

FLIP-FLOP NONINVARIANCE AND VOLTAGE-DRIVEN TOPOLOGICAL TRANSFORMATIONS OF THE SINGULAR VECTOR OPTICAL FIELDS GENERATED WITH A NEMATIC θ -CELL

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Abstract. Singular optical fields generated with the so-called θ -cell, assembled from substrates with linear (L) and circular (C) surface alignments for the nematic director, are studied theoretically using the Jones formalism and experimentally by measuring the azimuth and ellipticity maps of the exiting light beam. The theoretical consideration predicts topologically different distributions of the major axis of the polarization ellipse (called the polarization director) for the opposite directions (from L - to C -substrate and backward) of a normally propagating light beam, called the L - C and C - L geometries, respectively. We refer to this spatial nonreciprocity with respect to the direction of light propagation as the flip-flop noninvariance of the optical field. Due to the flip-flop noninvariance, the polarization order (topological strength) of the optical singularity in the center of the pattern is predicted to be $m=+1$ in the L - C , while $m=-1$ in the C - L geometry. It is concluded that the spatially distributed optical rotation coexisting with the linear birefringence is at the heart of the flip-flop noninvariance of the singular optical field generated by the θ -cell. The experiment confirms the theoretical prediction for three different settings of the incident light polarization (0° , 45° , 90°) with respect to the alignment direction of the L -substrate. Applying the voltage along the cell normal, we have studied the topological transformations of the exiting optical fields. The voltage-driven reorientation of the nematic director towards the cell normal is accompanied by the formation of discontinuities in the polarization director distributions. A special procedure is developed to account for discontinuities in the calculation of the polarization order. The experiment shows that the applied voltage reduces the optical rotation of the light polarization, thereby eliminating the difference between the optical fields of the beam observed in the two (L - C and C - L) geometries.

Keywords: vector beam, polarization order, nematic θ -cell, electric field, polarization mapping, topological defect, image processing, digital data processing

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1. Introduction

To be on the same page with a reader, we start the introduction with the definition of an optical vector beam. An optical vector beam refers to so-called structured light, owing to the spatially non-uniform distribution of the electric field polarization azimuth, typically centered on the beam axis [1,2], and is therefore called the cylindrical vector beam (CVB). Depending on the configuration of the polarization vector field around the beam axis, one

deals with at least two distinct optical vector field configurations of the CVBs corresponding to radial or azimuthal vector beams (RVB or AVB, respectively), which can be expressed as one of the Hermite-Gauss solutions of the Helmholtz equation [3]. It should be noted that the azimuth of the light polarization ellipse is not unidirectional; it is a double-headed, conventionally termed the director, in full analogy with the nematic director in liquid crystals. One of the key topological characteristics of the CVB is the polarization order (m), which can be either integer or half-integer, in direct analogy with the topological strength s of disclinations in nematics [4-6]. Both $|m|$ and $|s|$ are determined as the number of full (2π) rotations of the director on full traveling along a closed circuit encircling the singularity, with their sign being positive (+), if the director rotates in the traveling direction, and negative (-), if the director rotates in the opposite direction with respect to the traveling direction. Given that the topological strength of a singularity is a term traditionally used in the topology of vector singularities [4-6], the polarization order m will hereinafter also be termed the topological strength of the optical vector singularity (OVS). The OVSs with circular or radial polarization director fields are positive in sign by their topological strength, while the hyperbolic-like director fields are of negative topological strength. For the sake of clarity, it is worth noting that the Jones vector of a CVB is of the form [7]:

$$\vec{E} = \begin{bmatrix} \cos(m\varphi + \varphi_0) \\ \sin(m\varphi + \varphi_0) \end{bmatrix}, \quad (1)$$

where φ is the polar angle of a traveling point on a closed circuit, called the winding angle, and φ_0 is an initial offset angle. For $m = +1$ Eq. (1) describes the RVB if $\varphi_0 = 0$ and AVB if $\varphi_0 = \pi/2$. In a Cartesian coordinate system, for $m = -1$, Eq. 1 produces a family of hyperbolae with their axes mutually rotated with respect to each other for $\varphi_0 = 0$ and $\varphi_0 = \pi/2$ as will be shown below in Sec. 3.1.1.

The interest to the vector beams is caused by their application in the novel optical technologies. The vector beam is suitable for high-resolution microscopy with subwavelength tight focusing [8]. Additionally, vector beams remain stable when propagating through turbulence, preventing the loss of transmitted information [9]. These beams are quite effective for microparticle manipulation, enabling three-dimensional particle capture and serving as an optical "vacuum cleaner" to gather multiple particles [10]. Vector beams are also appealing for singular-optic communication, multi-channel encryption [11], and supporting high information capacity in optical communication [12].

One of the earliest methods for generating CVBs is the so-called intracavity mode selection [13,14]. A wide branch in the generation of vector beams is based on optical effects in anisotropic media, including birefringent [15] and dichroic materials [16,17]. A technique capable for the transformation of a linear polarization into vector-beam polarization using a $\lambda/2$ plate with spatially varying fast-axis orientations was designed in [18]. Extensive research has explored various methods for generating vector beams. Recent developments encompass traditional devices such as gratings [19,20], q-plates [21], interferometers [22], and spatial light modulators [23], as well as innovative technologies such as metasurfaces [24], nonlinear [25], and photonic crystals [26]. Recently, we have found that applying an electric field to the so-called nematic liquid-crystalline (LC) θ -cell with circular (C) and unidirectional (linear, L) director alignments on opposite substrates can change the

topological charge of the vortex beam from $l=1$ to $l=2$ [27]. It has been shown that the experimentally observed half-period shift of the interference fringes reflects the π optical phase shift in domains with opposite director-twist handedness on both sides of the nematic disclination separating these domains in the θ -cell [28].

The aim of this paper is to perform detailed theoretical and experimental studies of the singular optical vector fields generated with the θ -cell at different settings of the incident linear polarization with respect to the direction of the unidirectional rubbing of the L -substrate as well as at alternative directions of the light propagations from L - to C - (L - C geometry) and from C - to L -substrate (C - L geometry) without and with the applied electric field. Namely, we map the 2D distribution of the polarization state (azimuth and ellipticity), deriving it with the Jones matrix approach and experimentally measuring it using the imaging polarimeter. Owing to these results, we report on two remarkable findings. First, we demonstrate that the singular optical vector fields generated by the θ -cell are not invariant under the spatial flip-flop operation, which reverses the cell setting from L - C to C - L or vice versa. Our second finding is that the singular optical vector fields can be continuously transformed with the applied electric field to configurations with different topological strengths of optical vector singularities.

2. Experimental methods

The θ -cell was fabricated on the basis of thermotropic nematic liquid crystal 6CHBT (4-(trans4'-n-hexylcyclohexyl)-isothiocyanatobenzene, $C_6H_{13}C_7H_9PhNCS$) with the positive dielectric anisotropy $\Delta\epsilon = +6.5$ [29] and the threshold voltage $U_{th}=0.75$ V [30]. This LC material belongs to the homologous series of 4-(4-alkylcyclohexyl) isothiocyanatobenzenes (nCHBT). The alignment polymer (Kapton) layer was spin-coated over the transparent electrodes on the cell substrates for the planar alignment of the nematic director. The thickness (d) of the cell was fixed with cylindrical spacers dispersed in the UV-solidified glue and measured using the light interference technique to be $d=5.2$ μm . With the mechanical rubbing technique, the nematic director was aligned unidirectionally (linearly, L) and circularly (C) at the opposite bounding L - and C -substrates, respectively. Throughout the text, the linear rubbing direction is horizontal and along the X -axis of the Cartesian coordinate system (Fig. 1). The orientation of the incident linear polarization is defined with respect to the Y -axis, such that the polarizer orientation $P=0^\circ$ corresponds to the direction along the Y -axis, while the incident polarization, which is parallel to the X -axis and which is parallel to the rubbing direction of the L -substrate is denoted by $P=90^\circ$.

Our theoretical consideration presented in Sec. 3, reveals that the polarization state of the light wave exiting the θ -cell appears to be non-invariant with respect to the spatial flip-flop operation (180° rotation) around the Y -axis. In other words, it does matter from which side (of the L - or C -substrate) the light enters the cell. For this reason, to check the prediction, we measure the polarization state of the exiting light wave for the two geometries of the normally incident linearly polarized light wave. Namely, the L - C geometry corresponds to the light wave entering the cell through the L -substrate and propagating towards the C -substrate, while the opposite geometry, from the C - to L -substrate, is denoted as the C - L geometry.

The AC voltage of the root mean square value U (0, 1.5, and 3.0 V) and frequency $f=10$ kHz was applied to the transparent electrodes deposited on the inner surfaces of the glass cell substrates.

To build 2D maps of the light polarization ellipse parameters, namely, the orientation (azimuth) and the ellipticity angle of the exiting light wave, we employ our lab-made imaging

polarimetric setup (laser wavelength $\lambda = 0.6328 \mu\text{m}$) described earlier in Ref. [31]. The measurements were performed for three orientations of the polarizer $P = 0, 45$ and 90° counted from the Y -axis. During the measurements, the analyzer automatically rotates from 0° to 180° .

