (12)$$

where $z_{diff} = k_0 d_0^2$ denotes the diffraction length, and $z_{disp} = t_0^2 / k''$ is the dispersion length.

We can express the complex solution in Eq. (12) in terms of its modulus and phase as:

$$A(z, x, y, \xi) = |A(z, x, y, \xi)| \exp(i\phi(z, x, y, \xi)), \quad (13)$$

For the square modulus (which is proportional to the pulse intensity), we obtain:

$$|A(z, x, y, \xi)|^2 = \frac{A_0^2}{\left(1 + z^2 / z_{diff}^2\right) \sqrt{1 + z^2 / z_{disp}^2}} \times \exp\left(-\frac{x^2 + y^2}{d_0^2 \left(1 + z^2 / z_{diff}^2\right)} - \frac{\xi^2}{\xi_0^2 \left(1 + z^2 / z_{disp}^2\right)}\right). \quad (14)$$

Eq. (14) describes the spatial and temporal evolution of the squared pulse-envelope amplitude in a transparent medium, capturing both transverse broadening from Fresnel diffraction and longitudinal broadening from group-velocity dispersion.

4. Evolution of the optical force under diffraction and dispersion of a laser pulse

Using the exact diffraction and dispersion-induced evolution of the squared envelope amplitude given in Eq. (14), we determine the corresponding propagation of the longitudinal polarization-force density by substituting the squared modulus of the amplitude into Eq. (3). We first evaluate the derivative of Eq. (14) with respect to ξ :

$$\frac{\partial |A(z, x, y, \xi)|^2}{\partial \xi} = -\frac{2\xi}{\xi_0^2 \left(1 + z^2 / z_{disp}^2\right) \left(1 + z^2 / z_{diff}^2\right) \sqrt{1 + z^2 / z_{disp}^2}} \times \exp\left(-\frac{x^2 + y^2}{d_0^2 \left(1 + z^2 / z_{diff}^2\right)} - \frac{\xi^2}{\xi_0^2 \left(1 + z^2 / z_{disp}^2\right)}\right), \quad (15)$$

$$\vec{f} = f_z(z, x, y, \xi) \vec{z} = -A_0^2 \frac{4n_0 \chi^{(1)}}{c} \frac{\xi}{\xi_0^2} \frac{1}{\left(1 + z^2 / z_{disp}^2\right)^{\frac{3}{2}}} \times \frac{1}{\left(1 + z^2 / z_{diff}^2\right)} \exp\left(-\frac{x^2 + y^2}{d_0^2 \left(1 + z^2 / z_{diff}^2\right)} - \frac{\xi^2}{\xi_0^2 \left(1 + z^2 / z_{disp}^2\right)}\right) \vec{z}. \quad (16)$$

As Eq. (16) shows, the longitudinal force density is antisymmetric with respect to the pulse center $\xi = 0$ and changes sign across it. For the sign convention adopted here, the force on both sides of the pulse is directed toward the pulse center: the leading part of the pulse exerts a force toward decreasing ξ , whereas the trailing part exerts a force toward increasing ξ [8]. Since the longitudinal force density is a gradient field along the ξ direction, an associated potential-energy density can be introduced.

The corresponding differential change in potential-energy density (work) is introduced through:

$$dU(z, x, y, \xi) = \vec{f} \cdot d\vec{z} = -f_z^P(z, x, y, \xi) v_{gr} d\xi. \quad (17)$$

The minus sign arises from the conventional relation between force and work, according to which work is negative when the displacement is opposite to the force. Integrating Eq. (17) over the coordinate ξ , we obtain the corresponding potential-energy density:

$$U(z, x, y, \xi) = -A_0^2 \frac{4n_0 \chi^{(1)} v_{gr}}{c} \frac{1}{\left(1 + z^2 / z_{diff}^2\right) \sqrt{1 + z^2 / z_{disp}^2}} \times \exp\left(-\frac{x^2 + y^2}{d_0^2 \left(1 + z^2 / z_{diff}^2\right)} - \frac{\xi^2}{\xi_0^2 \left(1 + z^2 / z_{disp}^2\right)}\right). \quad (18)$$

Comparison of Eq. (18) with the squared envelope amplitude in Eq. (14) shows that the potential-energy density follows the inverted profile of the squared envelope amplitude. Under the present sign convention, it therefore forms an attractive potential profile for neutral polarizable particles.

