Ukrainian Journal of Physical Optics
ISSUE 3
2026

Symmetry-Tensorial Analysis of the Electro-Elastic Effect in Crystals: Comparing Similarities and Differences with the Pockels Effect

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Received: 29.05.2026

Abstract. In the present work, the electro-elastic effect has been considered for all Curie and point groups of crystal symmetry, except for the triclinic system, when an electric field is applied along the principal crystallographic directions. Relations for the change in acoustic wave velocities under the electric field are derived. It has been shown that, in many cases, the electro-elastic effect behaves similarly to the Pockels effect in optics. The rotation of the eigen vectors of the optical-frequency impermeability tensor caused by the linear electro-optic effect is analogous to the rotation of the eigen vectors of the Christoffel tensor in acoustics. In some symmetry point groups, the application of an electric field necessitates rewriting the elastic stiffness tensor in the rotated coordinate system. It has been found that if the non-orthogonality is induced by the electric field, the dependence of the respective acoustic wave velocities on the electric field is quadratic; otherwise, it is linear. By applying an electric field, one can control the angle of non-orthogonality. The angle of non-orthogonality can be induced by an electric field in all high-symmetry groups, including Curie groups and groups of cubic, middle, and orthorhombic systems, whenever the application of the electric field results in an abrupt lowering of symmetry, at least to the monoclinic system. In this case, the velocities of the AWs that acquire non-orthogonality depend quadratically on the electric field. In cases where the angle of non-orthogonality increases under the electric field, the acoustic wave velocities contain both linear and quadratic terms in the electric field strength.

Keywords: electro-elastic effect, acoustic wave velocities, Pockels effect, symmetry
UDC: 535.5, 534.22
DOI: 10.3116/16091833/Ukr.J.Phys.Opt.2026.03106

1. Introduction

Crystal acoustics is a branch of crystal physics that studies the propagation of acoustic waves (AWs) in crystals [1]. In many ways, crystal acoustics is related to crystal optics. However, since the constitutive tensor of elastic constants is a fourth-rank tensor, unlike the second-rank tensor of the optical frequency impermeability tensor, the phenomena of crystal acoustics are much more complex and multifaceted in their manifestation than those of crystal optics. The fundamental differences include the impossibility of AWs propagating in a vacuum, and transverse AWs in gases and liquids; the presence of three acoustic eigenwaves propagating in the medium (two transverse and one longitudinal) as opposed to only two transverse optical waves; the possibility of AWs being non-orthogonal, while electromagnetic waves can only be orthogonal; significant oblique of acoustic energy flow compared to optical Pointing vector; the possibility of up to 16 acoustic axes in crystals [1], while in crystal optics there can be a maximum of two, etc. However, many phenomena inherent in crystal optics also exist in crystal acoustics. These include, for example, acoustic activity and parametric crystal acoustics effects (we use this term by analogy with the effects of parametric optics, such as the Pockels, Kerr, electro- and piezo-gyration, Faraday, piezo-optics, etc.), which manifest as the rotation of the polarization plane and the change in the propagation velocity of acoustic eigenwaves under the action of external fields, respectively. In addition, crystal optics and crystal acoustics have effects that require consideration of both the media's acoustic and optical parameters. These phenomena include acousto-optic diffraction [2-4], Brillouin scattering [5], the photo-acoustic effect [6], etc.

The phenomena of parametric crystal acoustics manifest similarly to those of parametric crystal optics. Specifically, the analog of the well-known Pockels effect is the electro-elastic effect [7-9]; the Faraday effect corresponds to the rotation of the plane of polarization of AWs in a magnetic field [10,11]; the piezo-optical effect is akin to the elasto-acoustic effect [12-14], and so on. However, unlike the effects of parametric crystal optics, the phenomena of parametric crystal acoustics have been studied far less. It should be noted that the discovery of the Faraday effect for AWs confirmed the possibility of transverse AWs propagation in a superfluid ^{3}He-B [11]. The elasto-acoustic effect has been experimentally studied in rocks, where seismic activity, well stability, etc., can be assessed by changes in AW velocities under internal stresses. So these results were important for the oil industry and solving other geological problems [12,15]. The acousto-elastic effect is important for the development of high-precision pressure sensors and stable quartz resonators as well as for the nondestructive testing of structural materials (see e.g. [16]). The electro-elastic effect is used in delay lines, voltage sensors, piezoelectric resonators, etc.

The electro-elastic and piezo-elastic effects are quite small, being on the order of 10^{-3} of the AWs velocity when the bias field and external stress are on the order of 10^6 V/m and 10^6 N/m^2, respectively. Therefore, the development of methods for their study, in particular by searching for optimal geometries of their manifestation, is an important task. In this way it is impossible to do without preliminary symmetry-tensorial analysis of these effects and derivation of respective relations for the AWs velocities and the external actions. In the present work, we will concentrate our attention on the symmetry-tensorial analysis of electro-elastic effects and, starting from this, develop a systematic approach to studying the phenomena of parametric crystal acoustics, in particular the electro-elastic effect. The behavior of AWs will be compared with changes in the refractive indices of crystals caused by the Pockels effect [17].

2. Methods of analysis

The electro-elastic effect is described as the change in the elastic stiffness tensor (EST)

$$\bar{4}2m\,, \tag{1}$$

where $\bar{4}$ is the EST at zero bias field, and m - is this tensor under applied electric field E_m, $\Theta_{ijklm} = \Theta_{\lambda\mu m}$ – is the fifth-rank polar tensor with the internal symmetry [[V^2]^2]V. The internal symmetry of the $\Theta_{\lambda\mu m}$ tensor suggests that the electro-elastic effect can exist only in noncentrosymmetric media. The lowering of the point group of symmetry of media under the action of the polar vector of the electric field (Curie group of symmetry ∞mm) was determined following the Curie symmetry principle. The AWs velocities under the action of the electric field have been determined on the basis of the Christoffel equation

$$C_{ijkl} m_j m_k p_l = \rho v^2 p_i\,, \tag{2}$$

where $M_{il} = C_{ijkl} m_j m_k$ - is the second-rank Christoffel tensor, p_l, p_i - are the components of the unit displacement vector and m_j, m_k - are the components of the unit wavevector of the AWs. The angle of rotation of the eigenvectors of the Christoffel tensor is determined as

$$\tan 2\zeta_k = \frac{2M_{il}}{M_{ii} - M_{ll}}. \tag{3}$$

Our analysis will begin with the non-centrosymmetric Curie point groups of symmetry, and then proceed to the point groups of symmetry toward symmetry lowering. In the present work, we will not consider the geometries of electric field application at which the symmetry is lowered to the triclinic system, since in this case, due to the appearance of all components of the EST, the analysis is too cumbersome. In our analysis, the electric field coincides with the principal crystallographic direction.

3. Results and discussion

3.1. Curie groups of symmetry

The non-centrosymmetric Curie groups of symmetry include such groups as ∞, ∞mm, $\infty 2$, and $\infty\infty$. At least the polar ceramics [18] and chiral glasses [19,20] are representative solid-state materials of these symmetry groups. However, it has been found that the tensor $\Theta_{\lambda\mu m}$ does not contain nonzero components for the symmetry group $\infty\infty$. Notice that the third-rank polar tensor describing the linear electro-optic effect, which has initial symmetry $[V^2]V$, is also equal to zero for this symmetry group.

Let us begin our analysis with the Curie group of symmetry $\infty 2$. It has been found that despite the fact that the application of the electric field E_3 leads to the lowering of the symmetry $\infty 2 \to \infty$ the EST is not changed because there are no respective coupling components in the electro-elastic fifth-rank polar tensor. The electro-optic effect behaves in the same way. For group $\infty 2$, the electro-optic tensor possesses only two components:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{4}$$

where r_{ij} is the Pockels tensor component. Applying an electric field along the infinite-fold axis does not change the optical indicatrix due to the linear electro-optic effect.

Under the application of the electric field along the *X*-axis, the symmetry is reduced to the point group of symmetry 2, and the EST acquires the change:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & 0 & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & 0 & 0 & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}^0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5(C_{11}^0 - C_{12}^0) \end{bmatrix} \to \tag{5}$$

$$\to\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & \Theta_{141}E_1 & 0 & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & \Theta_{241}E_1 & 0 & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & \Theta_{341}E_1 & 0 & 0 \\ \Theta_{141}E_1 & \Theta_{241}E_1 & \Theta_{341}E_1 & C_{44}^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}^0 & \Theta_{651}E_1 \\ 0 & 0 & 0 & 0 & \Theta_{651}E_1 & C_{66}^0 \end{bmatrix}. \tag{5}$$

When the AWs propagate along the Z-axis, the Christoffel tensor can be written as:

$$\begin{bmatrix} C_{55}^0-\lambda & 0 & 0 \\ 0 & C_{44}^0-\lambda & \Theta_{431}E_1 \\ 0 & \Theta_{431}E_1 & C_{33}^0-\lambda \end{bmatrix}, \tag{6}$$

where λ are the eigenvalues of the Christoffel tensor. The non-orthogonality angle in this case is determined by the relation

$$\tan 2\zeta_X = \frac{2\Theta_{431}E_1}{C_{44}^0 - C_{33}^0} \tag{7}$$

As one can see, the angle of non-orthogonality is equal to zero at zero electric field and depends almost linearly (if the angle is small enough) on the electric field. The AW velocities can be written as:

$$\begin{gathered} \upsilon_{31} = \sqrt{\frac{C_{44}^0 + \Theta_{431}E_1}{\rho}} \approx \upsilon_{31}^0 + \frac{\Theta_{431}E_1}{2\rho\upsilon_{32}^0},\ \upsilon_{31}^0 = \sqrt{\frac{C_{44}^0}{\rho}}, \\ \upsilon_{33} \approx \upsilon_{33}^0 + \frac{(\Theta_{431}E_1)^2}{2\rho\upsilon_{33}^0(C_{33}^0 - C_{44}^0)},\ \upsilon_{33}^0 = \sqrt{\frac{C_{33}^0}{\rho}}, \\ \upsilon_{32} \approx \upsilon_{32}^0 - \frac{(\Theta_{431}E_1)^2}{2\rho\upsilon_{32}^0(C_{33}^0 - C_{44}^0)},\ \upsilon_{32}^0 = \sqrt{\frac{C_{44}^0}{\rho}}. \end{gathered} \tag{8}$$

where υ_{ij}^0 is the initial AWs velocities. The velocities of the two eigen waves depend on the electric field quadratically, but only the increment of υ_{31} AW linearly depends on the electric field. It should be noted that these eigen waves that quadratically depend on the electric field acquire non-orthogonality of polarization. The pure transverse wave υ_{31} depends linearly on the electric field. Therefore, the reason for the appearance of quadratic dependence of AWs velocities on the electric field at the linear electro-elastic effect is the induction of the rotation of the eigen vectors of the Christoffel tensor, i.e., induction of the non-orthogonality of the AWs by the electric field.

Notice that, upon application of the electric field along the X-axis, the optical indicatrix is rotated about the X-axis by the angle $\tan 2\xi_X = (2r_{41}E_1)/(B_3^0 - B_1^0)$ (where B_i^0 is the component of the optical impermeability tensor), while the change in the refractive index under the angle ξ_X is $n_3 = n_3^0 + 0.5(n_3^0)^3(r_{41}E_1)^2$, and is quadratically dependent on the electric field (here n_i^0 - is the initial value of the refractive index). Along the initial axes of the optical indicatrix, the refractive indices are not changed under the E_1 electric field. This effect is similar to the electro-elastic one.

Let us proceed to the group of symmetry ∞mm. Under the application of the electric field E_3 the symmetry is not lowered. The EST is the same as the initial one for the group of symmetry $\infty 2$ (See tensor (5)). All AWs are purely polarized waves both before and after the application of the electric field. The change of the AW velocities is as follows:

$$\begin{gathered}\upsilon_{11} \approx \upsilon_{11}^0 + \frac{\Theta_{113}E_3}{2\rho\upsilon_{11}^0},\ \upsilon_{11}^0 = \sqrt{\frac{C_{11}^0}{\rho}},\quad \upsilon_{13} \approx \upsilon_{13}^0 + \frac{\Theta_{443}E_3}{2\rho\upsilon_{13}^0},\quad \upsilon_{13}^0 = \sqrt{\frac{C_{44}^0}{\rho}},\\ \upsilon_{12} \approx \upsilon_{12}^0 + \frac{(2\Theta_{113} - \Theta_{123})E_3}{2\rho\upsilon_{12}^0},\quad \upsilon_{12}^0 = \sqrt{\frac{0.5(C_{11}^0 - C_{12}^0)}{\rho}}.\end{gathered} \tag{9}$$

Thus, the increment of the AWs velocities linearly depends on the electric field. The group of symmetry ∞mm is characterized by the linear electro-optic tensor:

$$\begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{42} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{10}$$

Application of the electric field along the infinity-fold axis leads to an increment in refractive indices that linearly depends on the electric field strength, too.

With the application of the electric field along the X-axis, the symmetry is lowered to group m. It leads to the change of the EST as:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & \Theta_{151}E_1 & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & 0 & \Theta_{251}E_1 & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & \Theta_{351}E_1 & 0 \\ 0 & 0 & 0 & C_{44}^0 & 0 & \Theta_{641}E_1 \\ \Theta_{151}E_1 & \Theta_{251}E_1 & \Theta_{351}E_1 & 0 & C_{44}^0 & 0 \\ 0 & 0 & 0 & \Theta_{641}E_1 & 0 & C_{66}^0 \end{bmatrix}. \tag{11}$$

At the propagation of the AWs along the Z-axis, the Christoffel tensor is as follows:

$$\begin{bmatrix} C_{55}^0 - \lambda & 0 & \Theta_{531}E_1 \\ 0 & C_{44}^0 - \lambda & 0 \\ \Theta_{531}E_1 & 0 & C_{33}^0 - \lambda \end{bmatrix}, \tag{12}$$

resulting in the non-orthogonality angle $\tan 2\zeta_Y = (2\Theta_{431}E_1)/(C_{55}^0 - C_{33}^0)$. The AWs velocities are determined by the relations:

$$\begin{gathered}\upsilon_{32} = \sqrt{\frac{C_{44}^0}{\rho}},\\ \upsilon_{33} \approx \upsilon_{33}^0 + \frac{(\Theta_{531}E_1)^2}{2\rho\upsilon_{33}^0(C_{33}^0 - C_{44}^0)},\ \upsilon_{33}^0 = \sqrt{\frac{C_{33}^0}{\rho}},\\ \upsilon_{31} \approx \upsilon_{31}^0 - \frac{(\Theta_{531}E_1)^2}{2\rho\upsilon_{31}^0(C_{33}^0 - C_{44}^0)},\ \upsilon_{31}^0 = \sqrt{\frac{C_{44}^0}{\rho}}.\end{gathered} \tag{13}$$

As one can see, those velocities that relate to the non-orthogonal AWs are proportional to the square of the electric field. The AW polarized parallel to the Y-axis is a pure transverse wave and does not depend on the electric field.

Under the same conditions, the indicatrix rotates about the Y-axis at the angle $\tan 2\xi_Y = \dfrac{2r_{42}E_1}{B_{33} - B_{11}}$ while the refractive indices increment is proportional to the square of electric field when the light propagates along initial axes of coordinate system $n_{1,3} = n^0_{1,3} \pm \dfrac{(r_{42}E_1)^2}{B^0_{11} - B^0_{33}}$. Thus, a direct analogy between the behavior of acoustic and optical waves in an electric field is observed.