The original software was specifically developed to process the stack of registered images (each image corresponds to a specific analyzer's azimuth). First, the pre-processing of the images is performed, namely smoothing, using three types of spatial filters - box filter, Gaussian blur, and median filter. The user can choose the filter type and the kernel size. Then, one pixel with the same coordinates in each image's region of interest (ROI) is selected to form the experimental dependence of intensity on the analyzer's azimuth. After that, this dependence is fitted by the least-squares method using the Levenberg-Marquardt algorithm, which yields the fitting parameters, from which the azimuth θ and ellipticity angle ε of the polarization state are calculated. The procedure is repeated for each pixels of ROI, resulting in two maps of azimuth and ellipticity angle. These maps are rendered in color using specially designed palettes. Based on the azimuth map, the vector map of the major-axis orientations of the polarization ellipses and the vector streamlines map of polarization azimuth are plotted. The vector map of the major-axis orientations of the ellipses is superimposed on the color maps of the azimuth and ellipticity angles. Both the azimuth and ellipticity angle maps are used to plot a vector map of polarization ellipses. The plotted maps are presented in Section 4.

3. Optical vector field generated with the θ -cell: Jones calculus

Due to the competing planar circular (C) and unidirectional linear (L) director alignments at the bounding C - and L -substrates (Fig. 1), the θ -cell optically behaves as a twisted nematic (TN) with adiabatic rotation of the light polarization.

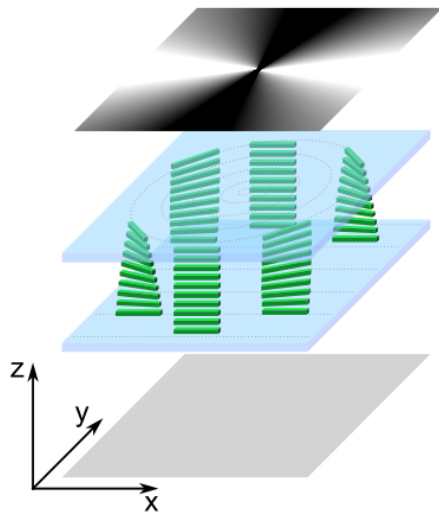


Fig. 1. Structure of the θ -cell (schematically).

The Jones vector of the light wave exiting such a TN cell with the director at the incoming substrate oriented at the azimuthal (in-plane) α with respect to the X -axis of the Cartesian coordinate system, is of the form

$$\begin{aligned} \vec{E} &= R(\alpha)R(\omega)J_N R(-\alpha)\vec{E}_{in} \\ &= \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{bmatrix} \begin{bmatrix} e^{-i\frac{2\pi}{\lambda}n_e d} & 0 \\ 0 & e^{-i\frac{2\pi}{\lambda}n_o d} \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \vec{E}_{in}, \end{aligned} \quad (2)$$

where $R(\alpha)$ is the rotation matrix by the angle α , $R(\omega)J_N$ is the Jones matrix of a TN cell in the adiabatic regime of light propagation, where $R(\omega)$ is the rotation matrix by the angle ω by which the director twists between the bounding substrates and J_N is the Jones matrix of a untwisted nematic slab [27,28,32-34] of the thickness d , with the extraordinary and ordinary refractive indices n_e and n_o , respectively and the director being parallel to the X -axis, $R(-\alpha)$ is the matrix inverse to $R(\alpha)$, and finally, $\vec{E}_{in}$ is the Jones vector of the incoming light. Such an alignment corresponds to choosing of the X -axis along the rubbing direction of the L -substrate. For the linearly polarized incoming light waves with the light polarization parallel and perpendicular to the X -axis whose Jones vectors, respectively, are $\vec{E}_{in}^{\parallel} = [1 \ 0]^T$ and $\vec{E}_{in}^{\perp} = [0 \ 1]^T$ (the uppercase index T denotes the transpose operation such that $\vec{E}_{in}^{\parallel}$ and $\vec{E}_{in}^{\perp}$ are vector-columns), from Eq. (2) we find the Jones vectors of the exiting light waves for the two settings of the incoming polarization

$$\vec{E}^{\parallel} = \begin{bmatrix} \cos(\alpha + \omega)\cos(\alpha)e^{-i\frac{2\pi}{\lambda}n_e d} + \sin(\alpha + \omega)\sin(\alpha)e^{-i\frac{2\pi}{\lambda}n_o d} \\ \sin(\alpha + \omega)\cos(\alpha)e^{-i\frac{2\pi}{\lambda}n_e d} - \cos(\alpha + \omega)\sin(\alpha)e^{-i\frac{2\pi}{\lambda}n_o d} \end{bmatrix}, \quad (3)$$

$$\vec{E}^{\perp} = \begin{bmatrix} \cos(\alpha + \omega)\sin(\alpha)e^{-i\frac{2\pi}{\lambda}n_e d} - \sin(\alpha + \omega)\cos(\alpha)e^{-i\frac{2\pi}{\lambda}n_o d} \\ \sin(\alpha + \omega)\sin(\alpha)e^{-i\frac{2\pi}{\lambda}n_e d} + \cos(\alpha + \omega)\cos(\alpha)e^{-i\frac{2\pi}{\lambda}n_o d} \end{bmatrix} \quad (4)$$

3.1. L - C geometry

For the case, when the incoming substrate is the L -substrate (L - C geometry) with the rubbing direction of the L -substrate parallel to the X -axis one has $\alpha = 0$ and then from Eqs. (3) and (4), we find

$$\vec{E}_{L-C}^{\parallel} = e^{-i\frac{2\pi}{\lambda}n_e d} \begin{bmatrix} \cos(\omega) \\ \sin(\omega) \end{bmatrix}, \quad (5)$$

$$\vec{E}_{L-C}^{\perp} = e^{-i\frac{2\pi}{\lambda}n_o d} \begin{bmatrix} -\sin(\omega) \\ \cos(\omega) \end{bmatrix}. \quad (6)$$

Therefore, for the L - C geometry, when the incoming linear light polarization is parallel to the rubbing direction of the L -substrate, according to Eq. (5), the light wave exits the cell as the extraordinary wave with the refractive index n_e , while, according to Eq. (6), the exiting wave is the ordinary wave with the refractive index n_o , when the incoming light polarization is perpendicular to the rubbing direction of the L -substrate. Fig. 2 illustrates that in the L - C geometry, the twist angle of the director between the substrates is $\omega = \varphi - \pi/2$ for probed points in the I -st and II -nd quadrants, defined by the polar angle $0 \leq \varphi \leq \pi$ of the radius vector $\vec{r}$ within the polar and Cartesian XY coordinate systems with the same origin. For the III -rd and IV -th quadrants, where $\pi \leq \varphi \leq 2\pi$, the angle of the director twist is $\omega = \varphi - 3\pi/2$. Equivalently, the III -rd and IV -th quadrants can be defined by $-\pi \leq \varphi \leq 0$, and then $\omega = \varphi + \pi/2$. The signs “+” and “-” in Fig. 2 indicate that the director

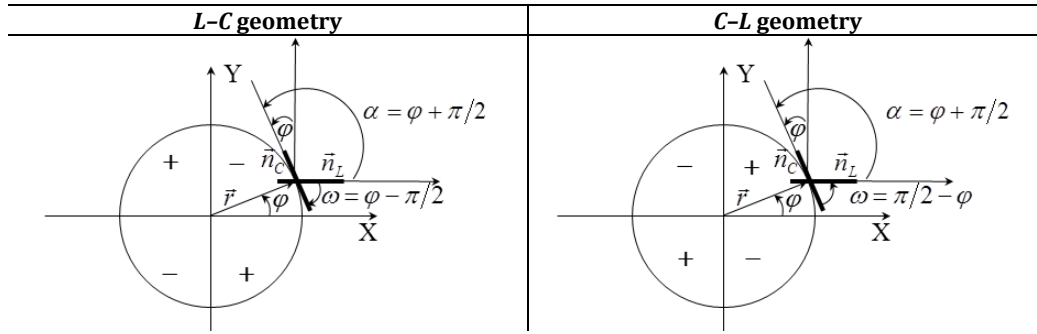


Fig. 2. To the definition of the angles α , ω and φ . Thick lines depict the director orientations $\vec{n}_L$ and $\vec{n}_C$ at the L - and C -substrates, respectively; $\vec{r}$ is the radius vector at the running angle φ of the polar coordinate system.

rotation from the L - to C -substrate is clockwise in the the I -st and III -rd quadrants and counterclockwise in the II -nd and IV -th quadrants. Such behavior of the optical rotation reflects the reciprocity of the optical structural rotation of the light polarization plane.

For the incoming light polarization $\vec{P} \parallel \vec{X}$ ($\vec{E}_{in}^{\parallel} = [1 \ 0]^T$), which corresponds to the polarizer setting $P=90^\circ$ in the experimental set-up and maps shown below the substitution of $\omega = \varphi - \pi/2$ in Eq. (5) gives the Jones vector of the exiting light wave with the polarization azimuth

$$\theta = \begin{cases} \varphi - \frac{\pi}{2}, & 0 < \varphi < \pi \\ \varphi - \frac{3\pi}{2}, & \pi < \varphi < 2\pi \end{cases}. \quad (7)$$

Eq. (7) corresponds to the family of concentric circles (Fig. 3a) as streamlines for the polarization vectors of the exiting light waves.

For the incoming light polarization $\vec{P} \perp \vec{X}$ ($\vec{E}_{in}^{\perp} = [0 \ 1]^T$), which corresponds to the polarizer setting $P=0^\circ$ in the experimental set-up and maps shown in Fig. 4 (for $P=90^\circ$ in Fig. 5) the azimuth of the Jones vector for $0 < \varphi < 2\pi$ is

$$\theta = \varphi, \quad (8)$$

which corresponds to the streamlines in the form of a family of radial straight lines (Fig. 3b).