5. Scaling of pulse broadening and force reduction

From Eq. (14), the transverse beam radius is found to evolve with the distance $z = v_{gr}t$ from the initial position as:

$$d(z) = d_0 \sqrt{1 + z^2 / z_{diff}^2}, \quad (19)$$

while the corresponding temporal pulse width evolves as:

$$\xi(z) = \xi_0 \sqrt{1 + z^2 / z_{disp}^2}, \quad (20)$$

At the pulse center, the peak squared envelope amplitude decreases during propagation according to:

$$|A(z)|^2 = \frac{A_0^2}{(1 + z^2 / z_{diff}^2) \sqrt{1 + z^2 / z_{disp}^2}}. \quad (21)$$

Because the longitudinal force in Eq. (16) is proportional to the gradient of the squared envelope amplitude (intensity), its peak value decreases with propagation distance by the same scaling factors as in Eqs. (19)-(21). The trapping potential in Eq. (18), which follows the inverted intensity profile, shows a similar reduction. Overall, Eqs. (16) and (18) show that dispersion and diffraction strongly limit long-range trapping.

In a gaseous medium, the diffraction length is typically shorter than the dispersion length; thus, diffraction dominates the intensity reshaping and consequently reduces the optical force and potential. Conversely, for femtosecond pulses propagating in solids (such as glass), the opposite trend may hold: the dispersion length can be shorter than the diffraction length, meaning that dispersion dominates the attenuation of the optical forces.

6. Order-of-magnitude estimations

In SI units, the quantity defined by Eq. (1) has the dimensions of force density (N m^{-3}), corresponding to a volume force acting on the dielectric medium. Although small in magnitude, such forces become significant in high-density media or when acting collectively on ensembles of neutral particles. For the experimental analysis, we choose the case where the diffraction length equals the dispersion length, $z_{diff} = z_{disp}$. This regime can be realized for propagation of a femtosecond pulse in air using the laser and medium parameters listed in Table 1.

Table 1. Laser and medium parameters used for the order-of-magnitude estimations.

Category	Parameter	Value
Laser source	Laser system	Ti:Sapphire laser
	Central wavelength	$\lambda = 800 \text{ nm}$
Dispersion	Pulse duration	$t_0 = 21 \text{ fs}$
	Group-velocity dispersion of air at $\lambda = 800 \text{ nm}$	$k \cong 2.1 \times 10^{-29} \text{ s}^2 / \text{cm}$
Diffraction	Carrier wavenumber at 800 nm	$k_0 = \frac{2\pi}{\lambda_0} = 7.85 \times 10^4 \text{ cm}^{-1}$
	Beam-spot diameter	$d_0 \cong 160 \mu\text{m}$

Using these diffraction and dispersion parameters, the condition $z_{diff} \cong z_{disp} \cong 20 \text{ cm}$ is satisfied. The peak intensity of the laser pulse must be less than or on the order of $I_0 \cong 10^7 \text{ W/cm}^2$. This ensures the pulse power remains below the critical power for self-focusing, corresponding to the linear propagation regime of the laser pulse in air.

Fig. 1 shows (a) the potential-energy density and (b) the longitudinal optical-force density for the initial Gaussian pulse in the near-field regime. When diffraction and second-order dispersive broadening are negligible, the force and potential profiles retain approximately their initial shapes and peak magnitudes, consistent with the approximation used in [8]. In Figs. 1 and 2, $r = \sqrt{x^2 + y^2}$ represents the radial transverse coordinate.

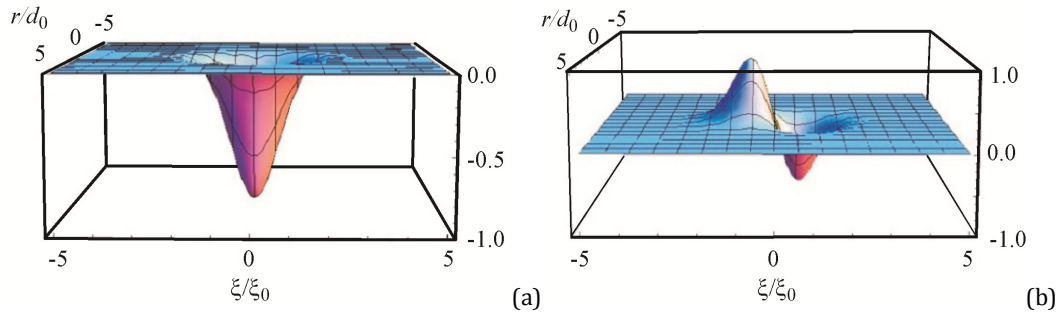


Fig. 1. Initial profiles of (a) the trapping potential and (b) the longitudinal force density for the initial Gaussian pulse at a propagation distance of $z = 0$.