For the group of symmetry ∞ application of the electric field along infinity-fold axis does not change the symmetry and the structure of the tensor of the elastic stiffness coefficient (the same structure as the initial tensor (5)), while the individual components are changed as $\Delta C_{12} = \Theta_{123}E_3 = (2\Theta_{113} - \Theta_{663})E_3$, $\Delta C_{33} = \Theta_{333}E_3$, $\Delta C_{11} = \Delta C_{22} = \Theta_{113}E_3$, $\Delta C_{13} = \Delta C_{23} = \Theta_{133}E_3$, $\Delta C_{44} = \Delta C_{55} = \Theta_{443}E_3$. The AWs velocities undergo a linear change with the electric field:

$$\begin{aligned}
\upsilon_{11} &\approx \upsilon^0_{11} + \frac{\Theta_{113}E_3}{2\rho\upsilon^0_{11}}, \; \upsilon^0_{11} = \sqrt{\frac{C^0_{11}}{\rho}}, \\
\upsilon_{13} &\approx \upsilon^0_{13} + \frac{\Theta_{443}E_3}{2\rho\upsilon^0_{13}}, \; \upsilon^0_{13} = \sqrt{\frac{C^0_{44}}{\rho}}, \\
\upsilon_{12} &\approx \upsilon^0_{11} + \frac{(2\Theta_{113} - \Theta_{123})E_3}{2\rho\upsilon^0_{12}}, \; \upsilon^0_{12} = \sqrt{\frac{0.5(C^0_{11} - C^0_{12})}{\rho}}.
\end{aligned} \tag{14}$$

The tensor describing the Pockels effect in the group of symmetry ∞ is as follows:

$$\begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ -r_{41} & r_{42} & 0 \\ r_{42} & r_{41} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{15}$$

Application of the electric field along the infinity-fold axis leads to a change in the principal refractive indices $n_{1,2} = n^0_{1,2} - \dfrac{1}{(n^0_{1,2})^3} r_{13}E_3$, $(n^0_1 = n^0_2)$ and $n_3 = n^0_3 - \dfrac{1}{(n^0_3)^3} r_{33}E_3$, and they behave analogously to the AWs in an electric field.

3.2. Crystals of the cubic system

In this paragraph, we will consider three point groups of symmetry, namely 432, $\bar{4}3m$, and 23. To such crystalline materials belong, e.g., langbeinites minerals (point group of symmetry 23) [21], $\mathrm{CsNbMoO_6}$ crystals (point group of symmetry $\bar{4}3m$) [22], $\mathrm{Ag_3AuTe_2}$ mineral [23].

Let us consider the point group of symmetry 432 and suppose that the electric field is applied parallel to the <001> direction. In this case, the change of the point group of symmetry under an electric field is 432→ 4, while the change of the EST looks like:

$$\begin{bmatrix} C^0_{11} & C^0_{12} & C^0_{12} & 0 & 0 & 0 \\ C^0_{12} & C^0_{11} & C^0_{12} & 0 & 0 & 0 \\ C^0_{12} & C^0_{12} & C^0_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C^0_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C^0_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C^0_{44} \end{bmatrix} \to \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{11} & C_{13} & 0 & 0 & -C_{16} \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ C_{16} & -C_{16} & 0 & 0 & 0 & C_{66} \end{bmatrix}, \tag{16}$$

However, due to relation $\Delta C_{ijkl} = C_{ijkl} - C^0_{ijkl} = \Theta_{ijklm}E_m$ only $C_{16} = \Theta_{163}E_3$ (the C^0_{12}, $C^0_{66} = C^0_{44}$. and C^0_{33} components remain unchanged), the new component of the EST, appears. Therefore, the EST for a crystal under an electric field E_3 has the form:

$$\begin{bmatrix} C^0_{11} & C^0_{12} & C^0_{12} & 0 & 0 & C_{16} \\ C^0_{12} & C^0_{11} & C^0_{12} & 0 & 0 & -C_{16} \\ C^0_{12} & C^0_{12} & C^0_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C^0_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C^0_{44} & 0 \\ C_{16} & -C_{16} & 0 & 0 & 0 & C^0_{44} \end{bmatrix}, \tag{17}$$

which disagrees with the structure of the EST (16) for the point group of symmetry 4. The matter is that rewriting of the tensor (17) in the coordinate system rotating around the Z-axis at an angle

$$\varphi_0 = \frac{1}{4}\arctan\frac{4C_{16}}{C^0_{11} - C^0_{12} - 2C^0_{44}} = \frac{1}{4}\arctan\frac{4\Theta_{163}E_3}{C^0_{11} - C^0_{12} - 2C^0_{44}} \tag{18}$$

one can reaches the condition when $C_{16} = C_{26} = 0$, and obtain the structure of the EST consistent with the point group of symmetry 4. Rotation of the coordinate system by an arbitrary angle φ yields:

$$\begin{aligned} &C_{11} = C^0_{11} + \Theta_{163}E_3 \sin 4\varphi - 0.5(C^0_{11} - C^0_{12} - 2C^0_{44})\sin^2 2\varphi, \quad C_{13} = C_{23}(\varphi) = C^0_{12}, \\ &C_{12} = 0.25(C^0_{11} + 3C^0_{12} - 2C^0_{44}) - \Theta_{163}E_3 \sin 4\varphi - 0.25(C^0_{11} - C^0_{12} - 2C^0_{44})\cos 4\varphi, \\ &C_{22} = C_{11} = C^0_{11} + \Theta_{163}E_3 \sin 4\varphi - 0.5(C^0_{11} - C^0_{12} - 2C^0_{44})\sin^2 2\varphi, \quad C_{33} = C^0_{11}, \\ &C_{16} = \Theta_{163}E_3 \cos 4\varphi - 0.25(C^0_{11} - C^0_{12} - 2C^0_{44})\sin 4\varphi, \\ &C_{26} = 0.25(C^0_{11} - C^0_{12} - 2C^0_{44})\sin 4\varphi - \Theta_{163}E_3 \cos 4\varphi, \quad C_{55} = C_{44} = C^0_{44}, \\ &C_{66} = 0.5(C^0_{11} - C^0_{12}) - \Theta_{163}E_3 \sin 4\varphi - 0.5(C^0_{11} - C^0_{12} - 2C^0_{44})\cos^2 2\varphi, \end{aligned} \tag{19}$$

i.e., the EST acquires a peculiar structure for the symmetry group 4. If one rotates the coordinate system by the angle φ_0, the so-called eigen coordinate system for the tensor (17) is reached. Therefore, the EST rotates under the application of the electric field E_3 by the angle determined by Eq. (18).

At the propagation of the AWs along the X-axis, perpendicular to the direction of the electric field application, the AWs velocities are:

$$\upsilon_{13} = \upsilon^0_{13} = \sqrt{\frac{C_{44}}{\rho}}, \tag{20}$$

$$v_{12} = v^0_{12} - \frac{\Theta^2_{163}E^2_3}{2\rho v^0_{12}(C^0_{11} - C^0_{44})}, \text{ where } v^0_{12} = \sqrt{\frac{C^0_{44}}{\rho}}, \tag{21}$$

$$v_{11}=v_{11}^{0}+\frac{\Theta_{163}^{2}E_{3}^{2}}{2\rho v_{11}^{0}(C_{11}^{0}-C_{44}^{0})},\ \text{where}\ \ v_{11}^{0}=\sqrt{\frac{C_{11}^{0}}{\rho}}, \tag{22}$$

It is seen (Eqs. (21,22)) that AW velocity increment is proportional to the square of the electric field strength. The non-orthogonality angle is determined as: $\tan 2\zeta_{Z}=\frac{2\Theta_{163}E_{3}}{C_{11}^{0}-C_{44}^{0}}$. Therefore, the velocities of those AWs that acquire non-orthogonality are proportional to the square of the electric field. However, when the AWs propagate along the X'-axis, which is rotated by an angle φ with respect to the *X*-axis, AWs velocities are proportional to the electric field to the first power:

$$\begin{aligned}\upsilon_{12}&=\sqrt{\frac{C_{66}}{\rho}}-\frac{C_{16}^{2}}{2\rho(C_{11}-C_{66})\sqrt{\frac{C_{66}}{\rho}}}\\&\approx\frac{1}{\sqrt{\rho}}\left(\begin{array}{c}\sqrt{0.5(C_{11}^{0}-C_{12}^{0})-0.5(C_{11}^{0}-C_{12}^{0}-2C_{44}^{0})\cos^{2}2\varphi}\\-\dfrac{\Theta_{241}E_{3}\sin 4\varphi}{2\sqrt{0.5(C_{11}^{0}-C_{12}^{0})-0.5(C_{11}^{0}-C_{12}^{0}-2C_{44}^{0})\cos^{2}2\varphi}}\end{array}\right),\end{aligned} \tag{23}$$

$$\begin{aligned}\upsilon_{11}&=\sqrt{\frac{C_{11}}{\rho}}+\frac{C_{16}^{2}}{2\rho(C_{11}-C_{66})\sqrt{\frac{C_{66}}{\rho}}}\\&\approx\frac{1}{\sqrt{\rho}}\left(\begin{array}{c}\sqrt{0.5(C_{11}^{0}-C_{12}^{0})-0.5(C_{11}^{0}-C_{12}^{0}-2C_{44}^{0})\cos^{2}2\varphi}\\+\dfrac{\Theta_{241}E_{3}\sin 4\varphi}{2\sqrt{0.5(C_{11}^{0}-C_{12}^{0})-0.5(C_{11}^{0}-C_{12}^{0}-2C_{44}^{0})\cos^{2}2\varphi}}\end{array}\right).\end{aligned} \tag{24}$$

Here, the subscript indices of v_{ij} indicate: *i* – direction of propagation of the acoustic wave and *j*- direction of its polarization. In Eqs. (23,24) angle φ depends on the electric field according to Eq. (18), and at the propagation of the AWs in the direction defined by the angle φ_{0}, their velocities do not change under the electric field.

The eigenvectors of the Christoffel tensor rotate about the [001] direction at an angle

$$\tan 2\zeta_{Z}=\frac{2\Theta_{163}E_{3}}{C_{11}^{0}-C_{44}^{0}}, \tag{25}$$

That means that PT and PL AWs acquire non-orthogonality, and the displacement vector rotates by an angle ζ_{Z} about the [001] axis with respect to the [100] direction. The predicted, by symmetry, acoustic axes for the transverse AWs parallel to [100], [010], and $\{111\}$ vanish under the application of an electric field. Only the acoustic axis parallel to the [001] direction remains.

It should be noted that the third-rank polar tensor describing the Pockels effect in optics is equal to zero for the point group of symmetry 432.

The group of symmetry $\bar{4}3m$ is characterized by four independent coefficients of $\Theta_{\lambda\mu m}$ tensor, namely Θ_{141}, Θ_{241}, Θ_{341} and Θ_{561}. Upon the application of an electric field parallel to the <001> direction, the change of the point group of symmetry under is $\bar{4}3m\rightarrow mm2$. Then the change of the EST is:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{12}^0 & 0 & 0 & 0 \\ C_{12}^0 & C_{11}^0 & C_{12}^0 & 0 & 0 & 0 \\ C_{12}^0 & C_{12}^0 & C_{11}^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}^0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44}^0 \end{bmatrix} \rightarrow \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}. \qquad (26)$$

It is seen (Eq. 26) that the new components should not appear in the EST. However, due to the application of an electric field and tensor coupling, new components appear: $C_{16} = C_{26} = \Theta_{241}E_3$, $C_{36} = \Theta_{141}E_3$, $C_{45} = \Theta_{561}E_3$. Thus, the tensor is as follows:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{12}^0 & 0 & 0 & C_{16} \\ C_{12}^0 & C_{11}^0 & C_{12}^0 & 0 & 0 & C_{16} \\ C_{12}^0 & C_{12}^0 & C_{11}^0 & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44}^0 & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{44}^0 & 0 \\ C_{16} & C_{16} & C_{36} & 0 & 0 & C_{44}^0 \end{bmatrix}, \qquad (27)$$

that is inconsistent with the symmetry point group $mm2$.

Under the rotation of the coordinate system around the Z-axis at an angle of ±45 deg the EST acquires the form:

$$\begin{aligned} &C_{11} = 2\Theta_{241}E_3 + 0.5(C_{11}^0 + C_{12}^0 + 2C_{44}^0), \\ &C_{12} = 0.5(C_{11}^0 - C_{12}^0 - 2C_{44}^0), \\ &C_{13} = C_{12}^0 + \Theta_{141}E_3, \\ &C_{22} = 0.5(C_{11}^0 + C_{12}^0 + 2C_{44}^0) - 2\Theta_{241}E_3, \\ &C_{13} = C_{12}^0 - \Theta_{141}E_3, \\ &C_{33} = C_{11}^0, \\ &C_{16} = C_{26} = C_{36} = C_{45} = 0, \\ &C_{44} = C_{44}^0 - \Theta_{561}E_3, \\ &C_{55}^* = C_{44} + \Theta_{561}E_3, \\ &C_{66} = 0.5(C_{11}^0 - C_{12}^0). \end{aligned} \qquad (28)$$

Note that the optical indicatrix, when an electric field in the same direction is applied, is also rotated by ±45° about the Z-axis.

For the case when AWs propagate along the X-axis, the AWs velocities are as follows:

$$\begin{aligned} &v_{13} = \sqrt{\frac{C_{44}^0}{\rho}}, \\ &v_{12} = \sqrt{\frac{C_{44}^0}{\rho}} - \frac{\Theta_{241}^2 E_3^2}{2\rho(C_{11}^0 - C_{44}^0)\sqrt{\frac{C_{44}^0}{\rho}}} = v_{12}^0 - \frac{\Theta_{241}^2 E_3^2}{2\rho v_{12}^0 (C_{11}^0 - C_{44}^0)}, \\ &v_{11} = \sqrt{\frac{C_{11}^0}{\rho}} + \frac{\Theta_{241}^2 E_3^2}{2\rho(C_{11}^0 - C_{44}^0)\sqrt{\frac{C_{11}^0}{\rho}}} = v_{11}^0 + \frac{\Theta_{241}^2 E_3^2}{2\rho v_{11}^0 (C_{11}^0 - C_{44}^0)}. \end{aligned} \qquad (29)$$

As one can see, the increment of AWs velocities is proportional to the square of the electric field strength. In optics, the Pockels effect does not lead to a change in the principal refractive indices in the *X*, *Y*, and *Z* directions. The relations for AWs velocities for the AWs propagating along the X'-axis (the bisectrices between *X* and *Y*-axes) are written as:

$$\upsilon_{13}=\sqrt{\frac{C_{44}+\Theta_{561}E_3}{\rho}}=\upsilon_{13}^0+\frac{\Theta_{561}E_3}{2\rho\upsilon_{13}^0}\text{, where}\quad \upsilon_{13}^0=\sqrt{\frac{C_{44}}{\rho}}\text{,} \tag{30}$$

$$\upsilon_{12}=\upsilon_{12}^0=\sqrt{\frac{0.5(C_{11}-C_{12})}{\rho}}\text{,} \tag{31}$$

$$\upsilon_{11}=\sqrt{\frac{C_{11}-\Theta_{561}E_3}{\rho}}=\upsilon_{11}^0-\frac{\Theta_{561}E_3}{2\rho\upsilon_{11}^0}\text{, where } \upsilon_{11}^0=\sqrt{\frac{C_{11}}{\rho}}\text{.} \tag{32}$$

In this case, the increment of the AWs velocities is linearly dependent on the electric field strength. The Pockels effect also results in a linear dependence of the refractive indices. At the propagation of AWs along the *X*-axis, the angle of non-orthogonality of AWs is determined as:

$$\tan 2\zeta_Z=\frac{2C_{16}}{C_{11}^0-C_{44}^0}=\frac{2\Theta_{241}E_3}{C_{11}^0-C_{44}^0}\text{.} \tag{33}$$

Therefore, the angle of non-orthogonality depends on the electric field. At the propagation of AWs along the X'-axis the angle of non-orthogonality is equal to zero.