It is worth noting that the transformation between the circular and radial vector fields is a well-known property of vector optical singularities [35-37]. Topologically they belong to the same class of singularities, described in the 2D space by the same homotopic group

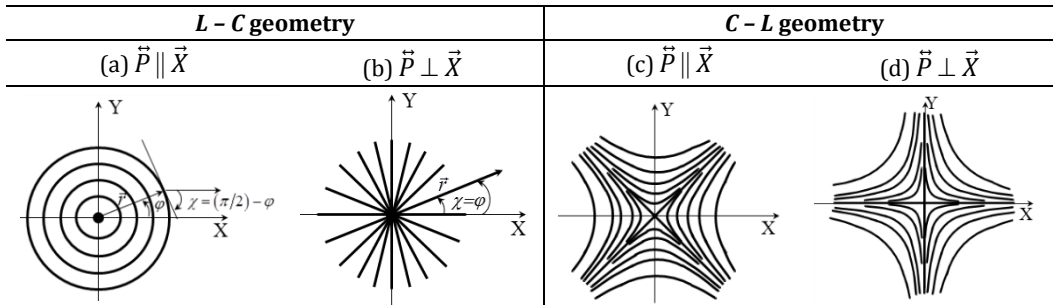


Fig. 3. Polarization streamlines for the exiting light waves for L - C (schematically): (a) $\vec{P} \parallel \vec{X}$ ($P=90^\circ$), (b) $\vec{P} \perp \vec{X}$ ($P=0^\circ$), and C - L geometries for the phase retardation $\cos \Delta\Phi = 0$: (c) $\vec{P} \parallel \vec{X}$ ($P=0^\circ$), (d) $\vec{P} \perp \vec{X}$ ($P=90^\circ$).

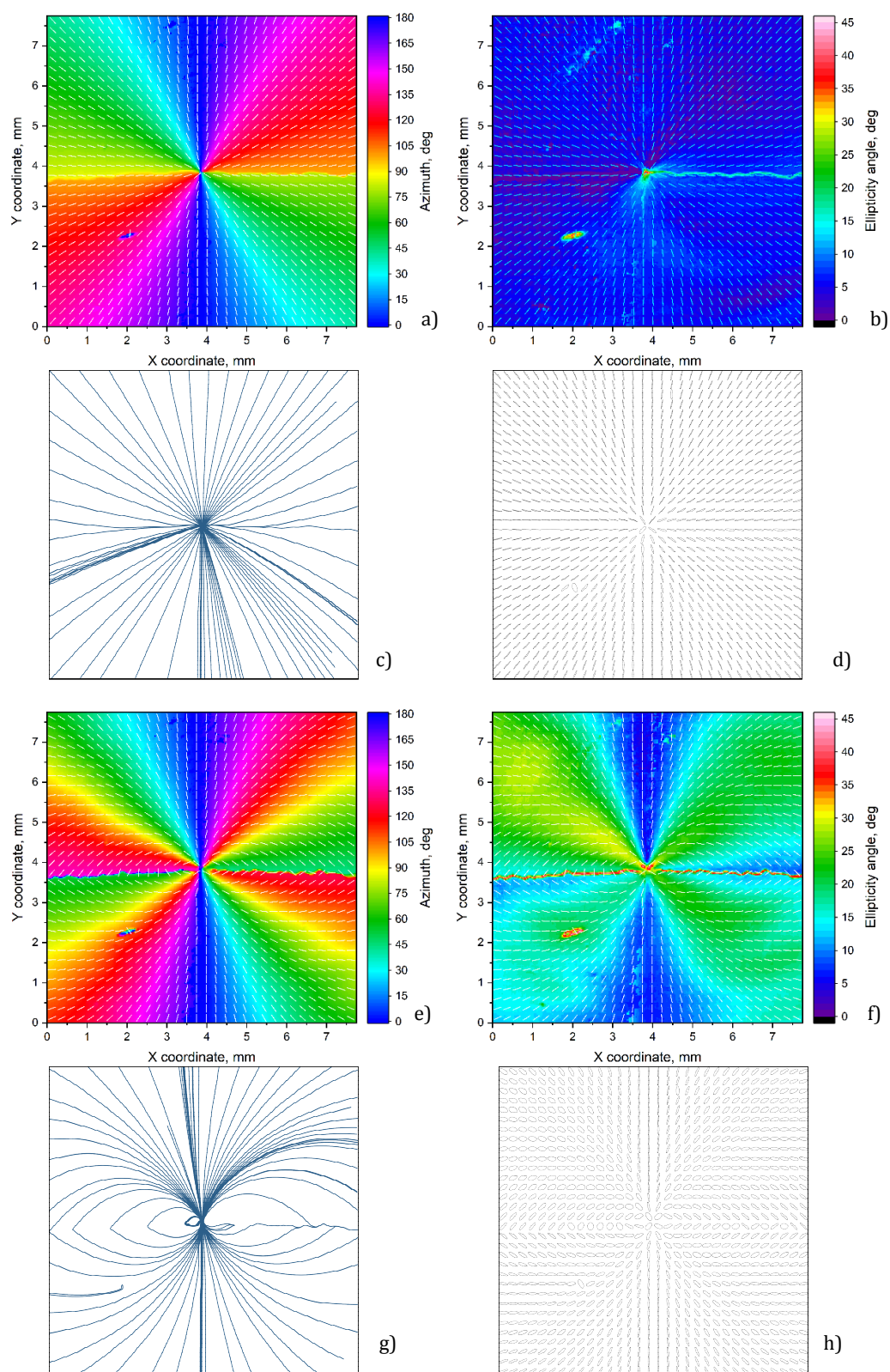


Fig. 4. Maps of the azimuth... (beginning, continued on the next page)

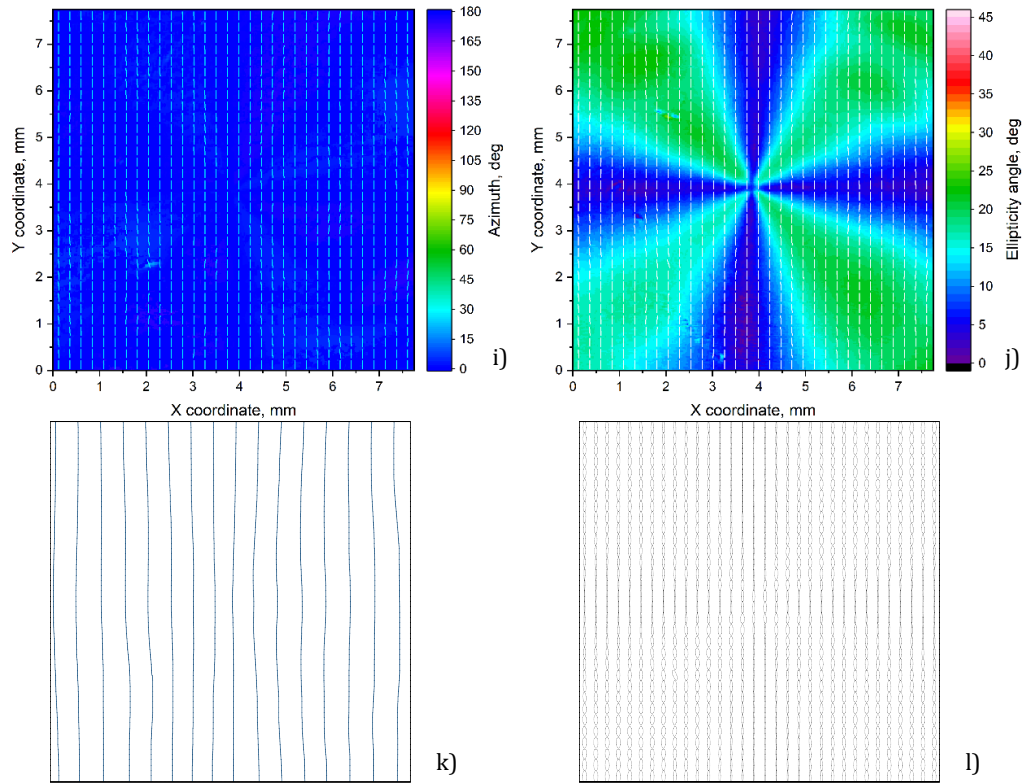


Fig. 4. Maps of the azimuth (a,e,i), ellipticity angle (b,f,j), azimuth streamlines (c,g,k) and polarization ellipses (d,h,l) in the beam cross-section for the light emergent from the L - C θ -cell. The incident linearly polarized light beam has a $P=0$ -degree orientation. $U = 0$ (a,b,c,d); 1.5 (e,f,g,h) and 3.0 V (i,j,k,l). The orientation of the major axes of the polarization ellipses is indicated by the white line segments.

$\pi_1(RP^1) = \mathbb{Z}$, where $RP^1 = S^1$ is the order parameter degeneracy (OPD) space, a circle S^1 and $\mathbb{Z}$ stands for the group of integers, whose elements correspond to the winding number of the polarization vector along a contour drawn around the singularity and consequently, they can be transformed into each other by continuous transformations of their vector fields. This topological property is in direct analogy with the topology of the director fields around disclinations in nematics, which can be continuously transformed from a circular to a radial configuration for a disclination of topological strength $s=+1$, as well as to those corresponding to $s=-1$ [4-6].