Fig. 2 presents graphs of (a) the trapping potential and (b) the optical force density at a propagation distance of $z_{diff} \cong z_{disp} \cong 20 \text{ cm}$. At this distance, the peaks of both potential and force are reduced by the scaling factor $A_z^2 \simeq A_0^2 / (2/\sqrt{2})$, in full agreement with Eq. (21). The transverse spot size of the potential and force profiles is enlarged by the factor $d(z_{diff}) = d_0/2$ (see Eq. (19)) at one diffraction-dispersion length $d(z_{diff})$, while the longitudinal spatial form at the same distance $\zeta(z_{disp})$ is broadened by the factor $\zeta(z_{disp}) = \zeta_0\sqrt{2}$ (see Eq. (20)).

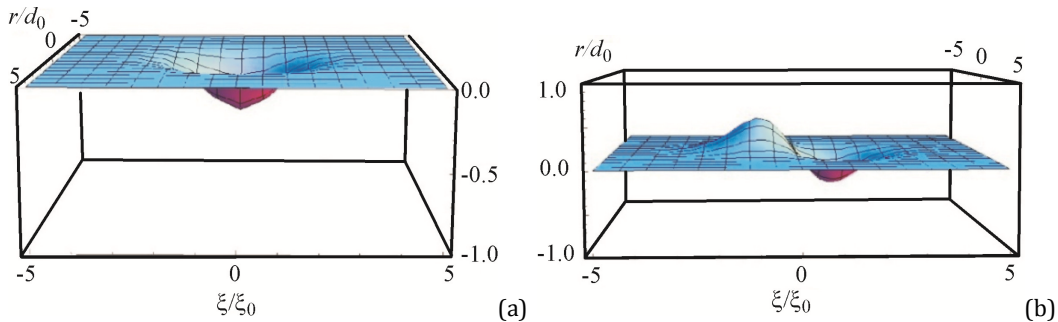


Fig. 2. Graphs of (a) the trapping potential and (b) the optical force density at a propagation distance of $z_{diff} \cong z_{disp} \cong 20 \text{ cm}$. The peak magnitudes of the potential and force are reduced by the factor $A_z^2 \simeq A_0^2 / (2/\sqrt{2})$, according to Eq. (21). The transverse spot size of the potential and force at one diffraction-dispersion length $d(z_{diff})$ is enlarged by the factor $d(z_{diff}) = d_0/2$ (see Eq. (19)), while the temporal pulse width at the same distance $\zeta(z_{disp})$ is broadened to $\zeta(z_{disp}) = \zeta_0\sqrt{2}$ (see Eq. (20)).

Figs. 1 and 2 show a significant reduction in both the force density and potential density magnitudes even after a single diffraction length. After several diffraction lengths, these magnitudes become too small for effective trapping under linear propagation conditions. Therefore, the near-field regime is most relevant for particle manipulation within the linear propagation regime. Trapping efficiency could be enhanced by exploiting nonlinear

propagation, for instance, by retaining the third-order nonlinear susceptibility χ^3 term in Eq. (1) or by operating in regimes where high laser intensity is sustained over extended distances.