For the last group of symmetry of the cubic system, i.e., 23 point group, the application of the electric field along the <001> direction leads to a change of symmetry 23→2 (2||*Z*). The EST is changed as:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{12}^0 & 0 & 0 & 0\\ C_{12}^0 & C_{11}^0 & C_{12}^0 & 0 & 0 & 0\\ C_{12}^0 & C_{12}^0 & C_{11}^0 & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44}^0 & 0 & 0\\ 0 & 0 & 0 & 0 & C_{44}^0 & 0\\ 0 & 0 & 0 & 0 & 0 & C_{44}^0 \end{bmatrix} \to \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16}\\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26}\\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36}\\ 0 & 0 & 0 & C_{44} & C_{45} & 0\\ 0 & 0 & 0 & C_{45} & C_{55} & 0\\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix}\text{.} \tag{34}$$

Note that $C_{16}=\Theta_{163}E_3, C_{26}=\Theta_{263}E_3, C_{36}=\Theta_{363}E_3, C_{45}=\Theta_{453}E_3$. The structure of the EST under the action of the electric field is

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{12}^0 & 0 & 0 & C_{16}\\ C_{12}^0 & C_{11}^0 & C_{12}^0 & 0 & 0 & C_{26}\\ C_{12}^0 & C_{12}^0 & C_{11}^0 & 0 & 0 & C_{36}\\ 0 & 0 & 0 & C_{44}^0 & C_{45} & 0\\ 0 & 0 & 0 & C_{45} & C_{44}^0 & 0\\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{44}^0 \end{bmatrix}\text{,} \tag{35}$$

The structure of this tensor does not correspond to the point group of symmetry 2. However, all components of the tensor (35) depend on the angle of rotation of the coordinate system φ around the *Z*-axis:

$$\begin{aligned}
C_{11} &= C_{11}^0 + (\Theta_{141} + \Theta_{241})E_3 \sin 2\varphi + 0.5(\Theta_{141} - \Theta_{241})E_3 \sin 4\varphi \\
&\quad -0.5(C_{11}^0 - C_{12}^0 - 2C_{44}^0)\sin^2 2\varphi, \\
C_{12} &= 0.25(C_{11}^0 + 3C_{12}^0 - 2C_{44}^0) - 0.25(C_{11}^0 - C_{12}^0 - 2C_{44}^0)\cos 4\varphi \\
&\quad -0.5(\Theta_{141} - \Theta_{241})E_3 \sin 4\varphi, \\
C_{16} &= 0.5(\Theta_{141} + \Theta_{241})E_3 \cos 2\varphi + 0.5(\Theta_{141} - \Theta_{241})E_3 \cos 4\varphi \\
&\quad -0.25(C_{11}^0 - C_{12}^0 - 2C_{44}^0)\sin 4\varphi, \\
C_{22} &= C_{11}^0 - (\Theta_{141} + \Theta_{241})E_3 \sin 2\varphi + 0.5(\Theta_{141} - \Theta_{241})E_3 \sin 4\varphi \\
&\quad -0.5(C_{11}^0 - C_{12}^0 - 2C_{44}^0)\sin^2 2\varphi, \\
C_{26} &= 0.5(\Theta_{141} + \Theta_{241})E_3 \cos 2\varphi - 0.5(\Theta_{141} - \Theta_{241})E_3 \cos 4\varphi \\
&\quad +0.25(C_{11}^0 - C_{12}^0 - 2C_{44}^0)\sin 4\varphi, \\
C_{66} &= 0.5(C_{11}^0 - C_{12}^0) - 0.5(C_{11}^0 - C_{12}^0 - 2C_{44}^0)\cos^2 2\varphi \\
&\quad -0.5(\Theta_{141} - \Theta_{241})E_3 \sin 4\varphi, \\
C_{13} &= C_{12}^0 + \Theta_{341}E_3 \sin 2\varphi, \quad C_{23} = C_{12}^0 - \Theta_{341}E_3 \sin 2\varphi, \\
C_{44} &= C_{44}^0 - \Theta_{561}E_3 \sin 2\varphi, \quad C_{55} = C_{44}^0 + \Theta_{561}E_3 \sin 2\varphi, \\
C_{33} &= C_{11}^0, \qquad C_{36} = \Theta_{341}E_3 \cos 2\varphi, \qquad C_{45} = \Theta_{561}E_3 \cos 2\varphi.
\end{aligned} \tag{36}$$

At the propagation of AWs along the *X*-axis, the AWs velocities are as follows:

$$\begin{aligned}
&v_{13} = \sqrt{\frac{C_{44}^0}{\rho}}, v_{12} = \sqrt{\frac{C_{44}^0}{\rho}} - \frac{\Theta_{141}^2 E_3^2}{2\rho(C_{11}^0 - C_{44}^0)\sqrt{C_{44}^0/\rho}} = v_{12}^0 - \frac{\Theta_{141}^2 E_3^2}{2\rho v_{12}^0 (C_{11}^0 - C_{44}^0)}, \\
&v_{11} = \sqrt{\frac{C_{11}^0}{\rho}} + \frac{\Theta_{141}^2 E_3^2}{2\rho(C_{11}^0 - C_{44}^0)\sqrt{C_{11}^0/\rho}} = v_{11}^0 + \frac{\Theta_{141}^2 E_3^2}{2\rho v_{11}^0 (C_{11}^0 - C_{44}^0)}.
\end{aligned} \tag{37}$$

The so-called eigen coordinate system of the elastic tensor for the point group of symmetry 2 possesses the rotation degree of freedom around the two-fold symmetry axis (*Z*-axis). Therefore, when the AWs are propagating along the X'-axis, which relates to the rotated coordinate system, the AWs velocities can be written as:

$$v_{13} = \sqrt{\frac{C_{44}^0 \pm \Theta_{561}E_3}{\rho}} \approx \sqrt{\frac{C_{44}^0}{\rho}} + \frac{\pm\Theta_{561}E_3}{2\rho\sqrt{C_{44}^0/\rho}} \approx v_{13}^0 + \frac{\pm\Theta_{561}E_3}{2\rho v_{13}^0}, \tag{38}$$

$$v_{12} = \sqrt{\frac{0.5(C_{11}^0 - C_{12}^0)}{\rho}}, \tag{39}$$

$$\begin{aligned}
v_{11} &= \sqrt{\frac{0.5(C_{11}^0 - C_{12}^0 - 2C_{44}^0) \pm (\Theta_{141} + \Theta_{241})E_3}{\rho}} \\
&\approx \sqrt{\frac{0.5(C_{11}^0 - C_{12}^0 - 2C_{44}^0)}{\rho}} \pm \frac{(\Theta_{141} + \Theta_{241})E_3}{2\sqrt{0.5\rho(C_{11}^0 - C_{12}^0 - 2C_{44}^0)}}.
\end{aligned} \tag{40}$$

As one can see from the propagation of the AWs along the *X*-axis of the cubic system, the velocity increment is proportional to the square of the electric field strength, while when they propagate along the rotated crystal physical axis of the monoclinic system, the increment linearly depends on the electric field.

At the zero-bias field and AWs propagation along the *X*-axis, the angle of non-orthogonality is equal to zero, while at the same condition but under the applied electric field E_3, the non-orthogonality angle is equal to $\tan 2\zeta_Z = (2\Theta_{163})/C_{11} - C_{66}$.

3.3. Tetragonal point groups of symmetry

The tetragonal system is abundant with crystals of different symmetries, e.g. KDP family crystals (point group of symmetry $\bar{4}2m$) [24], propionates (point group of symmetry 422 and 4) [25], $Li_2B_4O_7$ (point group of symmetry $4mm$) [26], $CdGa_2S_4$ (point group of symmetry $\bar{4}$) [27], etc.

Let us start our analysis from the point group of symmetry $4mm$ at the application of an electric field along the *Z* and *Y*-axes. At the E_3 electric field, the point group of symmetry, as well as the EST, are not changed. The EST has the view

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}. \tag{41}$$

The increments of the EST component are as follows: $\Delta C_{11} = \Delta C_{22} = \Theta_{113}E_3$, $\Delta C_{12} = \Theta_{123}E_3$, $\Delta C_{13} = \Delta C_{23} = \Theta_{133}E_3$, $\Delta C_{33} = \Theta_{333}E_3$, $\Delta C_{44} = \Delta C_{55} = \Theta_{443}E_3$ and $\Delta C_{66} = \Theta_{663}E_3$. At the propagation of the AWs along *X* or *Y*-axes, their velocities are written as:

$$\begin{aligned} v_{11} &= \frac{1}{\sqrt{\rho}}\sqrt{C_{11}^0 + \Theta_{113}E_3} \approx v_{11}^0 + \frac{\Theta_{113}E_3}{2\rho v_{11}^0}, \\ v_{12} &= \frac{1}{\sqrt{\rho}}\sqrt{C_{66}^0 + \Theta_{663}E_3} \approx v_{12}^0 + \frac{\Theta_{663}E_3}{2\rho v_{12}^0}, \\ v_{13} &= \frac{1}{\sqrt{\rho}}\sqrt{C_{44}^0 + \Theta_{443}E_3} \approx v_{13}^0 + \frac{\Theta_{443}E_3}{2\rho v_{13}^0}. \end{aligned} \tag{42}$$

Note that in optics, the change in the parameters of the optical indicatrix induced by the electric field is, in general, the same as in acoustics, i.e., the increments of refractive indices depend linearly on the electric field, and the optical indicatrix does not rotate under the application of an electric field. At the propagation of the AWs along *X* or *Y*-axes, the angle of non-orthogonality is equal to zero.

At the application of a bias field along the *Y*-axis, the symmetry is changed as $4mm \to m$, and the change of the EST is as follows:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & 0 & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & 0 & 0 & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}^0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66}^0 \end{bmatrix} \to \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & 0 & 0 \\ C_{12} & C_{22} & C_{23} & C_{24} & 0 & 0 \\ C_{13} & C_{23} & C_{33} & C_{34} & 0 & 0 \\ C_{14} & C_{24} & C_{34} & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & C_{56} \\ 0 & 0 & 0 & 0 & C_{56} & C_{66} \end{bmatrix}. \tag{43}$$

The induced by the electric field E_2 components of the EST are: $C_{14} = \Theta_{142}E_2$, $C_{24} = \Theta_{242}E_2$, $C_{34} = \Theta_{342}E_2$, and $C_{56} = \Theta_{562}E_2$. Therefore, as has been shown for other groups of symmetry, the tensor structure becomes:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & C_{14} & 0 & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & C_{24} & 0 & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & C_{34} & 0 & 0 \\ C_{14} & C_{24} & C_{34} & C_{44}^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}^0 & C_{56} \\ 0 & 0 & 0 & 0 & C_{56} & C_{66}^0 \end{bmatrix}, \tag{44}$$

that is not peculiar for the group of symmetry m ($m \perp X$), since some of the coefficients are left to be equal. At the propagation of the AWs along the X-axes, their velocities are written as:

$$\begin{aligned} v_{31} &= \sqrt{\frac{C_{44}^0}{\rho}}, \\ v_{33} &= \frac{1}{\sqrt{\rho}}\sqrt{C_{33}^0 + \frac{(\Theta_{342}E_2)^2}{(\mathrm{C}_{33}^0 - \mathrm{C}_{44}^0)}} \approx v_{33}^0 + \frac{(\Theta_{342}E_2)^2}{2v_{33}^0(\mathrm{C}_{33}^0 - \mathrm{C}_{44}^0)}, \\ v_{32} &= \frac{1}{\sqrt{\rho}}\sqrt{C_{44}^0 - \frac{(\Theta_{342}E_2)^2}{(\mathrm{C}_{33}^0 - \mathrm{C}_{44}^0)}} \approx v_{32}^0 - \frac{(\Theta_{342}E_2)^2}{2v_{33}^0(\mathrm{C}_{33}^0 - \mathrm{C}_{44}^0)}, \end{aligned} \tag{45}$$

being quadratically dependent on the electric field. The angle of non-orthogonality is defined by the relation $\tan 2\zeta_X = (2\Theta_{342}E_2 v)/(C_{44}^0 - C_{33}^0)$. For the EST, the mirror symmetry plane $m \perp X$ is equivalent to the two-fold symmetry axis $2 \parallel X$ [1]. The rotation of the coordinate system around the X-axis by an angle φ yields:

$$\begin{aligned} & C_{11} = C_{11}^0, C_{12} = C_{12}^0 \cos^2\varphi + C_{13}^0 \sin^2\varphi - \Theta_{142}E_2 \sin 2\varphi, \\ & C_{13} = C_{13}^0 \cos^2\varphi + C_{12}^0 \sin^2\varphi + \Theta_{142}E_2 \sin 2\varphi, \\ & C_{14} = 0.5(C_{13}^0 + C_{12}^0)\sin 2\varphi + \Theta_{142}E_2 \cos 2\varphi, \\ & C_{22} = \frac{1}{8}(3C_{11}^0 + 2C_{13}^0 + 3C_{33}^0 + 4C_{44}^0) - (\Theta_{242} + \Theta_{342})\sin 2\varphi - 0.5(\Theta_{242} - \Theta_{342})\sin 4\varphi \\ & \qquad + 0.5(C_{11}^0 - C_{33}^0)\cos 2\varphi + \frac{1}{8}(C_{11}^0 - 2C_{13}^0 + C_{33}^0 - 4C_{44}^0)\cos 4\varphi, \\ & C_{23} = C_{13}^0 + 0.5(\Theta_{242} - \Theta_{342})\sin 4\varphi + 0.25(C_{11}^0 - 2C_{13}^0 + C_{33}^0 - 4C_{44}^0)\sin^2 2\varphi, \\ & C_{24} = 0.5(\Theta_{242} + \Theta_{342})\cos 2\varphi + 0.5(\Theta_{242} - \Theta_{342})\cos 4\varphi \\ & \qquad + 0.25(C_{11}^0 - C_{33}^0)\sin 2\varphi + \frac{1}{8}(C_{11}^0 - 2C_{13}^0 + C_{33}^0 - 4C_{44}^0)\sin 4\varphi, \\ & C_{33} = \frac{1}{8}(3C_{11}^0 + 2C_{13}^0 + 3C_{33}^0 + 4C_{44}^0) + (\Theta_{242} + \Theta_{342})\sin 2\varphi - 0.5(\Theta_{242} - \Theta_{342})\sin 4\varphi \\ & \qquad - 0.5(C_{11}^0 - C_{33}^0)\cos 2\varphi + \frac{1}{8}(C_{11} - 2C_{13} + C_{33}^0 - 4C_{44})\cos 4\varphi, \\ & C_{34} = 0.5(\Theta_{242} + \Theta_{342})\cos 2\varphi - 0.5(\Theta_{242} - \Theta_{342})\cos 4\varphi \\ & \qquad + 0.25(C_{11}^0 - C_{33}^0)\sin 2\varphi - \frac{1}{8}(C_{11}^0 - 2C_{13}^0 + C_{33}^0 - 4C_{44}^0)\sin 4\varphi, \\ & C_{44} = C_{44}^0 + 0.5(\Theta_{242} - \Theta_{342})\sin 4\varphi + 0.25(C_{11}^0 - 2C_{13}^0 + C_{33}^0 - 4C_{44}^0)\sin^2 2\varphi, \\ & C_{55} = C_{44}^0 - (C_{44}^0 + C_{66}^0)\sin^2\varphi + \Theta_{562}E_2 \sin 2\varphi, \\ & C_{56} = \Theta_{562}E_2 \cos 2\varphi - 0.5(C_{44}^0 - C_{66}^0)\sin 2\varphi, \\ & C_{66} = C_{66}^0 + (C_{44}^0 - C_{66}^0)\sin^2\varphi - \Theta_{562}E_2 \sin 2\varphi. \end{aligned} \tag{46}$$

At the propagation of the AWs along the Z'-axis, the AWs velocities can be written as:

$$v_{31} \approx v_{31}^0 - \frac{C_{44}^0 + C_{66}^0 \sin^2\varphi + \Theta_{562} E_2 \sin 2\varphi}{2\rho v_{31}^0}, \ v_{31}^0 = \sqrt{\frac{C_{44}^0}{\rho}},$$

$$v_{32} \approx v_{32}^0 + \frac{0.5(\Theta_{242} - \Theta_{242}) E_2 \sin 4\varphi + 0.25(C_{11}^0 + 2C_{13}^0 + C_{33}^0 - 4C_{44}^0)\sin^2 2\varphi}{2\rho v_{32}^0},$$

$$v_{32}^0 = \sqrt{\frac{C_{44}^0}{\rho}},$$

$$v_{33} \approx \sqrt{\frac{\left(\begin{array}{l}\frac{1}{8}(3C_{11}^0 + 2C_{13}^0 + 3C_{33}^0 + 4C_{44}^0) - 0.5(C_{11}^0 - C_{33}^0)\cos 2\varphi \\ + \frac{1}{8}(C_{11}^0 - 2C_{13}^0 + C_{33}^0 - 4C_{44}^0)\cos 4\varphi\end{array}\right)}{\rho}} \tag{47}$$

$$+ \frac{(\Theta_{242} + \Theta_{342}) E_2 \sin 2\varphi - 0.5(\Theta_{242} - \Theta_{342}) E_2 \sin 4\varphi}{2\sqrt{\rho\left(\begin{array}{l}\frac{1}{8}(3C_{11}^0 + 2C_{13}^0 + 3C_{33}^0 + 4C_{44}^0) - 0.5(C_{11}^0 - C_{33}^0)\cos 2\varphi \\ + \frac{1}{8}(C_{11}^0 - 2C_{13}^0 + C_{33}^0 - 4C_{44}^0)\cos 4\varphi\end{array}\right)}}.$$

At the angle φ =0, the initial AW velocity is equal to $v_{33}^0 = \sqrt{C_{33}^0/\rho}$. From the relations (47), it follows that change in AWs velocities are linearly dependent on the electric field strength when the AWs propagate under the angle φ about the *Z*-axis (in the relations for v_{32} and v_{33} we have neglected the quadratic term as a small one). Let us recall that, in the Pockels effect induced by the electric field E_2, the optical indicatrix also rotates about the *X*-axis. However, the refractive indices along the *X*, *Y*, and *Z*–axes remain unchanged.