3.2. C - L geometry

When the light enters the cell through the C -substrate (C - L geometry), then $\alpha = \varphi + (\pi/2)$. For the rubbing direction of the L -substrate still being parallel to the X -axis from Eqs. (5) and (6), we find

$$\vec{E}_{C-L}^{\parallel} = \begin{bmatrix} \sin(\varphi + \omega) \sin(\varphi) e^{-i\frac{2\pi}{\lambda} n_e d} + \cos(\varphi + \omega) \sin(\varphi) e^{-i\frac{2\pi}{\lambda} n_o d} \\ -\cos(\varphi + \omega) \sin(\varphi) e^{-i\frac{2\pi}{\lambda} n_e d} + \sin(\varphi + \omega) \cos(\varphi) e^{-i\frac{2\pi}{\lambda} n_o d} \end{bmatrix}, \quad (9)$$

$$\vec{E}_{C-L}^{\perp} = \begin{bmatrix} -\sin(\varphi + \omega) \cos(\varphi) e^{-i\frac{2\pi}{\lambda} n_e d} + \cos(\varphi + \omega) \sin(\varphi) e^{-i\frac{2\pi}{\lambda} n_o d} \\ \cos(\varphi + \omega) \cos(\varphi) e^{-i\frac{2\pi}{\lambda} n_e d} + \sin(\varphi + \omega) \sin(\varphi) e^{-i\frac{2\pi}{\lambda} n_o d} \end{bmatrix}. \quad (10)$$

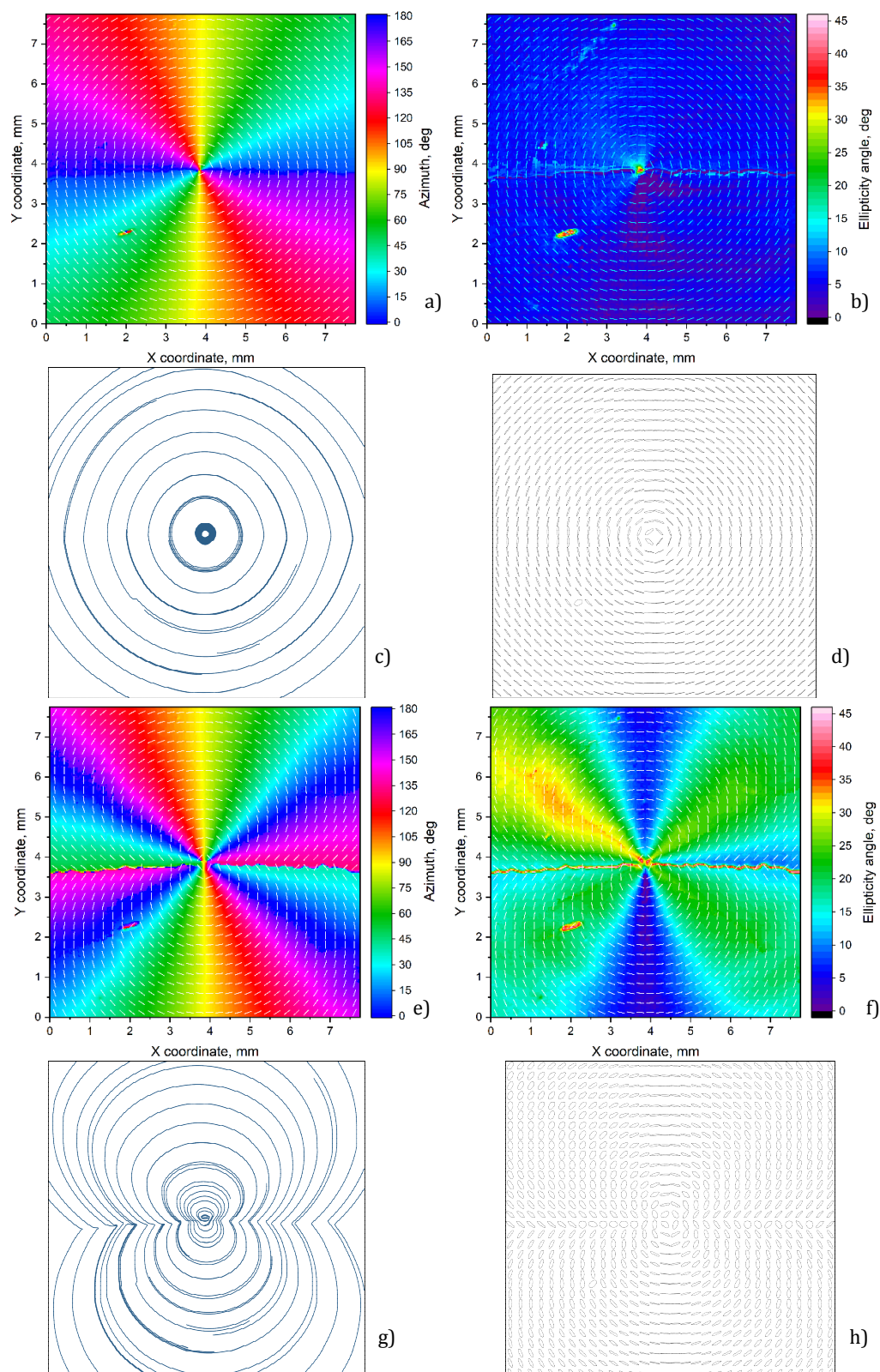


Fig. 5. Maps of the azimuth... (beginning, continued on the next page)

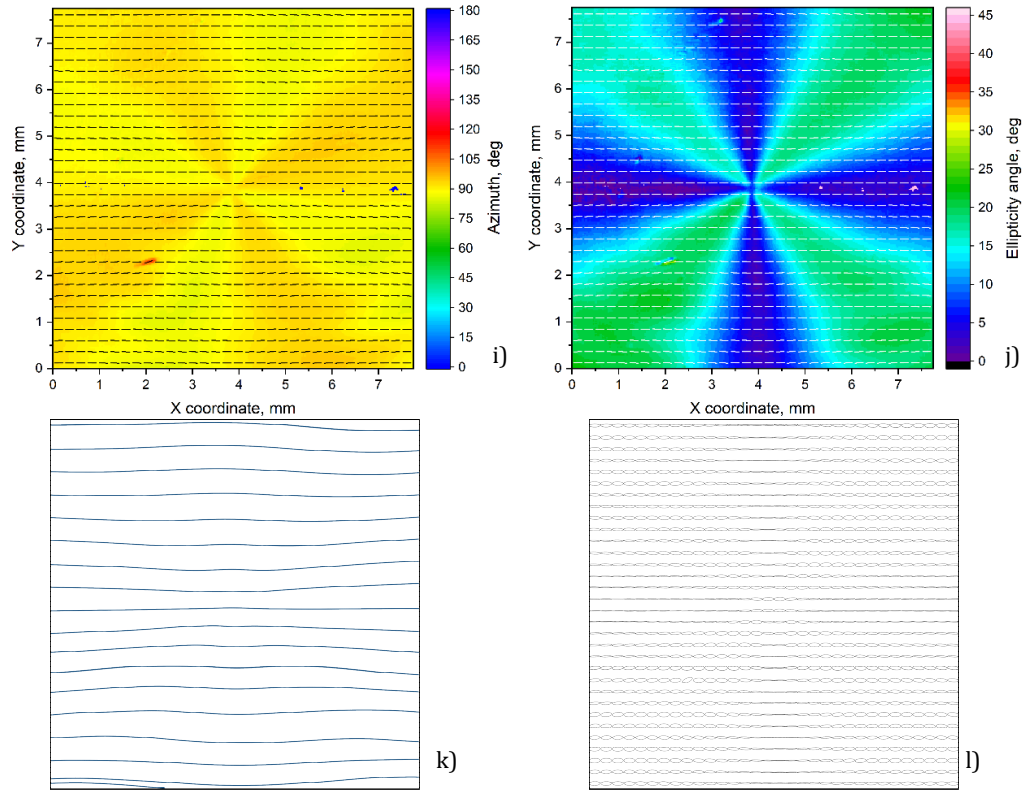


Fig. 5. Maps of the azimuth (a,e,i), ellipticity angle (b,f,j), azimuth streamlines (c,g,k) and polarization ellipses (d,h,l) in the beam cross-section for the light emergent from the *L-C* cell. The incident linearly polarized light beam has a $P=90$ -degree orientation. $U = 0$ (a,b,c,d); 1.5 (e,f,g,h) and 3.0 V (i,j,k,l). The orientation of the major axes of the polarization ellipses is indicated by the white line segments.

For the *C-L* geometry from Fig. 2b $\omega = \pi/2 - \varphi$ for probed points in the *I*-st and *II*-nd quadrants, defined by the polar angle $0 \leq \varphi \leq \pi$ of the radius vector $\vec{r}$ within the polar and Cartesian *XY* coordinate systems. For the *III*-rd and *IV*-th quadrants, where $\pi \leq \varphi \leq 2\pi$, the angle of the director twist is $\omega = 3\pi/2 - \varphi$. Comparison of Figs. 2a and 2b shows that the flipping/flopping of the cell from the *L-C* to *C-L* geometry, or backward, results in the alteration of sign of rotation of the nematic director between the two substrates, which in turn leads to the corresponding alteration of the sign of rotation of the light polarization plane and is in agreement with the property of reciprocity for structural rotation of the polarization plane due to the director twist [32,38], which is illustrated in Fig. 2. After the substitution of $\omega = \pi/2 - \varphi$ in Eq. (9) for $\vec{P} \parallel \vec{X}$ in the *I*-st and *II*-nd quadrants, where $0 \leq \varphi \leq \pi$, the Jones vector of the exiting waves becomes of the form

$$\vec{E}_{C-L} = \begin{bmatrix} e^{-i\frac{2\pi}{\lambda}n_d d} \sin(\varphi) \\ e^{-i\frac{2\pi}{\lambda}n_o d} \cos(\varphi) \end{bmatrix}. \quad (11)$$