Although the current analysis is limited to the linear propagation regime and describes the evolution of the optical force density of a particle ensemble, this density is not directly measurable. Therefore, we must derive the real force acting on the particles. For this purpose, we introduce the real force acting on the particles within the optical pulse at level e^{-1} from its maximum. Since the force density is an antisymmetric function, this real force is obtained by integrating twice the force density over the longitudinal half-distances $(\infty - \xi/2)$ and over the transverse (x, y) space [3, 8]:

$$F_z(z) = 2 \int \int \int_{\xi=-\infty}^{\xi=0} f_z(z, x, y, \xi) v_{gr} dx dy d\xi. \quad (22)$$

Substituting Eq. (16) into Eq. (22), we obtain the evolution of the real force $F_z(z)$ as a function of the propagation distance:

$$F_z(z) = \frac{4\pi n_0 \chi^{(1)} |A(z)|^2 d(z)^2}{c} = \frac{4\pi n_0 \chi^{(1)} A_0^2 d_0^2}{c \sqrt{1 + z^2 / z_{disp}^2}}. \quad (23)$$

As an unexpected result, only the deformation due to dispersion along the ξ direction affects the real force on the particles $F_z(z)$. This is because the fact that the averaged force depends on the product of the propagation-dependent parameters $|A(z)|^2 d(z)^2$. According to the solutions in Eq. (19) and Eq. (21), this product simplifies to:

$$|A(z)|^2 d(z)^2 = \frac{A_0^2 d_0^2}{\sqrt{1 + z^2 / z_{disp}^2}}. \quad (24)$$

Thus, as Eq. (23) shows, diffraction effects do not influence the averaged (real) optical polarization force. In contrast, dispersion significantly reduces this force by broadening the longitudinal pulse and reducing its temporal gradient.

7. Conclusions

In this work, we investigated the evolution of the longitudinal polarization-force density $f_z(z, x, y, \xi)$ and the real force $F_z(z)$ acting on an ensemble of particles, associated with ultrashort laser pulse propagation in a transparent medium. The analysis is based on the analytical solution of the linear (3+1) D paraxial equation, accounting for Fresnel diffraction and second-order group-velocity dispersion. This solution provides explicit expressions for the evolution of the pulse envelope, the corresponding longitudinal force-density and potential-energy-density profiles, and the averaged real force acting on the particles at level e^{-1} of the pulse maximum.

The resulting analytical expressions show how the transverse and longitudinal pulse broadening, driven by diffraction and dispersion, reduces the magnitudes of the associated optical-force and potential-energy-density distributions. In gaseous media, the relative importance of these effects is determined by the corresponding diffraction and dispersion lengths.

On the other hand, since the optical-force and potential-energy densities are not directly measurable, the actual macroscopic force that influences particle dynamics must be derived. Because the force density is an antisymmetric function along the ξ direction, this real force is

obtained by integrating twice the force density over the longitudinal half-distance $(\infty - \xi/2)^2$. and the entire transverse (x, y) plane [3, 8].

The averaged force depends directly on the product of the two propagation-dependent parameters $(|A(z)|^2 d(z)^2)$, rendering the averaged (real) optical polarization force completely unaffected by diffraction. This unexpected result stems from the fact that the diffraction-driven decrease in pulse amplitude (Eq. 21) is exactly compensated for by the simultaneous expansion of the transverse spot size (Eq. 19).

In this work, for the first time, analytical solutions describing the evolution of the longitudinal polarization density force and the averaged real force of laser pulses propagating over several diffraction and dispersion lengths in dispersive media are derived.

Consequently, these findings can be applied directly to optimize laser and particle characteristics, along with other optical parameters, to achieve an optimal particle-trapping regime within the pulse envelope.

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Data availability. No data was used for the research described in the article.

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Анотація. У цій роботі розв'язано $(3+1)D$ параксіальне рівняння з урахуванням дисперсії та дифракції другого порядку. Проаналізовано динаміку подовжніх

поляризаційних сил, що генеруються ультракороткими лазерними імпульсами. Отримано аналітичні залежності, що описують еволюцію огинаючої імпульсу, густини сили та потенціалу. У нашій попередній роботі поляризаційну силу досліджували в наближенні дисперсії першого порядку та за відсутності дифракції; було отримано стаціонарні розв'язки для густини сили та потенціалу в тривимірному просторі. Основною метою цієї роботи є дослідження часової еволюції поздовжньої поляризаційної сили та потенціалу залежно від дифракції та дисперсії фемтосекундного лазерного імпульсу. Результати дослідження є важливими для розробки та проектування лазерних прискорювачів і можуть знайти застосування в технологіях керування нейтральними частинками за допомогою лазерного випромінювання.

Ключові слова: поздовжня оптична сила притягання, нейтральні частинки, фемтосекундні лазерні імпульси, $(3+1)D$ параксіальне рівняння, дифракція, дисперсія