Let us analyze the symmetry group 422. Upon the application of the electric field along the *Z*-axis, the symmetry is reduced to the point group of symmetry 4. The change of the EST is as follows:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & 0 & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & 0 & 0 & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}^0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66}^0 \end{bmatrix} \to \begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & 0 & C_{16} \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & 0 & 0 & -C_{16} \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}^0 & 0 \\ C_{16} & -C_{16} & 0 & 0 & 0 & C_{66}^0 \end{bmatrix}. \tag{48}$$

The electric field E_3 induces two new components $C_{16} = \Theta_{163} E_3$ and $C_{26} = -C_{16} = -\Theta_{163} E_3$. The AWs velocities for the waves propagating along the *X*-axis can be written as:

$$v_{13} = \sqrt{\frac{C_{44}^0}{\rho}},$$

$$v_{11} \approx v_{11}^0 + \frac{\Theta_{163}^2 E_3^2}{2\rho v_{11}^0 (C_{11}^0 - C_{66}^0)}, v_{11}^0 = \sqrt{\frac{C_{11}^0}{\rho}} \tag{49}$$

$$v_{12} \approx v_{12}^0 - \frac{\Theta_{163}^2 E_3^2}{2\rho v_{12}^0 (C_{11}^0 - C_{66}^0)}, v_{12}^0 = \sqrt{\frac{C_{66}^0}{\rho}}.$$

The velocities depend quadratically on the electric field, whereas the Pockels effect does not change the optical indicatrix under the electric fieldE_3. At a certain angle φ_0, at the rotation of the coordinate system about the Z-axis (see Eq.(18)), theC_{16} and C_{26} tend to zero, while at the propagation of AWs along the direction defined by angleφ_0, the velocities do not change under the electric field. At the AWs propagating along the X-axis, the non-orthogonality angle is equal to $\tan 2\zeta_Z = \frac{2C_{16}}{C_{11}^0 - C_{66}^0} = \frac{2\Theta_{163}E_3}{C_{11}^0 - C_{66}^0}$ and depends on the electric field strength, while it is equal to zero when AWs propagate at an angle φ_0.

Upon the application of the electric field along the X-axis, the symmetry is reduced to the monoclinic point group of symmetry 2. The behavior of AWs velocities that propagate along the Z-axis is the same as in the case of the application of an electric field to the crystal with symmetry $4mm$ along the Y-axis. The AWs velocities are determined by Eq. (45).

At the application of the electric field to the crystals of the group of symmetry $\bar{4}2m$ along the Z-axis, the lowering of the symmetry is as follows: $\bar{4}2m \to mm2$. The tensor structure is changed as:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & 0 & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & 0 & 0 & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}^0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66}^0 \end{bmatrix} \to \begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & 0 & C_{16} \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & 0 & 0 & C_{16} \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44}^0 & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{44}^0 & 0 \\ C_{16} & C_{16} & C_{36} & 0 & 0 & C_{66}^0 \end{bmatrix}, \qquad (50)$$

where $C_{16} = C_{26} = \Theta_{163}E_3$, $C_{36} = \Theta_{363}E_3$, and $C_{45} = \Theta_{453}E_3$.

At the AW propagation along the X-axis their velocities are determined as

$$\begin{aligned} &\upsilon_{13}^0 = \sqrt{\frac{C_{44}^0}{\rho}}, \quad \upsilon_{11}^0 = \sqrt{\frac{C_{11}^0}{\rho}}, \quad \upsilon_{12}^0 = \sqrt{\frac{C_{66}^0}{\rho}}, \\ &\upsilon_{13} = \sqrt{\frac{C_{44}^0 - \Theta_{453}E_3}{\rho}} \approx \upsilon_{13}^0 - \frac{\Theta_{453}E_3}{2\rho\upsilon_{13}^0}, \\ &\upsilon_{11} = \sqrt{\frac{C_{11}^0}{\rho} + \frac{(\Theta_{163}E_3)^2}{2\rho(C_{11}^0 - C_{66}^0)}} \approx \upsilon_{11}^0 + \frac{(\Theta_{163}E_3)^2}{2\rho(C_{11}^0 - C_{66}^0)\upsilon_{11}^0}, \\ &\upsilon_{12} = \sqrt{\frac{C_{66}^0}{\rho} - \frac{(\Theta_{163}E_3)^2}{2\rho(C_{11}^0 - C_{66}^0)}} \approx \upsilon_{12}^0 - \frac{(\Theta_{163}E_3)^2}{2\rho(C_{11}^0 - C_{66}^0)\upsilon_{11}^0}, \end{aligned} \qquad (51)$$

and the increment of the velocity of two eigen AWs is proportional to the square of the electric field. The longitudinal and transverse AW polarized parallel to the Y-axis AWs acquired non-orthogonality $\tan 2\zeta_Z = (2\Theta_{163}E_3)/(C_{11}^0 - C_{66}^0)$. The transverse AWs polarized along the Z-axis remains a purely orthogonal AW.

If we rewrite the tensor (50) in the coordinate system ($X'Y'Z$) rotated by 45° around the Z-axis, which should be done in fact due to the different crystallographic lattice basis vectors of $\bar{4}2m$ and $mm2$ groups of symmetry, the components of the tensor will be written as:

$$\begin{gathered}C_{11}=0.5(\mathrm{C}_{11}^0+C_{12}^0+2C_{44}^0)+2\Theta_{163}E_3, C_{12}=0.5(\mathrm{C}_{11}^0-C_{12}^0-2C_{66}^0),\\ C_{13}=\mathrm{C}_{13}+\Theta_{363}E_3, C_{22}=0.5(\mathrm{C}_{11}^0+C_{12}^0+2C_{44}^0)-2\Theta_{163}E_3,\\ C_{23}=\mathrm{C}_{13}-\Theta_{363}E_3, C_{33}=\mathrm{C}_{11}^0, C_{26}=C_{36}=C_{45}=C_{16}=0,\\ C_{44}=\mathrm{C}_{44}^0-\Theta_{453}E_3, C_{55}=\mathrm{C}_{44}^0+\Theta_{453}E_3, C_{66}=0.5(\mathrm{C}_{11}^0-C_{12}^0),\end{gathered} \tag{52}$$

being proper for the $mm2$ point group of symmetry. The AWs velocities at their propagation along X'-axis become linearly dependent on the electric field

$$\begin{gathered}v_{11}=\sqrt{\frac{0.5(C_{11}^0+C_{12}^0+2C_{44}^0)}{\rho}}+\frac{\Theta_{163}E_3}{\sqrt{0.5\rho(C_{11}^0+C_{12}^0+2C_{44}^0)}},\\ v_{12}=\sqrt{\frac{0.5(C_{11}^0-C_{12}^0)}{\rho}},\quad v_{13}^0=\sqrt{\frac{C_{44}^0}{\rho}},\quad v_{13}\approx v_{13}^0+\frac{\Theta_{453}E_3}{2\rho v_{13}^0}.\end{gathered} \tag{53}$$

The non-orthogonality of AWs in this case is not manifested. Under the linear electro-optic effect, the optical indicatrix rotates by ±45 deg about the Z-axis, too. At the application of the electric field E_3, the refractive indices measured along the X' or Y' - axes also linearly depend on the electric field; however, along X, Y, and Z-axes, they do not change.

Upon the application of the electric field along the X-axis the symmetry is reduced from the point group of symmetry $\bar{4}2m$ to 2. The new components of EST that appear under electric field are: $C_{14}=\Theta_{141}E_1$, $C_{24}=\Theta_{241}E_1$, $C_{34}=\Theta_{341}E_1$, and $C_{56}=\Theta_{561}E_1$. The respective tensor look like:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & C_{14} & 0 & 0\\ C_{12}^0 & C_{11}^0 & C_{13}^0 & C_{24} & 0 & 0\\ C_{13}^0 & C_{13}^0 & C_{33}^0 & C_{34} & 0 & 0\\ C_{14} & C_{24} & C_{34} & C_{44}^0 & 0 & 0\\ 0 & 0 & 0 & 0 & C_{44}^0 & C_{56}\\ 0 & 0 & 0 & 0 & C_{56} & C_{66}^0 \end{bmatrix}. \tag{54}$$

Under the rotation coordinate system around X-axis on the angle φ it acquires the structure that is peculiar to the group of symmetry 2 ($2\|X$) with the components:

$$\begin{aligned} &C_{11}=C_{11}^0, C_{12}=C_{12}^0\cos^2\varphi+C_{13}^0\sin^2\varphi-\Theta_{141}E_1\sin 2\varphi,\\ &C_{13}=C_{13}^0\cos^2\varphi+C_{12}^0\sin^2\varphi+\Theta_{141}E_1\sin 2\varphi,\\ &C_{22}=\frac{1}{8}(3C_{11}^0+2C_{13}^0+3C_{33}^0+4C_{44}^0)-(\Theta_{241}+\Theta_{341})E_1\sin 2\varphi\\ &\qquad -0.5(\Theta_{241}-\Theta_{341})E_1\sin 4\varphi+0.5(C_{11}^0-C_{33}^0)\cos 2\varphi\\ &\qquad +\frac{1}{8}(C_{11}^0-2C_{13}^0+C_{33}^0-4C_{44}^0)\cos 4\varphi,\\ &C_{23}=C_{13}^0+0.5(\Theta_{241}-\Theta_{341})E_1\sin 4\varphi\\ &\qquad +0.25(C_{11}^0-2C_{13}^0+C_{33}^0-4C_{44}^0)\sin^2 2\varphi,\\ &C_{14}=0.5(C_{13}^0+C_{12}^0)\sin 2\varphi+\Theta_{141}E_1\cos 2\varphi,\\ &C_{24}=0.5(\Theta_{241}+\Theta_{341})E_1\cos 2\varphi+0.5(\Theta_{241}-\Theta_{341})E_1\cos 4\varphi\\ &\qquad +0.25(C_{11}^0-C_{33}^0)\sin 2\varphi+\frac{1}{8}(C_{11}^0-2C_{13}^0+C_{33}^0-4C_{44}^0)\sin 4\varphi,\end{aligned} \tag{55}$$

continued on the next page

$$\begin{aligned}
C_{34} &= 0.5(\Theta_{241}+\Theta_{341})E_1\cos 2\varphi - 0.5(\Theta_{241}-\Theta_{341})E_1\cos 4\varphi \\
&\quad +0.25(C_{11}^0-C_{33}^0)\sin 2\varphi - \frac{1}{8}(C_{11}^0-2C_{13}^0+C_{33}^0-4C_{44}^0)\sin 4\varphi, \\
C_{33} &= \frac{1}{8}(3C_{11}^0+2C_{13}^0+3C_{33}^0+4C_{44}^0)+(\Theta_{241}+\Theta_{341})E_1\sin 2\varphi \\
&\quad -0.5(\Theta_{241}-\Theta_{341})E_1\sin 4\varphi - 0.5(C_{11}^0-C_{33}^0)\cos 2\varphi \\
&\quad +\frac{1}{8}(C_{11}-2C_{13}+C_{33}^0-4C_{44})\cos 4\varphi, \\
C_{44} &= C_{44}^0+0.5(\Theta_{241}-\Theta_{341})E_1\sin 4\varphi + 0.25(C_{11}^0-2C_{13}^0+C_{33}^0-4C_{44}^0)\sin^2 2\varphi, \\
C_{55} &= C_{44}^0-(C_{44}^0+C_{66}^0)\sin^2\varphi+\Theta_{561}E_1\sin 2\varphi,\; C_{56}=\Theta_{561}E_1\cos 2\varphi \\
&\quad -0.5(C_{44}^0-C_{66}^0)\sin 2\varphi, \\
C_{66} &= C_{66}^0+(C_{44}^0-C_{66}^0)\sin^2\varphi-\Theta_{561}E_1\sin 2\varphi.
\end{aligned} \tag{55}$$

The optical indicatrix also rotates under the electric field E_1. The AWs velocities for the wave propagation along the Z-axis are as follows:

$$\begin{aligned}
\upsilon_{31} &= \sqrt{\frac{C_{44}^0}{\rho}}, \\
\upsilon_{33} &= \sqrt{\frac{C_{33}^0}{\rho}+\frac{(\Theta_{341}E_1)^2}{2\rho(C_{33}^0-C_{44}^0)}} \approx \upsilon_{33}^0+\frac{(\Theta_{341}E_1)^2}{2\rho(C_{33}^0-C_{44}^0)\upsilon_{33}^0},\; \upsilon_{33}^0=\sqrt{\frac{C_{33}^0}{\rho}}, \\
\upsilon_{32} &= \sqrt{\frac{C_{33}^0}{\rho}-\frac{(\Theta_{341}E_1)^2}{2\rho(C_{33}^0-C_{44}^0)}} \approx \upsilon_{32}^0-\frac{(\Theta_{341}E_1)^2}{2\rho(C_{33}^0-C_{44}^0)\upsilon_{33}^0},\; \upsilon_{32}^0=\sqrt{\frac{C_{44}^0}{\rho}},
\end{aligned} \tag{56}$$

while at the Pockels effect, the respective refractive indices do not undergo change. At the propagation of AWs under the angle φ with respect to the Z-axis in the YZ plane, their velocities are:

$$\begin{aligned}
v_{13} &= v_{13}^0-\frac{(C_{44}^0+C_{66}^0)\sin^2\varphi+\Theta_{561}E_1\sin 2\varphi}{2\rho v_{13}^0}, \qquad v_{13}^0=\sqrt{\frac{C_{44}^0}{\rho}}, \\
v_{23} &= v_{23}^0+\frac{(\Theta_{241}-\Theta_{341})E_1\sin 4\varphi+0.25(C_{11}^0+2C_{13}^0+C_{33}^0-4C_{44}^0)\sin^2 2\varphi}{2\rho v_{23}^0}, \quad v_{23}^0=\sqrt{\frac{C_{44}^0}{\rho}}, \\
v_{33} &= \sqrt{\frac{\begin{pmatrix}1/8(3C_{11}^0+2C_{13}^0+3C_{33}^0+4C_{44}^0)-0.5(C_{11}^0-C_{33}^0)\cos 2\varphi \\ +1/8(C_{11}^0-2C_{13}^0+C_{33}^0-4C_{44}^0)\cos 4\varphi\end{pmatrix}}{\rho}} \\
&\quad +\frac{(\Theta_{241}+\Theta_{341})E_1\sin 2\varphi-0.5(\Theta_{241}-\Theta_{341})E_1\sin 4\varphi}{2\sqrt{\rho\begin{pmatrix}1/8(3C_{11}^0+2C_{13}^0+3C_{33}^0+4C_{44}^0)-0.5(C_{11}^0-C_{33}^0)\cos 2\varphi \\ +1/8(C_{11}^0-2C_{13}^0+C_{33}^0-4C_{44}^0)\cos 4\varphi\end{pmatrix}}}.
\end{aligned} \tag{57}$$

It is seen that, as in the previous cases, the linear dependence of the increment AWs velocities on the electric field can be reached when the AWs propagate along the axes of a rotated coordinate system. In the relations for v_{23} and v_{33} we have neglected the quadratic term as a small one. In linear electro-optics, the dependence of refractive indices on the electric field is quadratic.