For the intermediate (different from $\varphi = 0; \pi/2$) directions, the exiting waves are elliptically polarized with the azimuth of the major axis of the polarization ellipse θ and ellipticity angle ϵ , defined as

$$\tan(2\theta) = \frac{\chi + \chi^*}{1 - |\chi|^2} = \frac{2\text{Re}(\chi)}{1 - |\chi|^2}, \quad (12)$$

$$\sin(2\varepsilon) = -i \frac{\chi - \chi^*}{1 + |\chi|^2} = \frac{2\text{Im}(\chi)}{1 + |\chi|^2}, \quad (13)$$

where $\chi = E_y / E_x$ is the so-called complex polarization variable, χ^* is its complex conjugate and $|\chi|$ is its absolute value. From Eq. (11) one finds $\chi = (\cos\Delta\Phi + i\sin\Delta\Phi)\cotan\varphi$, $\chi^* = (\cos\Delta\Phi - i\sin\Delta\Phi)\cotan\varphi$, $|\chi|^2 = \chi\chi^* = [\text{Re}(\chi)]^2 + [\text{Im}(\chi)]^2 = \cotan^2\varphi$, and thus

$$\tan 2\theta = \tan 2(\varphi \pm \pi/2) \cos\Delta\Phi, \quad (14)$$

$$\sin 2\varepsilon = \sin 2\varphi \sin\Delta\Phi. \quad (15)$$

Eq. (11) shows that for the C - L geometry, the exiting light wave is the linearly polarized ordinary wave with the refractive index n_o and its electric field vector parallel to the Y axis for $\varphi = 0$ and $\varphi = \pi$, i.e. in the direction along the X axis, while it is the linearly polarized extraordinary wave with the refractive index n_e and its electric field vector parallel to the X axis when $\varphi = \pm\pi/2$, i.e. in the direction along the Y axis. Equivalently the same conclusion follows from Eq. (11). At $\varphi = \pi/4$ one finds $\chi|_{\varphi=\pi/4} = \cos\Delta\Phi + i\sin\Delta\Phi$, $\chi^*|_{\varphi=\pi/4} = \cos\Delta\Phi - i\sin\Delta\Phi$, $|\chi|^2|_{\varphi=\pi/4} = 1$; then, from Eq. (11) one has $\tan 2\theta|_{\varphi=\pi/4} = [2\cos\Delta\Phi/(1-1)] \rightarrow \infty$ and thus $\theta|_{\varphi=\pi/4} = \pi/4$. Similarly for $\varphi = (\pi/2) + (\pi/4)$ one finds $\theta|_{\varphi=(\pi/2)+(\pi/4)} = -\pi/4$. The θ -streamlines for $\vec{P} \parallel \vec{X}$ in the C - L geometry are shown in Fig. 3c.

Observe that, according to the data presented in Fig. 3, the singular optical fields generated with the θ -cell are not invariant under the space flip-flop operation. By reversing the side from which the light enters the cell, the flip-flop operation alters the direction of the light propagation. For the reader's convenience, we recall that the cell setting, at which the normally incident light propagates from the L - to C -substrate is called the L - C geometry, and the reverse direction, from C - to L -substrate refers to the C - L geometry. In the L - C geometry, the θ -streamlines are circular for $\vec{P} \parallel \vec{X}$ (Fig. 3a) and radial for $\vec{P} \perp \vec{X}$ (Fig. 3a), both corresponding to the positive topological strength $s = +1$ of the vector optical singularity, while in the C - L geometry the hyperbolic configurations (Figs. 3c, d) of θ -streamlines reveal the negative $s = -1$ topological strength. Optical singularities with the circular/radial polarization director fields of positive topological strength and with the hyperbolic polarization director fields of negative topological strength, the so-called V-points were generated in [37], using the Spatial Light Modulator.

It is worth highlighting, that for the C - L geometry according to Eqs. (11-15), within the given setting $\vec{P} \parallel \vec{X}$ ($\vec{P} \perp \vec{X}$) the polarization state of the exiting waves varies from being linearly polarized extraordinary (ordinary) light wave along the X axis at $\varphi = 0; \pi$ to an elliptically polarized wave with the polarization state (azimuth and ellipticity) depending on the phase retardation $\Delta\Phi = 2\pi\Delta n d/\lambda$ at the intermediate angles $\varphi \neq 0, \pi, \pm\pi/2$ toward the linearly polarized ordinary (extraordinary) light wave along the Y axis at $\varphi = \pm\pi/2$. Such a behavior is distinctly different from that for the L - C geometry, for which, according to

Eqs. (5,6), at any φ , the exiting wave is linearly polarized extraordinary wave for $\vec{P} \parallel \vec{X}$ and ordinary wave for $\vec{P} \perp \vec{X}$. Therefore, we are led to conclude that in addition to the difference in the distribution of the azimuth of the polarization director (Fig. 3), the L - C and C - L geometries also differ in the angular distribution of the ellipticity. Such a remarkable property of the θ -cell is yet another manifestation of its spatial flip-flop noninvariance. Indeed, in the L - C geometry the ellipticity of the exiting wave is zero at any φ , while for the C - L geometry the ellipticity of the exiting wave is a function of the probing angle φ and the linear phase retardation $\Delta\Phi$.

The flip-flop noninvariance of the θ -cell is easily understood if one takes into account that in each diametric direction, the θ -cell behaves as a TN cell obeying the adiabatic Mauguine regime of light propagation. For a conventional TN cell with the twisting angle of the director $0 \leq \omega \leq \pi/2$, the exiting light wave is a *linearly polarized extraordinary* wave if the incident light polarization is parallel to the director at the entrance substrate, i.e. the angle between the polarization and the director at the cell entrance is $\psi = 0$. At $\psi = \pi/2$, the light exists as an *ordinary linearly polarized* wave. At intermediate settings $0 < \psi < \pi/2$, the exiting light wave is *elliptically polarized* with the ellipticity proportional to $\sin\Delta\Phi$. Coming back to the θ -cell one finds that in the L - C geometry for a given setting $\vec{P} \parallel \vec{X}$ ($\vec{P} \perp \vec{X}$) one has $\psi = 0$ ($\psi = \pi/2$) for any running angle φ , but not the intermediate values of ψ , and, consequently, in the L - C geometry, the exiting wave remains linearly polarized for any running φ at $\psi = 0$ or $\psi = \pi/2$. On the contrary, in the C - L geometry, due to the circular configuration of the director alignment at the entrance C -substrate, the angle ψ between the nematic director and incident light polarization is a function of the running angle φ , namely $\psi = \varphi + \pi/2$, thereby ψ takes all the intermediate values for different φ . Intermediate values $\psi \neq 0, \pm\pi/2, \pi$ imply elliptical polarization of the exiting light wave at $\Delta\Phi \neq \pi m$ for $m \in \mathbb{Z}$ being an integer number. This key difference between the C - L and L - C geometries is at the heart of the flip-flop noninvariance of the θ -cell.

According to Eq. (15) for the C - L geometry, at $\Delta\Phi = m(\pi/2)$, and $\varphi = k(\pi/4)$ with m and k being odd integer numbers, one has $\sin 2\varepsilon = 1$, which indicates that the exiting waves are circularly polarized, while at even values of m for any φ one has $\sin 2\varepsilon = 0$ and, thus, the exiting waves are linearly polarized. Our measurements show that for the θ -cell studied in this paper, at zero applied electric field, the measured ellipticity angle ε is almost zero (Figs. 7,8), which indicates that $\sin \Delta\Phi \approx 0$ and thus the exiting waves are quasi-linearly polarized. This is the very situation depicted in Figs. 3c, d. With our universal polarimeter, the value of $\Delta\Phi$ for the studied θ -cell can be measured sending on the cell a circularly polarized incident light wave, measuring the ellipticity of the exiting wave along the Y -axis, where the director field is not twisted and then calculating the nematic birefringence. These measurements give the birefringence value $\Delta n \approx 0.13$ for the used nematic, which in turn, for the sample thickness $d = 5.2\mu$ and light wavelength $\lambda = 0.6328\mu$ results in $\Delta\Phi \approx 2\pi \times 1.068$ and thus, the calculated value $\sin \Delta\Phi \approx 0.1$ is in agreement with the corresponding statement given above.

The value of $\Delta\Phi$ can be changed by varying the sample thickness d , wavelength λ , or birefringence Δn . For the θ -cell, the variation of d requires preparation of cells of different discrete thicknesses and, consequently, cannot be continuous. The variation of the light wavelength requires different laser(-s); will be accompanied by the variation of Δn and thus, implies some experimental complications. In turn, Δn can be changed either by varying the temperature or applying electric/magnetic field to the cell. In the next section, we study the evolution the polarization maps under the electric field applied along the cell normal.

4. θ -cell under the electric field

4.1 L - C geometry

In the L - C geometry, the normally incident laser beam enters the θ -cell through the unidirectionally rubbed L -substrate and exits from the circularly rubbed C -substrate. Fig. 4a-d, shows that at zero applied voltage $U=0$ V and $P=0^\circ$ ($\vec{E}_{in} \parallel \vec{V}$), the ellipticity angle is almost zero (Fig. 4b,d), corresponding to the exiting linearly polarized RVB (Fig. 4a,c), which is in agreement with the theoretical predictions schematically depicted in Fig. 3b. In the case of linear polarization, which is everywhere restricted to the X - Y plane, the consideration reduces to the 2D plane, perpendicular to the disclination line and consequently the OPD space of the polarization director reduces to RP^1 , which geometrically (by the shape) is a circle S^1 of the dimensionality 1D such that the central defect is akin to a V-point [37]. Notice that because the polarization vector is a director, the jump of the optical phase by π on the X -axis does not show up as a defect on the azimuth map (Fig. 4a,c) and consequently the central singularity exists as an isolated defect.

The application of the voltage $U=1.5$ V, which is well above the threshold voltage $U_{th}=0.75$ V, drastically transforms the configuration of the optical field behind the cell (see the azimuth and ellipticity distributions in Figs. 4e-h), bringing at least two new qualitative features.

First, at the applied voltage $U=1.5$ V, the exiting light becomes elliptically polarized, which indicates that the OPD space of the optical field exiting the cell is not anymore restricted to the RP^1 , but is lifted to the 2D dimensionality, which in terms of the light polarization director is the projective plane RP^2 , in analogy with the OPD space for the nematic director, though also being not restricted to this analogy because of the new-coming polarization feature, the ellipticity. The OPD space for the elliptically polarized light is the Poincaré sphere. As a result of the transformation of the OPD space from RP^1 at $U=0$ V to the Poincaré sphere at $U=1.5$ V, the notion of the topological strength of the singularity at the center of the optical pattern is not of the same topological nature in these two cases. Namely, these singularities are formed in different OPD spaces. An analogy with nematic disclinations could be useful for the sake of clarity. In nematics, a line disclination with the radial or circular director distribution is characterized by the topological strength $s=+1$, if the director is restricted to stay perpendicular to the disclination axis. This imposed restriction reduces the dimensionality of the physical space under consideration to the 2D plane, which is perpendicular to the disclination axis, and the OPD space for the nematic director reduces to the RP^1 , which is geometrically a circle S^1 , similarly to the case of the radial distribution of the light polarization director alluded to above. However, once the nematic director is allowed to escape into the third dimension (into the 3D physical space), its OPD space is

lifted to the projective plane RP^2 , the disclination core becomes non-singular and, strictly speaking, the notion of the topological strength of the defect should be applied with care. The problem is that in the 3D physical space with the RP^2 OPD space, the topological charge of a disclination of integer strength is zero, implying that the director field can be transformed into the uniform state, eliminating the disclination by continuous transformations, for which the topological strength of the defect loses its physical sense. Such a transformation from a defect to a uniform (defect-free) state might lead to the impression that the law of conservation of topological strength is violated. However, it should be noted that the law of conservation of topological strength is applicable to the transformations within the same OPD space and deserves additional consideration when the OPD space is lifted to higher dimensionality. Therefore, as such, the topological strength of a disclination defined within the RP^1 OPD space topologically is not exactly equivalent to that defined within the RP^2 OPD space. Coming back to the light singularities, we are led to conclude that similarly the polarization order m of the singularity at the center of the pattern formed by linearly polarized waves within the OPD space RP^1 at $U=0$ V is not of the same nature as that for the case of the elliptically polarized waves within the Poincaré sphere as the OPD space at 1.5 V.

Second, in the case of zero applied voltage $U=0$ V, the phase π -jump along the X -axis does not result in a defect line and, thus, the singularity at the center of the radial distribution shown in Fig. 4c (azimuth map) is an isolated defect in the field of the polarization director. A closed loop around the core of the central defect does not cross any discontinuity, and the polarization order can be calculated as [4,5,38]

$$m = \frac{1}{2\pi} \int_0^{2\pi} \frac{d\theta(\varphi)}{d\varphi} d\varphi, \quad (16)$$

which, for the case of $\theta(\varphi)$ being a continuous function, simplifies to the form

$$m = \frac{\Delta\theta}{2\pi}. \quad (17)$$

For the radial distribution shown in Fig. 4c, in accordance with Eq.(8) one has $\theta(\varphi) = \varphi$, $\Delta\theta = \theta(2\pi) - \theta(0) = 2\pi$, with $\theta(0) = 0$, $\theta(2\pi) = 2\pi$, and according to Eq. (17), one finds $m = +1$.

On the contrary, at the applied voltage $U=1.5$ V, the measured polarization director shown in Fig. 4g approaches the X -axis at tilt angles of $\theta = \pm\pi/4$ (Fig. 4e,g), respectively on its sides, consequently containing a discontinuity jump $|\Delta\theta| = \pi/2$ on the X -axis. As a result, $\theta(\varphi)$ is not a continuous function and the calculation of the polarization order m cannot be performed in the same way as it was done for the isolated defect at $U=0$ V. If a closed loop around the core of a defect crosses a discontinuity, the procedure of the calculation of the winding number (polarization order or topological strength) of a defect splits into the following stages: 1) a closed loop with a specified traveling direction is encircled around the central singularity, 2) the traveling loop is sectioned into segments as the regions of the traveling angle φ , where $\theta(\varphi)$ is continuous and discontinuous, 3) $\Delta\theta_{cont}$ is calculated as the sum of differences between the $\theta(\varphi)$ values at the end and beginning of each segment of continuity and $\Delta\theta_{discont}$ as the sum of discontinuity jumps 4) the total value $\Delta\theta = \Delta\theta_{cont} + \Delta\theta_{discont}$ is calculated and substituted in Eq. (17) to get m (see Fig. 6a-c).

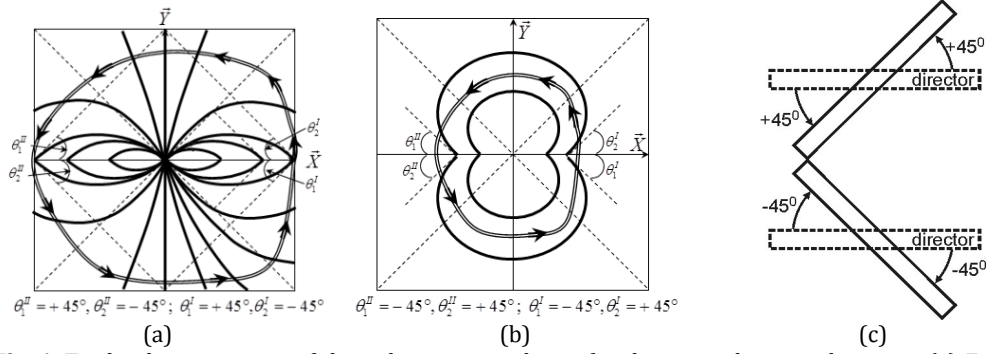


Fig. 6. To the determination of the polarization order m for the azimuth maps shown in: (a) Fig. 4g, (b) Fig. 5g (schematically), (c) scheme for determining the polarization director rotation.

Applying the above-described procedure to the data shown in Figs. 4e,g, we define two regions of $\theta(\varphi)$ continuity: $0 < \varphi < \pi$ and $\pi < \varphi < 2\pi$, and two discontinuity directions: $\varphi = 0$ and $\varphi = \pi$ (Fig. 6a). It is clear that the directions $\varphi = 0$ and $\varphi = 2\pi$ are equivalent and do not require a separate consideration. Fig. 6a illustrates the procedure of the determination of the polarization order m for the azimuth map shown in Fig. 4g. We chose the contra-clockwise traveling direction (from the positive branch of the X -axis towards the positive branch of the Y -axis) to be positive for the angles φ and θ along the loop (arrowed double line in Fig. 6a). For the above defined segments $0 < \varphi < \pi$ and $\pi < \varphi < 2\pi$ of the $\theta(\varphi)$ continuity, the calculations are straightforward and give 135° in each of the four quadrants of the coordinate system. Then, $\Delta\theta_{cont} = 4 \times 135^\circ = 3\pi$. The value of the discontinuity jump is calculated as the sum $\Delta\theta_{discont} = \Delta\theta_{discont}^I + \Delta\theta_{discont}^{II}$ of jumps $\Delta\theta_{discont}^I$ and $\Delta\theta_{discont}^{II}$, respectively, on the positive and negative branches of the X -axis. From Fig. 6a one finds $\Delta\theta_{discont}^I = \theta_2^I - \theta_1^I = -45^\circ - 45^\circ = -\pi/2$ and $\Delta\theta_{discont}^{II} = \theta_2^{II} - \theta_1^{II} = -45^\circ - 45^\circ = -\pi/2$. Then, $\Delta\theta_{discont} = -\pi/2 - \pi/2 = -\pi$ and $\Delta\theta = 3\pi - \pi = 2\pi$. Finally, from Eq. 17, we find that for the central singularity in the azimuth field shown in Fig. 4g, the polarization order is $m = +1$.

Therefore, we are led to conclude that although, the voltage-induced transformations of the light polarization director for the initial RVB go through the formation of the discontinuity (on the X -axis), superimposed on the optical singularity at the center of the pattern (Fig. 4g), the total polarization order (topological strength) of the central singularity appears to be preserved, though is not required to be due to the formation of the discontinuity. The specificity of the situation is that both the central singularity and the discontinuity on the X -axis carry their own contributions to the total topological strength of the central singularity. It is just the case, that the jump $\Delta\theta_{discont} = -\pi$ is of the opposite sign to the excess accumulation of $+\pi$ in $\Delta\theta_{cont} = 2\pi + \pi = 3\pi$. There is no topological restriction applied directly to the sign of $\Delta\theta_{discont}$. The restriction is that the topological path between the two states separated by the discontinuity should be the shortest. In the present case, the shortest topological path corresponds to the negative sign of $\Delta\theta_{discont}$. An alternative topological path $\theta_1^I = +45^\circ \rightarrow +90^\circ \rightarrow -90^\circ \rightarrow \theta_2^I = -45^\circ$, which is equivalent to the path $\theta_1^I = +45^\circ \rightarrow +90^\circ \rightarrow \theta_2^I = +135^\circ$ would result in $\Delta\theta_{discont}^I = \theta_2^I - \theta_1^I = +135^\circ - (+45^\circ) = +\pi/2$, is evidently longer and, thus, corresponds to an unphysical solution. It will be shown below

that for the initially circular distribution of the light polarization director corresponding to an AVB registered at $P = 90^\circ$ ($\vec{E}_{in} \parallel \vec{Y}$), the field-induced transformations are accompanied by the discontinuity of the positive sign in $\Delta\theta_{discont} = +\pi$.