Let us consider the point group of symmetry 4. When the electric field is applied along the Z-axis, the symmetry is not reduced, while new components of the tensor appear:

$\Delta C_{16} = \Theta_{163}E_3$, $\Delta C_{26} = \Theta_{263}E_3$, and the existing components change their value as $\Delta C_{11} = \Theta_{113}E_3$, $\Delta C_{12} = \Theta_{123}E_3$, $\Delta C_{13} = \Theta_{133}E_3$, $\Delta C_{33} = \Theta_{333}E_3$, $\Delta C_{44} = \Delta C_{55} = \Theta_{443}E_3$, and $\Delta C_{66} = \Theta_{663}E_3$. The change of the tensor of elastic stiffness coefficients is as follows:

$$\begin{bmatrix} C^0_{11} & C^0_{12} & C^0_{13} & 0 & 0 & C^0_{16} \\ C^0_{12} & C^0_{11} & C^0_{13} & 0 & 0 & -C^0_{16} \\ C^0_{13} & C^0_{13} & C^0_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C^0_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C^0_{44} & 0 \\ C^0_{16} & -C^0_{16} & 0 & 0 & 0 & C^0_{66} \end{bmatrix} \rightarrow \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{11} & C_{13} & 0 & 0 & -C_{16} \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ C_{16} & -C_{16} & 0 & 0 & 0 & C_{66} \end{bmatrix}. \tag{58}$$

The AWs velocities for the waves propagating along the *X*-axis are:

$$\begin{aligned} &\upsilon_{13} = \sqrt{\frac{C_{44} + \Theta_{443}E_3}{\rho}} \approx \upsilon^0_{13} + \frac{\Theta_{443}E_3}{2\rho\nu^0_{13}}, && \upsilon^0_{13} = \sqrt{\frac{C_{44}}{\rho}}, \\ &\upsilon_{11} \approx \upsilon^0_{11} + \frac{\Theta_{113}E_3}{2\rho\upsilon^0_{11}} + \frac{(\Theta_{163}E_3)^2}{2\rho\upsilon^0_{11}(C^0_{11} - C^0_{66} + (\Theta_{113} - \Theta_{663})E_3)}, && \upsilon^0_{11} = \sqrt{\frac{C^0_{11}}{\rho}}, \\ &\upsilon_{12} \approx \upsilon^0_{12} + \frac{\Theta_{663}E_3}{2\rho\upsilon^0_{11}} + \frac{(\Theta_{163}E_3)^2}{2\rho\upsilon^0_{11}(C^0_{11} - C^0_{66} + (\Theta_{113} - \Theta_{663})E_3)}, && \upsilon^0_{12} = \sqrt{\frac{C^0_{66}}{\rho}}. \end{aligned} \tag{59}$$

Under the application of the electric field E_3, the optical indicatrix does not rotate. Besides, the increment in refractive index is linear in the electric field strength. The angle of non-orthogonality before the application of the electric field is determined by the relation $\tan 2\zeta_Z = \frac{2C^0_{16}}{C^0_{11} - C^0_{66}}$, and after its application, by $\tan 2\zeta_Z = \frac{2(C^0_{16} + \Theta_{163}E_3)}{C^0_{11} - C^0_{66} + (\Theta_{113} - \Theta_{663})E_3}$. Thus, it is the first example of a possible operation with the existing angle of non-orthogonality with the electric field.

Application of the electric field to the crystals of the point group of symmetry $\bar{4}$ along the *Z*-axis leads to the lowering of the symmetry to the monoclinic group 2. The elastic stiffness is changed as:

$$\begin{bmatrix} C^0_{11} & C^0_{12} & C^0_{13} & 0 & 0 & C^0_{16} \\ C^0_{12} & C^0_{11} & C^0_{13} & 0 & 0 & -C^0_{16} \\ C^0_{13} & C^0_{13} & C^0_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C^0_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C^0_{44} & 0 \\ C^0_{16} & -C^0_{16} & 0 & 0 & 0 & C^0_{66} \end{bmatrix} \rightarrow \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C^0_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C^0_{66} \end{bmatrix}. \tag{60}$$

The increment of the tensor component due to electric field application are: $\Delta C_{16} = -C_{26} = \Theta_{163}E_3$, $\Delta C_{11} = -\Delta C_{22} = \Theta_{113}E_3$, $\Delta C_{12} = \Theta_{123}E_3$, $\Delta C_{44} = -C_{55} = \Theta_{443}E_3$ and $\Delta C_{13} = \Theta_{133}E_3$. Besides, new components are induced: $C_{36} = \Theta_{363}E_3$, and $C_{45} = \Theta_{453}E_3$. It is obvious that the tensor (60) has the rotational degree of freedom about the *Z*-axis. In linear electro-optics, the optical indicatrix rotates at an angle determined by the ratio of the electro-optic tensor components r_{63} / r_{13}. The AWs velocities for the AWs propagating along the *X*-axis are as follows:

$$\begin{aligned}
&\upsilon_{13}=\sqrt{\frac{C_{44}+\Theta_{443}E_3}{\rho}}\approx\upsilon_{13}^0+\frac{\Theta_{443}E_3}{2\rho v_{13}^0}, && \upsilon_{13}^0=\sqrt{\frac{C_{44}}{\rho}},\\
&\upsilon_{11}\approx\upsilon_{11}^0+\frac{\Theta_{113}E_3}{2\rho\upsilon_{11}^0}+\frac{(\Theta_{163}E_3)^2}{2\rho\upsilon_{11}^0(C_{11}^0-C_{66}^0+\Theta_{113}E_3)}, && \upsilon_{11}^0=\sqrt{\frac{C_{11}^0}{\rho}},\\
&\upsilon_{12}\approx\upsilon_{12}^0+\frac{(\Theta_{163}E_3)^2}{2\rho\upsilon_{11}^0(C_{11}^0-C_{66}^0+\Theta_{113}E_3)}, && \upsilon_{12}^0=\sqrt{\frac{C_{66}^0}{\rho}}.
\end{aligned}\tag{61}$$

It is seen that in point groups 4 and $\overline{4}$, applying an electric field along the Z-axis yields a quadratic dependence of the increment in AWs velocities on the electric field strength, whereas in the linear electro-optic effect, the increment in refractive indices depends linearly on the electric field strength in these cases. The non-orthogonality angle is determined by the relation $\tan 2\zeta_Z=\frac{2C_{16}}{C_{11}-C_{66}}=\frac{2(C_{16}^0+\Theta_{163}E_3)}{C_{11}^0+\Theta_{113}E_3-C_{66}^0}$. It is a similar situation with the group of symmetry mentioned above, 4.

3.4. Orthorombic point groups of symmetry

Within the orthorhombic system, only two non-centrosymmetric point symmetry groups belong, namely 222 and $mm2$. To these symmetry groups belong langbeinites in ferroelastic phase, Rochelle salt, crystals of A_2BX_4 family in the ferroelectric phase, etc [28-30]. Upon the application of the electric field along the Z-axis in crystals that belong to the group of symmetry 222, the symmetry is changed as $222\rightarrow 2$, while the EST acquires the change:

$$\begin{bmatrix}
C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & 0 & 0\\
C_{12}^0 & C_{22}^0 & C_{23}^0 & 0 & 0 & 0\\
C_{13}^0 & C_{23}^0 & C_{33}^0 & 0 & 0 & 0\\
0 & 0 & 0 & C_{44}^0 & 0 & 0\\
0 & 0 & 0 & 0 & C_{55}^0 & 0\\
0 & 0 & 0 & 0 & 0 & C_{66}^0
\end{bmatrix}\rightarrow
\begin{bmatrix}
C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & 0 & C_{16}\\
C_{12}^0 & C_{22}^0 & C_{23}^0 & 0 & 0 & C_{26}\\
C_{13}^0 & C_{23}^0 & C_{33}^0 & 0 & 0 & C_{36}\\
0 & 0 & 0 & C_{44}^0 & C_{45} & 0\\
0 & 0 & 0 & C_{45} & C_{55}^0 & 0\\
C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66}^0
\end{bmatrix}.\tag{62}$$

The new components of this tensor that are induced by the electric field are: $C_{16}=\Theta_{163}E_3$, $C_{26}=\Theta_{263}E_3$, $C_{36}=\Theta_{363}E_3$, and $C_{45}=\Theta_{453}E_3$. The AWs velocities at the wave propagation along the X-axis are:

$$\begin{aligned}
&\upsilon_{11}\approx\upsilon_{11}^0+\frac{(\Theta_{163}E_3)^2}{2\rho\upsilon_{11}^0(C_{11}^0-C_{66}^0)}, && \upsilon_{11}^0=\sqrt{C_{11}^0/\rho},\\
&\upsilon_{12}\approx\upsilon_{12}^0-\frac{(\Theta_{163}E_3)^2}{2\rho\upsilon_{11}^0(C_{11}^0-C_{66}^0)}, && \upsilon_{12}^0=\sqrt{C_{66}^0/\rho}\,,\\
&\upsilon_{13}=\sqrt{C_{44}/\rho}\,.
\end{aligned}\tag{63}$$

The tensor (62) has a rotational degree of freedom about the Z-axis, and the velocities quadratically depend on the electric field strength. A similar behavior is observed for the optical indicatrix under an electric field. The induced angle of non-orthogonality is determined as $\tan 2\zeta_Z=(2\Theta_{163}E_3)/(C_{11}^0-C_{66}^0)$.

For the point group of symmetry $mm2$ at the application of the electric field along the Z-axis, the crystal symmetry and the structure of the EST is not changed, and it is the same as the tensor for the group of symmetry 222 (Eq. 62), while existing components of the tensor

acquire the change: $\Delta C_{11}=\Theta_{113}E_3$, $\Delta C_{22}=-\Theta_{113}E_3$, $\Delta C_{33}=\Theta_{333}E_3$, $\Delta C_{12}=\Theta_{123}E_3$, $\Delta C_{13}=\Theta_{133}E_3$, $\Delta C_{23}=\Theta_{233}E_3$, $\Delta C_{44}=\Theta_{443}E_3$, $\Delta C_{55}=\Theta_{553}E_3$, $\Delta C_{66}=\Theta_{663}E_3$.
The AWs velocities at their propagation along the X-axis are determined as:

$$\begin{aligned} \upsilon_{13}&=\sqrt{\frac{C_{55}+\Theta_{553}E_3}{\rho}}\approx\upsilon_{13}^0+\frac{\Theta_{553}E_3}{2\rho\nu_{13}^0}, \qquad \upsilon_{13}^0=\sqrt{\frac{C_{55}}{\rho}},\\ \upsilon_{11}&=\sqrt{\frac{C_{11}+\Theta_{113}E_3}{\rho}}\approx\upsilon_{11}^0+\frac{\Theta_{113}E_3}{2\rho\nu_{11}^0}, \qquad \upsilon_{11}^0=\sqrt{\frac{C_{11}}{\rho}},\\ \upsilon_{12}&=\sqrt{\frac{C_{66}+\Theta_{663}E_3}{\rho}}\approx\upsilon_{12}^0+\frac{\Theta_{663}E_3}{2\rho\nu_{12}^0}, \qquad \upsilon_{12}^0=\sqrt{\frac{C_{66}}{\rho}}. \end{aligned} \tag{64}$$

It is seen that it is one of the cases in which the increase in AWs velocities is linearly dependent on the electric field strength when the AWs propagate along the initial axes of the coordinate system. The same is true concerning the linear electro-optic effect. The AWs at zero electric field, as well as under its application, are purely transverse and purely longitudinal.

At the application of the electric field along the X-axis, the symmetry is reduced to the point group m, and the tensor of elastic stiffness is changed as:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & 0 & 0\\ C_{12}^0 & C_{22}^0 & C_{23}^0 & 0 & 0 & 0\\ C_{13}^0 & C_{23}^0 & C_{33}^0 & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44}^0 & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55}^0 & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}^0 \end{bmatrix} \to \begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & C_{15} & 0\\ C_{12}^0 & C_{22}^0 & C_{23}^0 & 0 & C_{25} & 0\\ C_{13}^0 & C_{23}^0 & C_{33}^0 & 0 & C_{35} & 0\\ 0 & 0 & 0 & C_{44}^0 & 0 & C_{46}\\ C_{15} & C_{25} & C_{35} & 0 & C_{55}^0 & 0\\ 0 & 0 & 0 & C_{46} & 0 & C_{66}^0 \end{bmatrix}. \tag{65}$$

The AWs velocities at the wave propagation along the Z-axis are:

$$\begin{aligned} \upsilon_{31}&\approx\upsilon_{31}^0-\frac{(\Theta_{351}E_1)^2}{2\rho\upsilon_{31}^0(C_{33}^0-C_{55}^0)}, \qquad \upsilon_{31}^0=\sqrt{C_{55}^0/\rho},\\ \upsilon_{32}&=\sqrt{\mathrm{C}_{44}/\rho},\\ \upsilon_{33}&\approx\upsilon_{33}^0+\frac{(\Theta_{351}E_1)^2}{2\rho\upsilon_{33}^0(C_{33}^0-C_{55}^0)}, \qquad \upsilon_{33}^0=\sqrt{C_{33}^0/\rho}. \end{aligned} \tag{66}$$

Tensor (65) acquires a rotational degree of freedom around the X-axis, and the velocities are proportional to the square of the electric field. The angle of non-orthogonality is as follows $\tan 2\zeta_Y=2\Theta_{351}E_1/\left(C_{55}^0-C_{33}^0\right)$. It is clear that the square dependence of AWs velocities concerns only those waves that acquire non-orthogonality.