Azimuth (Figs. 4i,k) and ellipticity (Figs. 4j,l) maps, measured at the higher applied voltage $U=3.0$ V, reveal that the exiting light wave is quasi-linearly polarized with the polarization director being parallel to the polarization direction of the incident light and of small ellipticity angle ($\sim 20^\circ$) in the diagonal directions (Fig. 4j,l), while vanishing towards the X and Y coordinate axes. Such a polarization state of the exiting light indicates that at $U=3.0$ V, the optic axis direction averaged through the cell thickness is almost parallel to the light propagation direction, which is in agreement with the expectation that above the threshold voltage ($U > U_{th}$) on increasing the applied voltage, the nematic director reorients toward the cell normal. Indeed, the estimation above showed that at zero applied field the phase retardation by the cell is about $\Delta\Phi \approx 2\pi \times 1.1$ and it is reasonable that at $U=3.0$ V the maximal ellipticity angle of 25° measured in diagonal directions (Fig. 4j) falls in the first order of the phase retardation and, thus, corresponds to the effective birefringence of 0.008, which in turn gives the tilt angle (averaged through the cell thickness) of the nematic director with respect to the cell normal of about 5° . Such a small birefringence results in the Mauguine number $Ma = 2d\Delta n_{eff}/\lambda \approx 0.13 \ll 1$, which indicates that at $U=3.0$ V, the Mauguine regime is not obeyed anymore, and, thus, the rotation of the light polarization is negligible, which, in turn, indicates that no optical dislocation on the X -axis should be expected, which is in agreement with Figs. 4i,k. In addition, such a small angle of 5° for the director tilt with respect to the cell normal implies that, topologically, the nematic director escaped into the third dimension and becomes quasi-homeotropic (quasi-perpendicular) with respect to the substrate surfaces. Although the elimination (continuous or discontinuous) of the nematic defects (line disclination and point defect at the C -substrate) on increasing the voltage, deserves special topological consideration, it is out of the scope of this paper and will be continued elsewhere. Regardless, the experiment reveals that at a voltage of $U=3.0$ V and higher, the nematic director field is quasi-homeotropic and defect-free, which explains the absence of any optical singularities in the field of the polarization director (azimuth) (Fig. 4k).

At $P = 90^\circ$ ($\vec{E}_{in} \perp \vec{Y}$) and zero applied voltage ($U=0$ V), Fig. 5a-d, the azimuth map (Fig. 5a,c) of the exiting beam clearly displays the circular light polarization field, which is a typical signature of an AVB with $m=+1$, again being in good agreement with the theoretical predictions (Fig. 3a). On increasing the voltage to $U=1.5$ V (Fig. 5e-h) the initially circular polarization director stream lines registered at $U=0$ V (Fig. 5e-h) transform into the family of curves (Fig. 5e,g) akin to those called nephroids in Geometry. They have pairs of cusps, which form a discontinuity on the X -axis. The calculations of the light polarization order m of the central singularity for data shown in Fig. 5e,g, are performed following the procedure described above for the case $P=0^\circ$ ($\vec{E}_{in} \parallel \vec{Y}$). As before, we define two regions of $\theta(\varphi)$ continuity: $0 < \varphi < \pi$ and $\pi < \varphi < 2\pi$, and two discontinuity directions: $\varphi=0$ and $\varphi=\pi$ (Fig. 6b). The calculations of $\Delta\theta_{cont}$ again give 135° in each of the four quadrants of the coordinate system, such that $\Delta\theta_{cont} = 4 \times 135^\circ = 3\pi$. For the discontinuities from Fig. 6b, we find: $\Delta\theta_{discont}^I = \theta_2^I - \theta_1^I = +45^\circ - (-45^\circ) = +\pi/2$ and $\Delta\theta_{discont}^{II} = \theta_2^{II} - \theta_1^{II} = +45^\circ - (-45^\circ) = +\pi/2$.

Then, $\Delta\theta_{discont} = \Delta\theta_{discont}^I + \Delta\theta_{discont}^{II} = +\pi/2 + \pi/2 = +\pi$ and $\Delta\theta = 3\pi + \pi = 4\pi$. After the substitution of this data in Eq. 17, we find that for the central singularity in the polarization director field, shown in Fig. 5g, the total polarization order (topological strength) is $m = \frac{\Delta\theta}{2\pi} = +2$.

The latter result shows that for the case $P = 90^\circ$ ($\vec{E}_{in} \parallel \vec{Y}$), under the electric field, applied to the nematic θ -cell, the light polarization order (topological strength) of the optical singularity at the center of the pattern increases from $m = +1$ for the initial AVB, at $U = 0$ V, to $m = +2$ for the nephroid-like polarization director field at $U = 1.5$ V, shown in Fig. 5g. Although the topological strength of the central singularity is not preserved, there is neither a contradiction to the law of conservation of the topological strength nor to the homotopy classification of vector singularities. First, it should be noted that at the field-induced reorientation of the nematic director, the OPD space of the optical field transforms from RP^1 at $U = 0$ V to RP^2 , in terms of the light polarization director, and, more generally to the Poincaré sphere for the optical field as a whole. We have argued above, that the topological strength of a defect calculated within different OPD spaces is not obliged to be preserved. Second, the transformation of the initially circular AVB configuration of the polarization director field at $U = 0$ V into the nephroid-like distribution is not continuous but is accompanied by the formation of a discontinuity. It is just the case announced above when the shortest topological path across the discontinuity corresponds to the positive contribution of $\Delta\theta_{discont} = +\pi$.

At higher applied voltage, $U = 3.0$ V the azimuth (Figs. 5i,k) and ellipticity (Figs. 5j,l) maps reveal that the exiting light wave is quasi-linearly polarized with the polarization being parallel to the polarization direction of the incident light and of small ellipticity angle in the diagonal directions (Fig. 4j,l), while vanishing towards the X and Y coordinate axes. Such a polarization state of the exiting light is an additional confirmation that at $U = 3.0$ V, the optic axis direction averaged through the cell thickness is almost parallel to the light propagation direction, which is because, on increasing the applied voltage well above the threshold voltage, the nematic director reorients toward the cell normal. As a result, the light polarization director field is uniform and free of singularities (Fig. 5i,k).

The transformations of the optical field under the applied voltage at $P = 45^\circ$ are shown in Figs. 7a-l. At zero applied voltage, a singularity with polarization order $m = +1$ is observed (Fig. 7a-d). Upon applying a voltage of 1.5 V, a discontinuity is formed on the X -axis (Fig. 7e-h). Calculations of the polarization order of the central singularity, following the above described procedure, accounting for the contribution of the discontinuity, result in the total value $m = +1$. As in the previous cases, at $U = 3.0$ V, the maps for the optic field of the exiting wave (Fig. 7i,k) do not display any singularities and reveal the polarization state, which is close to that of the incident light, thereby signaling that, on average through the cell thickness, the light propagates almost parallel to the optic axis, which, in turn, is the nematic director, being reoriented almost homeotropically (normally) with respect to the cell substrates. The ellipticity angle is about 45° in the vertical directions while vanishing towards the direction of the X coordinate axis (Fig. 7j,l). The residual ellipticity of about 45° suggests that the nematic director is along the cell normal in the middle plane of the cell but gradually deviates from it towards the substrate planes on approaching them.

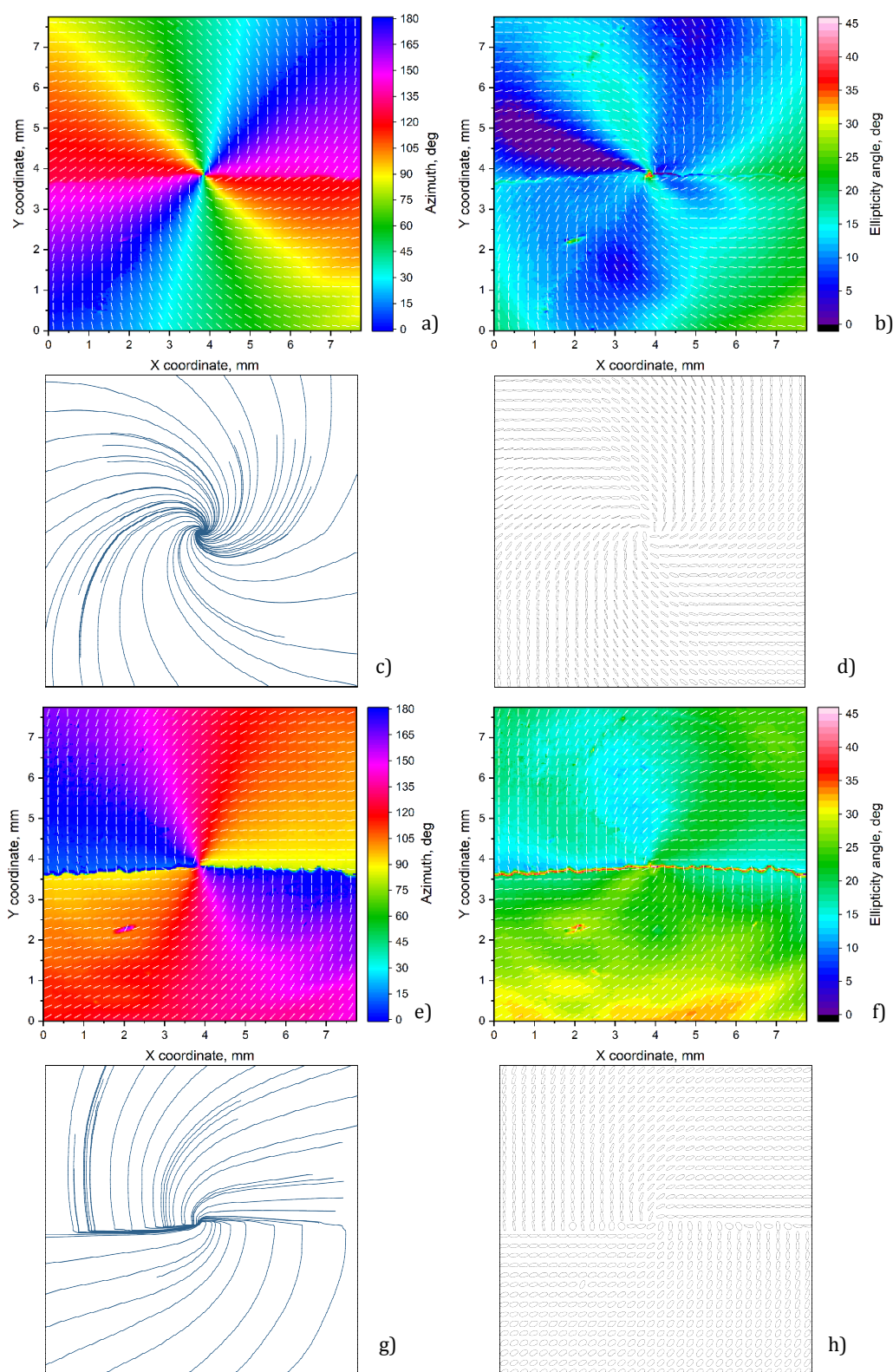


Fig. 7. Maps of the azimuth... (begining, continued on the next page)