3.5. Hexagonal point groups of symmetry

Let us consider the group of symmetry $6mm$. Under the electric field E_3 the point group of symmetry and the structure of the EST are not changed. However, the existing tensor components acquire an increment: $C_{11}=C_{22}=C_{11}^0+\Theta_{113}E_3$, $C_{33}=C_{33}^0+\Theta_{333}E_3$, $C_{66}=0.5(C_{11}^0-C_{12}^0)+(2\Theta_{113}-\Theta_{123})E_3$, $C_{12}=C_{12}^0+\Theta_{123}E_3$, $C_{13}=C_{23}=C_{13}^0+\Theta_{133}E_3$, $C_{55}=C_{44}=C_{44}^0+\Theta_{443}E_3$,. The velocities of the AWs propagating along the X-axis are determined as:

$$\begin{aligned} \upsilon_{11} &= \upsilon_{11}^{0} + \frac{\Theta_{113}E_3}{2\rho\upsilon_{11}^{0}}, & \upsilon_{11}^{0} &= \sqrt{C_{11}/\rho}, \\ \upsilon_{13} &= \upsilon_{13}^{0} + \frac{\Theta_{443}E_3}{2\rho\upsilon_{13}^{0}}, & \upsilon_{13}^{0} &= \sqrt{C_{44}/\rho}, \\ \upsilon_{12} &= \upsilon_{12}^{0} + \frac{(2\Theta_{113}-\Theta_{123})E_3}{2\rho\upsilon_{12}^{0}}, & \upsilon_{12}^{0} &= \sqrt{\frac{0.5(C_{11}-C_{12})}{\rho}}. \end{aligned} \tag{67}$$

At the application of the electric field E_1, since the point group of symmetry is reduced to the group m, the new components of the EST appear: $C_{15}=\Theta_{151}E_1$, $C_{25}=\Theta_{251}E_1$, $C_{35}=\Theta_{351}E_1$, $C_{46}=\Theta_{461}E_1$ and the tensor acquires the view:

$$\begin{bmatrix} C_{11}^{0} & C_{12}^{0} & C_{13}^{0} & 0 & C_{15} & 0 \\ C_{12}^{0} & C_{11}^{0} & C_{13}^{0} & 0 & C_{25} & 0 \\ C_{13}^{0} & C_{13}^{0} & C_{33}^{0} & 0 & C_{35} & 0 \\ 0 & 0 & 0 & C_{44}^{0} & 0 & C_{46} \\ C_{15} & C_{25} & C_{35} & 0 & C_{44}^{0} & 0 \\ 0 & 0 & 0 & C_{46} & 0 & C_{66}^{0} \end{bmatrix}. \tag{68}$$

For the agreement of this tensor with the tensor peculiar to the group of symmetry m, it is necessary to rotate the coordinate system around the Y-axis by the angle φ. Then the components of this tensor are expressed as:

$$\begin{aligned} C_{11} &= \frac{1}{8}(3C_{11}^{0}+2C_{13}^{0}+3C_{33}^{0}+4C_{44}^{0})+(\Theta_{151}+\Theta_{351})E_1\sin 2\varphi+0.5(\Theta_{151}-\Theta_{151})E_1\sin 4\varphi \\ &\quad +0.5(C_{11}^{0}-C_{33}^{0})\cos 2\varphi+\frac{1}{8}(C_{11}^{0}-2C_{13}^{0}+C_{33}^{0}-4C_{44}^{0})\cos 4\varphi, \\ C_{12} &= C_{13}^{0}\sin^2\varphi+C_{12}^{0}\cos^2\varphi+\Theta_{251}E_1\sin 2\varphi, \\ C_{13} &= C_{13}^{0}-0.5(\Theta_{151}-\Theta_{351})E_1\sin 4\varphi+0.25(C_{11}^{0}-2C_{13}^{0}+C_{33}^{0}-4C_{44}^{0})\sin^2 2\varphi, \\ C_{15} &= 0.25(\Theta_{251}-\Theta_{351})+0.5(\Theta_{151}+\Theta_{351})E_1\cos 2\varphi+0.25(2\Theta_{151}-\Theta_{251}-\Theta_{351})E_1\cos 4\varphi \\ &\quad -0.25(C_{11}^{0}-C_{12}^{0})\sin 2\varphi-\frac{1}{8}(C_{11}^{0}-C_{12}^{0}-4C_{44}^{0})\sin 4\varphi, \\ C_{22} &= C_{11}^{0}, \\ C_{25} &= \Theta_{251}E_1\cos 2\varphi+0.5(C_{11}^{0}-C_{12}^{0})\sin 2\varphi, \\ C_{33} &= \frac{1}{8}(3C_{11}^{0}+2C_{13}^{0}+3C_{33}^{0}+4C_{44}^{0})-(\Theta_{151}+\Theta_{351})E_1\sin 2\varphi+0.5(\Theta_{151}-\Theta_{351})E_1\sin 4\varphi \\ &\quad -0.5(C_{11}^{0}-C_{33}^{0})\cos 2\varphi+\frac{1}{8}(C_{11}-2C_{13}+C_{33}^{0}-4C_{44})\cos 4\varphi, \\ C_{35} &= 0.25(\Theta_{351}-\Theta_{251})+0.5(\Theta_{151}+\Theta_{351})E_1\cos 2\varphi-0.25(2\Theta_{151}-\Theta_{251}-\Theta_{351})E_1\cos 4\varphi \\ &\quad -0.25(C_{11}^{0}-C_{12}^{0})\sin 2\varphi+\frac{1}{8}(C_{11}^{0}-C_{12}^{0}-4C_{44}^{0})\sin 4\varphi, \\ C_{44} &= C_{44}^{0}+\Theta_{461}E_1\sin 2\varphi+0.5(C_{11}^{0}-C_{12}^{0}-2C_{44}^{0})\sin^2\varphi, \\ C_{46} &= 0.25(C_{12}^{0}-C_{11}^{0}+2C_{44}^{0})\sin 2\varphi+\Theta_{461}E_1\cos 2\varphi, \\ C_{55} &= 0.5(C_{11}^{0}-C_{12}^{0})-(\Theta_{151}+\Theta_{251})E_1\sin 4\varphi+0.5(C_{11}^{0}-C_{12}^{0}+2C_{44}^{0})\cos^2 2\varphi, \\ C_{66} &= C_{44}^{0}+\Theta_{461}E_1\sin 2\varphi+0.5(C_{11}^{0}-C_{12}^{0}-2C_{44}^{0})\cos^2\varphi. \end{aligned} \tag{69}$$

The AWs velocities for the waves propagating along the *Z*-axis are:

$$\begin{aligned}
&\upsilon_{32}=\sqrt{\frac{\mathrm{C}_{44}^{0}}{\rho}},\\
&\upsilon_{33}=\sqrt{\frac{\mathrm{C}_{33}^{0}}{\rho}+\frac{(\Theta_{351}\,\mathrm{E}_1)^2}{2\rho(\mathrm{C}_{33}^{0}-\mathrm{C}_{44}^{0})}}\approx\upsilon_{33}^{0}+\frac{(\Theta_{351}\,\mathrm{E}_1)^2}{2\rho(\mathrm{C}_{33}^{0}-\mathrm{C}_{44}^{0})\upsilon_{33}^{0}},\quad \upsilon_{33}^{0}=\sqrt{\frac{\mathrm{C}_{33}^{0}}{\rho}},\\
&\upsilon_{31}=\sqrt{\frac{\mathrm{C}_{44}^{0}}{\rho}-\frac{(\Theta_{351}\,\mathrm{E}_1)^2}{2\rho(\mathrm{C}_{33}^{0}-\mathrm{C}_{44}^{0})}}\approx\upsilon_{31}^{0}-\frac{(\Theta_{351}\,\mathrm{E}_1)^2}{2\rho(\mathrm{C}_{33}^{0}-\mathrm{C}_{44}^{0})\upsilon_{31}^{0}},\quad \upsilon_{31}^{0}=\sqrt{\frac{\mathrm{C}_{44}^{0}}{\rho}}
\end{aligned}\tag{70}$$

The non-orthogonality angle is determined as: $\tan 2\zeta_Y = 2\Theta_{351}E_1 / \left(C_{33}^0 - C_{44}^0\right)$ When the AWs propagate along the Z'-axis of the rotated coordinate system by the angle φ, their velocities are as follows:

$$\begin{aligned}
v_{31}=&\sqrt{\frac{0.5(C_{11}^0-C_{12}^0)+0.5(C_{11}^0-C_{12}^0+2C_{44}^0)\cos^2 2\varphi}{\rho}}\\
&-\frac{(\Theta_{151}+\Theta_{251})E_1\sin 4\varphi}{2\sqrt{0.5\rho(C_{11}^0-C_{12}^0+(C_{11}^0-C_{12}^0+2C_{44}^0)\cos^2 2\varphi)}},\\
v_{32}=&v_{32}^0-\frac{\Theta_{461}E_1\sin 2\varphi+0.5(C_{11}^0-C_{12}^0-2C_{44}^0)\sin^2\varphi}{2\rho v_{32}^0},\quad v_{32}^0=\sqrt{\frac{C_{44}^0}{\rho}},\\
v_{33}=&\sqrt{\frac{\begin{pmatrix}1/8(3C_{11}^0+2C_{13}^0+3C_{33}^0+4C_{44}^0)-0.5(C_{11}^0-C_{33}^0)\cos 2\varphi\\ +1/8(C_{11}^0-2C_{13}^0+C_{33}^0-4C_{44}^0)\cos 4\varphi\end{pmatrix}}{\rho}}\\
&+\frac{0.5(\Theta_{151}-\Theta_{351})E_1\sin 4\varphi-(\Theta_{151}+\Theta_{351})E_1\sin 2\varphi}{2\sqrt{\rho\begin{pmatrix}1/8(3C_{11}^0+2C_{13}^0+3C_{33}^0+4C_{44}^0)-0.5(C_{11}^0-C_{33}^0)\cos 2\varphi\\ +1/8(C_{11}^0-2C_{13}^0+C_{33}^0-4C_{44}^0)\cos 4\varphi\end{pmatrix}}}.
\end{aligned}\tag{71}$$

In the relations for v_{31} and v_{33} we have neglected the quadratic term as a small one. The non-orthogonality angle is equal $\tan 2\zeta_Y = 2C_{35}/\left(C_{55}-C_{44}\right)$.

For the symmetry group 622 at the application of the electric field along the *Z*-axis, the symmetry is lowered to the point group 6. However, the AWs velocities do not change under the electric field E_3. This case is similar to that observed in the Curie symmetry group ∞2. Therefore, in acoustics, one deals only with two axial symmetry groups in which applying the electric field along a higher-symmetry axis does not change AWs velocities due to the linear effect. In the Pockels effect, such groups also include 422 and 32.

Let us consider the point group of symmetry $\bar{6}2m$. Upon the application of the electric field along the *Z*-axis, the symmetry is lowered to the group $3m$, and the EST acquires the change:

$$\begin{bmatrix}
C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & 0 & 0\\
C_{12}^0 & C_{11}^0 & C_{13}^0 & 0 & 0 & 0\\
C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0\\
0 & 0 & 0 & C_{44}^0 & 0 & 0\\
0 & 0 & 0 & 0 & C_{44}^0 & 0\\
0 & 0 & 0 & 0 & 0 & 0.5(C_{11}^0-C_{12}^0)
\end{bmatrix}
\rightarrow
\begin{bmatrix}
C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & C_{15} & 0\\
C_{12}^0 & C_{11}^0 & C_{13}^0 & 0 & -C_{15} & 0\\
C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0\\
0 & 0 & 0 & C_{44}^0 & 0 & C_{46}\\
C_{15} & -C_{15} & 0 & 0 & C_{44}^0 & 0\\
0 & 0 & 0 & C_{46} & 0 & 0.5(C_{11}^0-C_{12}^0)
\end{bmatrix},\tag{72}$$

where $C_{15} = -C_{25} = \Theta_{153}E_3$ and $C_{46} = -2\Theta_{153}E_3$. The obtained tensor is peculiar to the crystallographic setting in the $3m$ point group of symmetry when $m \perp X$. Let the AWs propagate along the *X*-axis. In this case, the Christoffel tensor can be written as:

$$\begin{bmatrix} C_{11}^0 - \lambda & 0 & \Theta_{153}E_3 \\ 0 & C_{66}^0 - \lambda & 0 \\ \Theta_{153}E_3 & 0 & C_{55}^0 - \lambda \end{bmatrix}, \tag{73}$$

while the angle of non-orthogonality is determined by the relation: $\tan 2\zeta_Y = \dfrac{\Theta_{153}E_3}{C_{11}^0 - C_{55}^0}$. The AWs velocities are as follows:

$$\begin{aligned} &\upsilon_{12} = \sqrt{\frac{0.5(C_{11} - C_{12})}{\rho}}, \\ &\upsilon_{11} \approx \upsilon_{11}^0 + \frac{(\Theta_{153}E_3)^2}{2\rho(C_{11}^0 - C_{44}^0)\upsilon_{11}^0},\ \upsilon_{11}^0 = \sqrt{\frac{C_{11}^0}{\rho}}, \\ &\upsilon_{31} \approx \upsilon_{31}^0 - \frac{(\Theta_{153}E_{31})^2}{2\rho(C_{11}^0 - C_{44}^0)\upsilon_{31}^0},\ \upsilon_{31}^0 = \sqrt{\frac{C_{44}^0}{\rho}}. \end{aligned} \tag{74}$$

At the application of the electric field along the *X*-axis, the symmetry is changed as: $\bar{6}2m \to mm2$. The EST components acquire the change: $\Delta C_{11} = \Theta_{111}E_1$, $\Delta C_{22} = \Theta_{221}E_1$, $\Delta C_{12} = -(\Theta_{111} + \Theta_{221})E_1$, $\Delta C_{13} = \Theta_{131}E_1$, $\Delta C_{23} = -\Theta_{131}E_1$, $\Delta C_{55} = -\Delta C_{44} = -\Theta_{441}E_1$, $\Delta C_{66} = -(\Theta_{111} + \Theta_{221})E_1$. It should be noted that the EST written in the lattice vector basis of the group of symmetry $\bar{6}2m$ does not correspond to the tensor for the basis of the group of symmetry $mm2$, since the *Z*-axis of group $mm2$ is parallel to the *X*-axis of group $\bar{6}2m$. To align the *Z*-axis properly, one should rotate the tensor about the *Y*-axis by 90 deg. Finally, the tensor has the structure:

$$\begin{bmatrix} C_{33} & C_{23} & C_{13} & 0 & 0 & 0 \\ C_{23} & C_{22} & C_{12} & 0 & 0 & 0 \\ C_{13} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix}. \tag{75}$$

The AWs velocities are as follows:

$$\begin{aligned} &\upsilon_{11} = \sqrt{\frac{C_{33}^0}{\rho}}, \\ &\upsilon_{13} \approx \upsilon_{13}^0 - \frac{\Theta_{441}E_1}{2\rho\upsilon_{13}^0},\ \upsilon_{13}^0 = \sqrt{\frac{C_{44}^0}{\rho}}, \\ &\upsilon_{12} \approx \upsilon_{12}^0 + \frac{\Theta_{441}E_1}{2\rho\upsilon_{12}^0},\ \upsilon_{12}^0 = \sqrt{\frac{C_{44}^0}{\rho}}. \end{aligned} \tag{76}$$

As one can see, the non-orthogonality does not exist before and after the application of the electric field, and the increment in AWs velocities depends linearly on the electric field.

For the group of symmetry 6, application of the electric field E_3 does not lead to a symmetry change. The existing components of the EST change their values as: $C_{11}=C_{22}=C_{11}^0+\Theta_{113}E_3$, $C_{12}=C_{12}^0+\Theta_{123}E_3$, $C_{33}=C_{33}^0+\Theta_{333}E_3$, $C_{13}=C_{23}=C_{13}^0+\Theta_{133}E_3$, $C_{55}=C_{44}=C_{44}^0+\Theta_{443}E_3$, $C_{66}=0.5(C_{11}^0-C_{12}^0)+(2\Theta_{113}-\Theta_{123})E_3$. AWs velocities for the waves propagating along the X-axis are:

$$\begin{aligned}
&\upsilon_{11}=\upsilon_{11}^0+\frac{\Theta_{113}E_3}{2\rho\upsilon_{11}^0},\quad \upsilon_{11}^0=\sqrt{\frac{C_{11}}{\rho}},\\
&\upsilon_{13}=\upsilon_{13}^0+\frac{\Theta_{443}E_3}{2\rho\upsilon_{13}^0},\quad \upsilon_{13}^0=\sqrt{\frac{C_{44}}{\rho}},\\
&\upsilon_{12}=\upsilon_{12}^0+\frac{(2\Theta_{113}-\Theta_{123})E_3}{2\rho\upsilon_{12}^0},\quad \upsilon_{12}^0=\sqrt{\frac{0.5(C_{11}-C_{12})}{\rho}}.
\end{aligned}\tag{77}$$

The AWs are purely transverse or longitudinal, regardless of whether an electric field E_3 is applied.

For the group of symmetry $\bar{6}$, application of the electric field E_3 leads to the lowering of symmetry to the group 3. Under the application of an electric field, new components appear: $C_{24}=-C_{14}=-\Theta_{143}E_3$, $C_{25}=-C_{15}=-\Theta_{153}E_3$, $C_{46}=-\Theta_{153}E_3$, $C_{56}=\Theta_{143}E_3$. The peculiarity of the point group of symmetry 3 is that one can get rid of the C_{15} or C_{14} tensor components by rotation of the coordinate system around the Z-axis by a certain angle φ:

$$\begin{aligned}
&C_{14}(\varphi)=C_{14}\cos 3\varphi+C_{25}\sin 3\varphi,\\
&C_{25}(\varphi)=-C_{14}\sin(3\varphi)+C_{25}\cos(3\varphi).
\end{aligned}\tag{78}$$

If $\tan(3\varphi)=C_{25}/C_{14}$ then $C_{25}=0$, while if $\tan(3\varphi)=-C_{14}/C_{25}$, one has $C_{14}=0$. If the AWs are propagating along the X-axis and $C_{25}=0$ their velocities are determined as

$$\begin{aligned}
&\upsilon_{11}=\sqrt{\frac{C_{11}}{\rho}},\\
&\upsilon_{13}\approx\upsilon_{13}^0+\frac{(\Theta_{143}E_3)^2}{2\rho\upsilon_{13}^0(C_{44}-0.5(C_{11}-C_{12}))},\ \upsilon_{13}^0=\sqrt{\frac{C_{44}^0}{\rho}},\\
&\upsilon_{12}\approx\upsilon_{12}^0-\frac{(\Theta_{143}E_3)^2}{2\rho\upsilon_{12}^0(C_{44}-0.5(C_{11}-C_{12}))},\ \upsilon_{12}^0=\sqrt{\frac{0.5(C_{11}-C_{12})}{\rho}}.
\end{aligned}\tag{79}$$

with the angle of non-orthogonality being equal to $\tan 2\zeta_X=\frac{2\Theta_{143}}{C_{66}^0-C_{55}^0}$. When $C_{14}=0$:

$$\begin{aligned}
&\upsilon_{12}=\sqrt{\frac{0.5(C_{11}-C_{12})}{\rho}}\\
&\upsilon_{11}\approx\upsilon_{11}^0+\frac{(\Theta_{153}E_3)^2}{2\rho\upsilon_{11}^0(C_{11}-C_{44})},\ \upsilon_{11}^0=\sqrt{\frac{C_{11}^0}{\rho}},\\
&\upsilon_{13}\approx\upsilon_{13}^0-\frac{(\Theta_{153}E_3)^2}{2\rho\upsilon_{13}^0(C_{11}-C_{44})},\ \upsilon_{13}^0=\sqrt{\frac{C_{44}^0}{\rho}}
\end{aligned}\tag{80}$$

In this case, under the electric field, the AWs acquire non-orthogonality: $\tan 2\zeta_Y=\frac{2\Theta_{153}}{C_{11}^0-C_{55}^0}$.

3.6. Trigonal point groups of symmetry

This group of symmetry includes well-known crystalline materials such as quartz [31], calcite, lithium niobate, lead germanate in the ferroelectric phase, etc., as well as many other materials. Let us proceed with our analysis from the point group of symmetry 32. The EST for this point group of symmetry can be presented as:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & C_{14}^0 & 0 & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & -C_{14}^0 & 0 & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0 \\ C_{14}^0 & -C_{14}^0 & 0 & C_{44}^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}^0 & C_{14}^0 \\ 0 & 0 & 0 & 0 & C_{14}^0 & 0.5(C_{11}^0 - C_{12}^0) \end{bmatrix}. \tag{81}$$

Application of the electric field along the Z-axis results in the symmetry lowering to the point group 3 and the appearance of new components: $C_{15} = \Theta_{153} E_3$, $C_{25} = -\Theta_{153} E_3$, and $\Delta C_{46} = -\Theta_{153} E_3$. The respective tensor change the structure:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & C_{14}^0 & -C_{25} & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & -C_{14}^0 & C_{25} & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0 \\ C_{14}^0 & -C_{14}^0 & 0 & C_{44}^0 & 0 & C_{25} \\ -C_{25} & C_{25} & 0 & 0 & C_{44}^0 & C_{14}^0 \\ 0 & 0 & 0 & C_{25} & C_{14}^0 & 0.5(C_{11}^0 - C_{12}^0) \end{bmatrix}. \tag{82}$$

The peculiarity of the point group of symmetry consists in the possibility of presenting of this tensor in the view of two different structures with the help of its rewriting in the rotated coordinate system by the Z-axis at an angle φ

$$\begin{aligned} C_{14}(\varphi) &= C_{14}^0 \cos 3\varphi - \Theta_{153} E_3 \sin 3\varphi, \\ C_{25}(\varphi) &= -C_{14}^0 \sin 3\varphi - \Theta_{153} E_3 \cos 3\varphi \end{aligned}. \tag{83}$$

When $C_{25}(\varphi) = 0$, then $\tan 3\varphi = -\Theta_{153} E_3 / C_{14}^0$, while when $C_{14}(\varphi) = 0$, then $\tan 3\varphi = C_{14}^0 / \Theta_{153} E_3$. Then the structures are:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & C_{14} & 0 & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & -C_{14} & 0 & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0 \\ C_{14} & -C_{14} & 0 & C_{44}^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}^0 & C_{14} \\ 0 & 0 & 0 & 0 & C_{14} & 0.5(C_{11}^0 - C_{12}^0) \end{bmatrix} \text{ or } \begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & -C_{25} & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & 0 & C_{25} & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^0 & 0 & C_{25} \\ -C_{25} & C_{25} & 0 & 0 & C_{44}^0 & 0 \\ 0 & 0 & 0 & C_{25} & 0 & 0.5(C_{11}^0 - C_{12}^0) \end{bmatrix}, \tag{84}$$

respectively. Let us consider the case when $C_{14}(\varphi)=0$ that is realized under the condition $C_{25}\big|_{C_{14}=0} = -C^0_{14}\sin\arctan C^0_{14}/\Theta_{153}E_3 - \Theta_{153}E_3\cos\arctan C^0_{14}/\Theta_{153}E_3$. Then, supposing that the AWs propagate along the X' -axis (X'- axis appears after rotation of the coordinate system by the angle φ around the Z-axis), the Christoffel tensor will be as follows:

$$\begin{bmatrix} C^0_{11}-\lambda & 0 & C_{25}\big|_{C_{14}=0} \\ 0 & C^0_{66}-\lambda & 0 \\ C_{25}\big|_{C_{14}=0} & 0 & C^0_{44}-\lambda \end{bmatrix}. \tag{85}$$

The non-orthogonality angle is determined as $\tan 2\zeta_{Y'} = \dfrac{2C_{25}\big|_{C_{14}=0}}{C^0_{11}-C^0_{44}}$ being dependent on the electric field. The AWs velocities are determined by the relations:

$$\begin{gathered} \upsilon_{12} = \sqrt{\frac{0.5(C^0_{11}-C^0_{12})}{\rho}},\quad \upsilon_{11} \approx \upsilon^0_{11} + \frac{(C_{25}\big|_{C_{14}=0})^2}{2\rho\upsilon^0_{11}(C^0_{11}-C^0_{44})}, \\ \upsilon_{13} \approx \upsilon^0_{13} - \frac{(C_{25}\big|_{C_{14}=0})^2}{2\rho\upsilon^0_{13}(C^0_{11}-C^0_{44})},\quad \upsilon^0_{11} = \sqrt{\frac{C^0_{11}}{\rho}},\quad \upsilon^0_{13} = \sqrt{\frac{C^0_{44}}{\rho}}, \end{gathered} \tag{86}$$

where indices 1,2,3 correspond to the rotated coordinate system $X'Y'Z$. When $C_{25}(\varphi)=0$, and propagation of the AWs along the X'' - axis, the Christoffel tensor is written as:

$$\begin{bmatrix} C^0_{11}-\lambda & 0 & 0 \\ 0 & C^0_{66}-\lambda & C_{14}\big|_{C_{25}=0} \\ 0 & C_{14}\big|_{C_{25}=0} & C^0_{44}-\lambda \end{bmatrix}. \tag{87}$$

where $C_{14}\big|_{C_{25}=0} = C^0_{14}\cos\arctan(-\Theta_{153}E_3/C^0_{14}) - \Theta_{153}E_3\sin\arctan(-\Theta_{153}E_3/C^0_{14})$. The AWs velocities can be written as:

$$\begin{gathered} \upsilon_{11} = \sqrt{\frac{C^0_{11}}{\rho}},\quad \upsilon_{13} \approx \upsilon^0_{13} + \frac{(C_{14}\big|_{C_{25}=0})^2}{2\rho\upsilon^0_{13}(C^0_{44}-0.5(C^0_{11}-C^0_{12}))},\quad \upsilon^0_{13} = \sqrt{\frac{C^0_{44}}{\rho}}, \\ \upsilon_{12} \approx \upsilon^0_{12} - \frac{(C_{14}\big|_{C_{25}=0})^2}{2\rho\upsilon^0_{13}(C^0_{44}-0.5(C^0_{11}-C^0_{12}))},\quad \upsilon^0_{12} = \sqrt{\frac{0.5(C_{11}-C_{12})}{\rho}} \end{gathered}, \tag{88}$$

where indices 1,2,3 correspond to the rotated coordinate system $X''Y''Z$.

At the application of the electric field along the X-axis, the symmetry is changed as: $32 \rightarrow 2$, which yields a change in the structure of the EST:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & 0 & 0 \\ C_{12} & C_{22} & C_{23} & C_{24} & 0 & 0 \\ C_{13} & C_{23} & C^0_{33} & C_{34} & 0 & 0 \\ C_{14} & C_{24} & C_{34} & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & C_{56} \\ 0 & 0 & 0 & 0 & C_{56} & C_{66} \end{bmatrix}. \tag{89}$$

where $\Delta C_{11} = \Theta_{111}E_1$, $\Delta C_{22} = \Theta_{221}E_1$, $\Delta C_{12} = \Theta_{121}E_1$, $\Delta C_{13} = \Theta_{131}E_1$ $\Delta C_{23} = \Theta_{231}E_1$,

$\Delta C_{14}=\Theta_{141}E_1$, $\Delta C_{24}=\Theta_{241}E_1$, $\Delta C_{44}=\Theta_{441}E_1$, $\Delta C_{55}=\Theta_{551}E_1$, $\Delta C_{66}=\Theta_{661}E_1$, $\Delta C_{56}=\Theta_{561}E_1$, $C_{34}=\Theta_{341}E_1$. For the AWs propagating along the *Z*-axis, the Christoffel tensor can be written as:

$$\begin{bmatrix} C_{55}-\lambda & 0 & \\ 0 & C_{44}-\lambda & C_{34} \\ 0 & C_{34} & C_{33}-\lambda \end{bmatrix}. \tag{90}$$

The AWs velocities are as follows:

$$\begin{gathered} \upsilon_{31}\approx\upsilon_{31}^0+\frac{\Theta_{551}E_1}{2\rho\upsilon_{31}^0},\ \upsilon_{31}^0=\sqrt{\frac{C_{44}^0}{\rho}}, \\ \upsilon_{33}\approx\upsilon_{33}^0+\frac{(\Theta_{341}E_1)^2}{C_{33}^0-C_{44}^0-\Theta_{441}E_1},\ \upsilon_{33}^0=\sqrt{\frac{C_{33}^0}{\rho}}, \\ \upsilon_{32}\approx\upsilon_{32}^0-\frac{(\Theta_{341}E_1)^2}{C_{33}^0-C_{44}^0-\Theta_{441}E_1},\ \upsilon_{32}^0=\sqrt{\frac{C_{44}^0}{\rho}}. \end{gathered} \tag{91}$$

Let us consider the point group of symmetry $3m$ ($m\perp Y$). The EST is as follows:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & -C_{25}^0 & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & 0 & C_{25}^0 & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^0 & 0 & C_{25}^0 \\ -C_{25}^0 & C_{25}^0 & 0 & 0 & C_{44}^0 & 0 \\ 0 & 0 & 0 & C_{25}^0 & 0 & 0.5(C_{11}^0-C_{12}^0) \end{bmatrix}. \tag{92}$$

Under the application of the electric field E_3, the symmetry and the structure of this tensor remain unchanged. However, the existing tensor components undergo changes: $\Delta C_{11}=\Theta_{113}E_3$, $\Delta C_{33}=\Theta_{333}E_3$, $\Delta C_{12}=\Theta_{123}E_3$, $\Delta C_{13}=\Delta C_{23}=\Theta_{133}E_3$, $\Delta C_{44}=\Delta C_{55}=\Theta_{443}E_3$, $\Delta C_{66}=\Theta_{663}E_3$, $\Delta C_{46}=-\Theta_{153}E_3$, $C_{15}=\Theta_{153}E_3$, $C_{25}=-\Theta_{153}E_3$. At the propagation of AWs along the *X*-axis, the Christoffel tensor looks as:

$$\begin{bmatrix} C_{11}^0-\lambda & 0 & C_{15}^0+\Theta_{153}E_3 \\ 0 & C_{66}^0-\lambda & 0 \\ C_{15}^0+\Theta_{153}E_3 & 0 & C_{44}^0-\lambda \end{bmatrix}, \tag{93}$$

that result in AWs velocities:

$$\begin{gathered} \upsilon_{12}\approx\upsilon_{11}^0+\frac{\Theta_{663}E_3}{2\rho\upsilon_{12}^0},\ \upsilon_{12}^0=\sqrt{\frac{0.5(C_{11}^0-C_{12}^0)}{\rho}}, \\ \upsilon_{11}\approx\upsilon_{11}^0+\frac{\Theta_{113}E_3}{2\rho\upsilon_{11}^0}+\frac{(C_{15}^0-\Theta_{153}E_3)^2}{2\rho\upsilon_{11}^0(C_{11}^0-C_{44}^0+\Theta_{113}E_3-\Theta_{443}E_3)},\ \upsilon_{11}^0=\sqrt{\frac{C_{11}^0}{\rho}}, \\ \upsilon_{13}\approx\upsilon_{13}^0+\frac{\Theta_{443}E_3}{2\rho\upsilon_{13}^0}-\frac{(C_{25}^0-\Theta_{153}E_3)^2}{2\rho\upsilon_{13}^0(C_{11}^0-C_{44}^0+\Theta_{113}E_3-\Theta_{443}E_3)},\ \upsilon_{13}^0=\sqrt{\frac{C_{44}^0}{\rho}}, \end{gathered} \tag{94}$$

and non-orthogonality of AWs υ_{11} and υ_{13} defined as: $\tan 2\zeta_Y=\dfrac{2(C_{15}^0+\Theta_{153}E_3)}{C_{11}^0-C_{44}^0+(\Theta_{113}-\Theta_{443})E_3}$.

As one can see (Eq.(94)), in this case, both linear and quadratic terms exist in the increment of AWs velocities.

At the application of the electric field along the X-axis the symmetry is changed as $3m \rightarrow m$. The EST acquires the structure

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & C_{15} & 0 \\ C_{12} & C_{22} & C_{23} & 0 & C_{25} & 0 \\ C_{13} & C_{23} & C_{33} & 0 & C_{35} & 0 \\ 0 & 0 & 0 & C_{44} & 0 & C_{25} \\ C_{15} & C_{25} & C_{35} & 0 & C_{55} & 0 \\ 0 & 0 & 0 & C_{25} & 0 & C_{66} \end{bmatrix}. \tag{95}$$

The EST components depend on the electric field as: $\Delta C_{11} = \Theta_{111} E_1$, $\Delta C_{13} = \Theta_{131} E_1$, $\Delta C_{15} = \Theta_{151} E_1$, $\Delta C_{22} = \Theta_{221} E_1$, $\Delta C_{25} = \Theta_{251} E_1$, $\Delta C_{44} = \Theta_{441} E_1$, $\Delta C_{55} = -\Theta_{441} E_1$, $\Delta C_{66} = -(\Theta_{111} + \Theta_{221}) E_1$, $\Delta C_{12} = -(\Theta_{111} + \Theta_{221}) E_1$, $\Delta C_{23} = -\Theta_{131} E_1$, $\Delta C_{46} = (\Theta_{151} - \Theta_{251}) E_1$, $C_{35} = \Theta_{351} E_1$. The Christoffel tensor for the AWs propagating along the Z-axis is as follows:

$$\begin{bmatrix} C_{55} - \lambda & 0 & C_{35} \\ 0 & C_{44} - \lambda & 0 \\ C_{35} & 0 & C_{33}^0 - \lambda \end{bmatrix}. \tag{96}$$

The angle of non-orthogonality can be written as $\tan 2\zeta_Y = \dfrac{2\Theta_{351} E_1}{C_{55}^0 - C_{33}^0 - \Theta_{441} E_1}$. The AWs velocities are determined by the relations:

$$\begin{gathered} \upsilon_{31} \approx \upsilon_{31}^0 - \frac{\Theta_{441} E_1}{2\rho \upsilon_{31}^0} - \frac{(\Theta_{351} E_1)^2}{2\rho \upsilon_{31}^0 (C_{33}^0 - C_{44}^0 + \Theta_{441} E_1)}, \quad \upsilon_{32} \approx \upsilon_{32}^0 + \frac{\Theta_{441} E_1}{2\rho \upsilon_{32}^0}, \\ \upsilon_{33} \approx \upsilon_{33}^0 + \frac{(\Theta_{351} E_1)^2}{2\rho \upsilon_{13}^0 (C_{33}^0 - C_{44}^0 + \Theta_{441} E_1)}, \quad \upsilon_{31}^0 = \sqrt{\frac{C_{44}^0}{\rho}}, \quad \upsilon_{32}^0 = \sqrt{\frac{C_{44}^0}{\rho}}, \quad \upsilon_{33}^0 = \sqrt{\frac{C_{33}^0}{\rho}}. \end{gathered} \tag{97}$$

The last point group of symmetry under consideration in the present work is group 3. The EST for this group of symmetry is quite complicated, i.e.:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & C_{14}^0 & -C_{25}^0 & 0 \\ C_{12}^0 & C_{11}^0 & C_{13}^0 & -C_{14}^0 & C_{25}^0 & 0 \\ C_{13}^0 & C_{13}^0 & C_{33}^0 & 0 & 0 & 0 \\ C_{14}^0 & -C_{14}^0 & 0 & C_{44}^0 & 0 & C_{25}^0 \\ -C_{25}^0 & C_{25}^0 & 0 & 0 & C_{44}^0 & C_{14}^0 \\ 0 & 0 & 0 & C_{25}^0 & C_{14}^0 & 0.5(C_{11}^0 - C_{12}^0) \end{bmatrix}. \tag{98}$$

Under the electric field applied along the Z-axis, the symmetry and the tensor structure remain the same:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13} & C_{14} & -C_{25} & 0 \\ C_{12}^0 & C_{11}^0 & C_{13} & -C_{14} & C_{25} & 0 \\ C_{13} & C_{13} & C_{33}^0 & 0 & 0 & 0 \\ C_{14} & -C_{14} & 0 & C_{44} & 0 & C_{25} \\ -C_{25} & C_{25} & 0 & 0 & C_{44} & C_{14} \\ 0 & 0 & 0 & C_{25} & C_{14} & 0.5(C_{11}^0 - C_{12}^0) \end{bmatrix}, \tag{99}$$

while the most components are changed as: $\Delta C_{13} = \Delta C_{23} = \Theta_{133} E_3$, $\Delta C_{44} = \Delta C_{55} = \Theta_{443} E_3$,

$\Delta C_{14}=\Theta_{143}E_3$, $\Delta C_{24}=-\Theta_{143}E_3$, $\Delta C_{15}=\Theta_{153}E_3$, $\Delta C_{25}=-\Delta C_{15}=-\Theta_{153}E_3$ $\Delta C_{46}=-\Theta_{153}E_3$, $\Delta C_{56}=\Theta_{143}E_3$. When $\tan 3\varphi=(C_{15}^0-\Theta_{153}E_3)/(C_{14}^0+\Theta_{143}E_3)$, then $C_{25}(\varphi)=0$. At the propagation of the AWs along the *X*-axis, in this case, the Christoffel tensor is as follows

$$\begin{bmatrix} C_{11}^0-\lambda & 0 & 0 \\ 0 & C_{66}^0-\lambda & C_{56} \\ 0 & C_{56} & C_{44}^0-\lambda \end{bmatrix}. \tag{100}$$

The non-orthogonality angle is equal to $\tan 2\zeta_X=\dfrac{2(C_{14}^0+\Theta_{143}E_3)}{C_{66}^0-C_{44}^0+(2\Theta_{113}-\Theta_{443}-\Theta_{123})E_3}$. The velocities of the AWs are as follows:

$$\begin{aligned} &\upsilon_{11}=\sqrt{C_{11}/\rho}, \\ &\upsilon_{13}\approx\upsilon_{13}^0+\frac{\Theta_{443}E_3}{2\rho\upsilon_{13}^0}+\frac{(C_{14}^0+\Theta_{143}E_3)^2}{2\rho\upsilon_{13}^0(C_{44}^0+\Theta_{443}E_3-0.5(C_{11}^0-C_{12}^0))}, \quad \upsilon_{13}^0=\sqrt{\frac{C_{44}^0}{\rho}}, \\ &\upsilon_{12}\approx\upsilon_{12}^0-\frac{(C_{14}^0+\Theta_{143}E_3)^2}{2\rho\upsilon_{13}^0(C_{44}^0+\Theta_{443}E_3-0.5(C_{11}^0-C_{12}^0))}, \qquad \upsilon_{12}^0=\sqrt{\frac{0.5(C_{11}-C_{12})}{\rho}}. \end{aligned} \tag{101}$$

3.7. Monoclinic point groups of symmetry

Non-centrosymmetric monoclinic crystals include materials such as Triglycine sulfate, Rochelle salt, crystals of the $Sn_2P_2S_6$ family in the ferroelectric phase, etc. [32-34].

Let us consider the point group of symmetry 2 (2‖*Z*). Under the application of the electric field parallel to the *Z*-axis, the symmetry and the structure of the EST are not changed:

$$\begin{bmatrix} C_{11}^0 & C_{12}^0 & C_{13}^0 & 0 & 0 & C_{16}^0 \\ C_{12}^0 & C_{22}^0 & C_{23}^0 & 0 & 0 & C_{26}^0 \\ C_{13}^0 & C_{23}^0 & C_{33}^0 & 0 & 0 & C_{36}^0 \\ 0 & 0 & 0 & C_{44}^0 & C_{45}^0 & 0 \\ 0 & 0 & 0 & C_{45}^0 & C_{55}^0 & 0 \\ C_{16}^0 & C_{26}^0 & C_{36}^0 & 0 & 0 & C_{66}^0 \end{bmatrix} \to \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix}. \tag{102}$$

However, all components of this tensor acquire an increment due to the electro-elastic effect: $\Delta C_{11}=\Theta_{113}E_3$, $\Delta C_{12}=\Theta_{123}E_3$, $\Delta C_{13}=\Theta_{133}E_3$, $\Delta C_{16}=\Theta_{163}E_3$, $\Delta C_{22}=\Theta_{223}E_3$, $\Delta C_{23}=\Theta_{233}E_3$, $\Delta C_{26}=\Theta_{263}E_3$, $\Delta C_{33}=\Theta_{333}E_3$, $\Delta C_{36}=\Theta_{363}E_3$, $\Delta C_{44}=\Theta_{443}E_3$, $\Delta C_{55}=\Theta_{553}E_3$, $\Delta C_{66}=\Theta_{663}E_3$, $\Delta C_{45}=\Theta_{453}E_3$. At the propagation of the AWs along the X-axis, the Christoffel tensor can be written as:

$$\begin{bmatrix} C_{11}-\lambda & C_{16} & 0 \\ C_{16} & C_{66}-\lambda & 0 \\ 0 & 0 & C_{55}-\lambda \end{bmatrix}, \tag{103}$$

and the non-orthogonality angle is defined by the relation $\tan 2\zeta_Z=\dfrac{2(C_{16}^0+\Theta_{163}E_3)}{C_{11}^0-C_{66}^0+(\Theta_{113}-\Theta_{663})E_3}$. Therefore, the application of the electric field leads to the increment of the angle of non-orthogonality. The relations for AWs velocities are as follows:

$$\begin{aligned}
&\upsilon_{13}=\sqrt{\frac{C_{55}^{0}+\Theta_{553}E_{3}}{\rho}}\approx\upsilon_{13}^{0}+\frac{\Theta_{553}E_{3}}{2\rho\upsilon_{13}^{0}},\ \upsilon_{13}^{0}=\sqrt{\frac{C_{55}^{0}}{\rho}},\\
&\upsilon_{11}\approx\upsilon_{11}^{0}+\frac{\Theta_{113}E_{3}}{2\rho\upsilon_{11}^{0}}+\frac{(C_{16}^{0}+\Theta_{163}E_{3})^{2}}{2\rho\upsilon_{11}^{0}(C_{11}^{0}-C_{66}^{0}+(\Theta_{113}-\Theta_{663})E_{3})},\ \upsilon_{11}^{0}=\sqrt{\frac{C_{11}^{0}}{\rho}},\\
&\upsilon_{12}\approx\upsilon_{12}^{0}+\frac{\Theta_{663}E_{3}}{2\rho\upsilon_{12}^{0}}-\frac{(C_{16}^{0}+\Theta_{163}E_{3})^{2}}{2\rho\upsilon_{12}^{0}(C_{11}^{0}-C_{66}^{0}+(\Theta_{113}-\Theta_{663})E_{3})},\ \upsilon_{12}^{0}=\sqrt{\frac{C_{66}^{0}}{\rho}}.
\end{aligned}\tag{104}$$

For the point group of symmetry m ($m\perp Z$) under the application of the electric field along the X-axis, the structure of the EST is the same as it is determined by Eq. (102). The point group of symmetry under the electric field is not changed, while the components of the EST are changed in a similar manner as in point group 2. At the propagation of AWs along the Z-axis, the Christoffel tensor has the form:

$$\begin{bmatrix} C_{55}-\lambda & C_{45} & 0\\ C_{45} & C_{44}-\lambda & 0\\ 0 & 0 & C_{33}-\lambda \end{bmatrix},\tag{105}$$

while the angle of non-orthogonality can be written as: $\tan 2\zeta_{Z}=\dfrac{2(C_{45}^{0}+\Theta_{451}E_{1})}{C_{55}^{0}-C_{33}^{0}+(\Theta_{551}-\Theta_{331})E_{1}}$.

The relations for the AWs velocities can be written as:

$$\begin{aligned}
&\upsilon_{33}\approx\upsilon_{33}^{0}+\frac{\Theta_{331}E_{1}}{2\rho\upsilon_{33}^{0}}, && \upsilon_{33}^{0}=\sqrt{\frac{C_{33}^{0}}{\rho}},\\
&\upsilon_{13}\approx\upsilon_{13}^{0}+\frac{\Theta_{551}E_{1}}{2\rho\upsilon_{13}^{0}}+\frac{(C_{45}^{0}+\Theta_{451}E_{1})^{2}}{2\rho\upsilon_{13}^{0}(C_{55}^{0}-C_{44}^{0}+(\Theta_{551}-\Theta_{441})E_{1})}, && \upsilon_{13}^{0}=\sqrt{\frac{C_{55}^{0}}{\rho}},\\
&\upsilon_{23}\approx\upsilon_{23}^{0}+\frac{\Theta_{441}E_{1}}{2\rho\upsilon_{23}^{0}}-\frac{(C_{45}^{0}+\Theta_{451}E_{1})^{2}}{2\rho\upsilon_{23}^{0}(C_{55}^{0}-C_{44}^{0}+(\Theta_{551}-\Theta_{441})E_{1})}, && \upsilon_{23}^{0}=\sqrt{\frac{C_{44}^{0}}{\rho}}.
\end{aligned}\tag{106}$$

As one can see, the angle of non-orthogonality changes its value under the application of the electric field, and the increment in the AWs velocities is proportional to the linear and quadratic powers of the electric field.

4. Conclusions

As a result of the present work, we have obtained relations for AWs velocities for all point groups of crystal symmetry, except for the triclinic system under an electric field applied along the principal crystallographic directions. The Curie symmetry groups have also been considered. The main conclusions of the present work are as follows:

1. In many examples, the electro-elastic effect behaves similarly to the Pockels effect in optics. The rotation of the eigen vectors of the optical-frequency impermeability tensor caused by the linear electro-optic effect is analogous to the rotation of the eigen vectors of the Christoffel tensor in acoustics. However, in acoustics this rotation induces or increases the angle of non-orthogonality, whereas in optics the eigen waves remain orthogonal. Applying the electric field along high-symmetry axes in axial symmetry groups such as ∞2 and 622 leads to the same behavior of AWs velocities in acoustics as in optics, i.e., the linear term with respect to the electric field is zero. However, in acoustics this does not hold for point groups of symmetry 422 and 32.

2. In the symmetry groups 432, $\bar{4}3m$, and 23 under an electric field parallel to the <001> direction, the coordinate system in which the EST is written rotates. The same rotation occurs in the symmetry groups: $4mm$ under the electric field E_2; 422 under the electric fields E_3 and E_1; and $6mm$ under the electric field E_1. A 45° rotation appears in the point groups $\bar{4}2m$ and $\bar{4}3m$ under the electric field E_3, and a 90° rotation occurs in the symmetry group $\bar{6}2m$ about the Y axis under the electric field E_1.

3. If the non-orthogonality is induced by the electric field, the dependence of the respective AWs velocities on the electric field is quadratic; otherwise, it is linear. By applying an electric field, one can control the angle of non-orthogonality. The angle of non-orthogonality can be induced by an electric field in all high-symmetry groups, including Curie groups and groups of cubic, middle, and orthorhombic systems, whenever the application of the electric field results in an abrupt lowering of symmetry, at least to the monoclinic system. In this case, the velocities of the AWs that acquire the non-orthogonality depend quadratically on the electric field. Additionally, one can operate on the existing angle of non-orthogonality by using the electric field E_3 in the point groups of symmetry 4, $\bar{4}$, 3, $3m$, and 2, and by using the electric field E_1 in the groups of symmetry $3m$ and m. In cases where the angle of non-orthogonality increases under the electric field, the AWs velocities contain both linear and quadratic terms in the electric field strength.

Funding and acknowledgment. Authors acknowledge the National Research Foundation of Ukraine for financial support of the present study (project 2025.07/0388 “Revealing and systematic study of the effects of parametric crystal acoustics”).

Conflict of interest. Authors declare no conflict of interest.

Authors’ contribution. Authors contribution. Mys O. – derivation of acoustic wave velocities; Vasylkiv Yu. - tensors obtaining; Vlokh R. - conceptualization, data analysis, writing of the paper draft, supervision.

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doi: 0.3116/16091833/Ukr.J.Phys.Opt.2026.03106

Анотація. *У цій роботі розглянуто електропружний ефект для всіх груп симетрії Кюрі та точкових груп кристалів, за винятком триклінної системи, коли електричне поле прикладається вздовж головних кристалографічних напрямків. Виведено співвідношення для зміни швидкостей акустичних хвиль під дією електричного поля.*

Було показано, що в багатьох випадках електропружний ефект поводиться подібно до ефекту Поккельса в оптиці. Обертання власних векторів тензора діелектричної непроникності, спричинене лінійним електрооптичним ефектом, аналогічне обертанню власних векторів тензора Крістоффеля в акустиці. У деяких точкових групах симетрії прикладення електричного поля призводить до необхідності записування тензора пружної модулів в повернутій системі координат. Було виявлено, що якщо неортогональність індукується електричним полем, залежність відповідних швидкостей акустичних хвиль від електричного поля є квадратичною; в іншому випадку вона є лінійною. Прикладаючи електричне поле, можна контролювати кут неортогональності. Кут неортогональності може бути індукований електричним полем у всіх групах високої симетрії, включаючи групи Кюрі та групи кубічної, середніх та орторомбічної сингоній, коли прикладання електричного поля призводить до різкого зниження симетрії, принаймні до моноклінної сингонії. У цьому випадку швидкості акустичних хвиль, які набувають неортогональності, залежать квадратично від електричного поля. У випадках, коли кут неортогональності змінюється під дією електричного поля, швидкості акустичних хвиль містять як лінійні, так і квадратичні члени за напруженістю електричного поля.

Ключові слова: *електропружний ефект, швидкості акустичних хвиль, ефект Поккельса, симетрія*