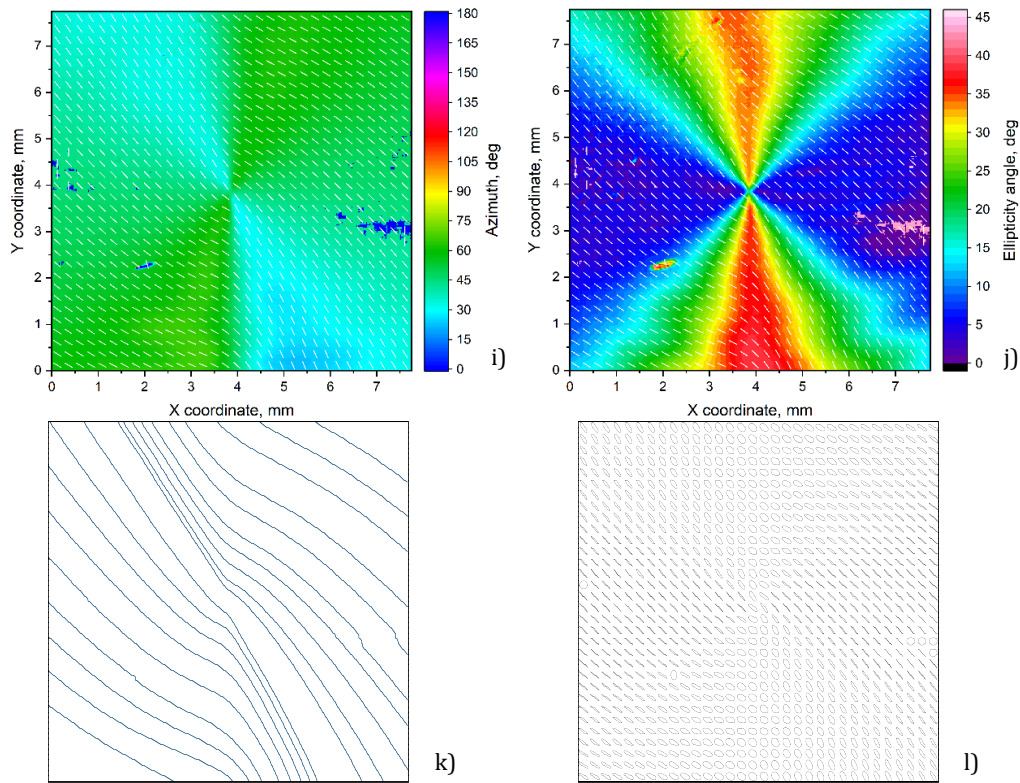


Fig. 7. Maps of the azimuth (a,e,i), ellipticity angle (b,f,j), azimuth streamlines (c,g,k) and polarization ellipses (d,h,l) in the beam cross-section for the light emergent from the L - C θ -cell. The incident linearly polarized light beam has a $P=45$ -degree orientation. $U = 0$ (a,b,c,d); 1.5 (e,f,g,h) and 3.0 V (i,j,k,l). The orientation of the major axes of the polarization ellipses is indicated by the white line segments.

4.2. C - L geometry

In this section, we consider the results of the experimental study of the θ -cell in the C - L geometry such that the normally incident laser beam enters the θ -cell through the circularly rubbed C -substrate and exits from the unidirectionally rubbed L -substrate. The maps of the azimuth of the major axis (polarization director) of the polarization ellipse and the angle of ellipticity for the case when the polarization of the incident beam is $P=0^\circ$ ($\vec{E}_{in} \parallel \vec{Y}$) are presented in Fig. 8. The distribution of the light polarization director shown in Fig. 8a,c (azimuth map), represents a typical configuration, which in Topology of defects in ordered media, nematic disclinations [4-6] in particular, is classified as such that corresponds to a vector singularity with the light polarization order $m=-1$ (topological strength $s=-1$ for nematic disclinations), which is in agreement with the theoretical prediction depicted in Fig. 3d. Remarkably, the field of the polarization director distribution, registered in the C - L geometry (Fig. 8a,c), topologically differs from that obtained in the L - C geometry (Fig. 4a,c) at the same other conditions and forming the RVB with the vector singularity of the polarization order $m=+1$. The topological difference between the optical field configurations in the L - C and C - L geometries is the experimental evidence for the flip-flop non-invariance of the optical field generated with the θ -cell.

At the applied voltage $U = 1.5$ V, the light polarization director field drastically transforms (Fig. 8 e-h) such that in the sectors around the Y -axis it becomes reminiscent of that for an RVB

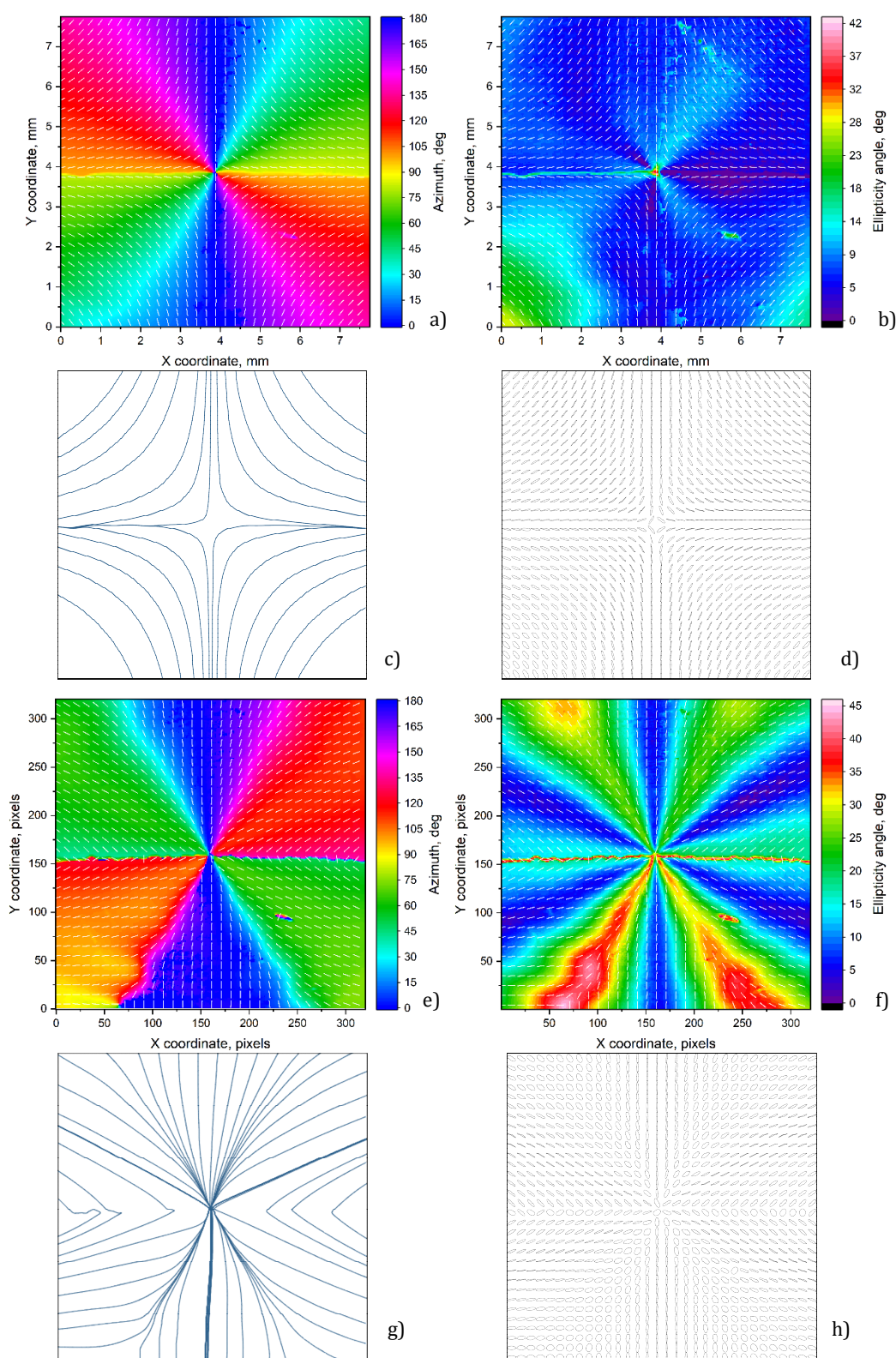


Fig. 8. Maps of the azimuth... (beginning, continued on the next page)

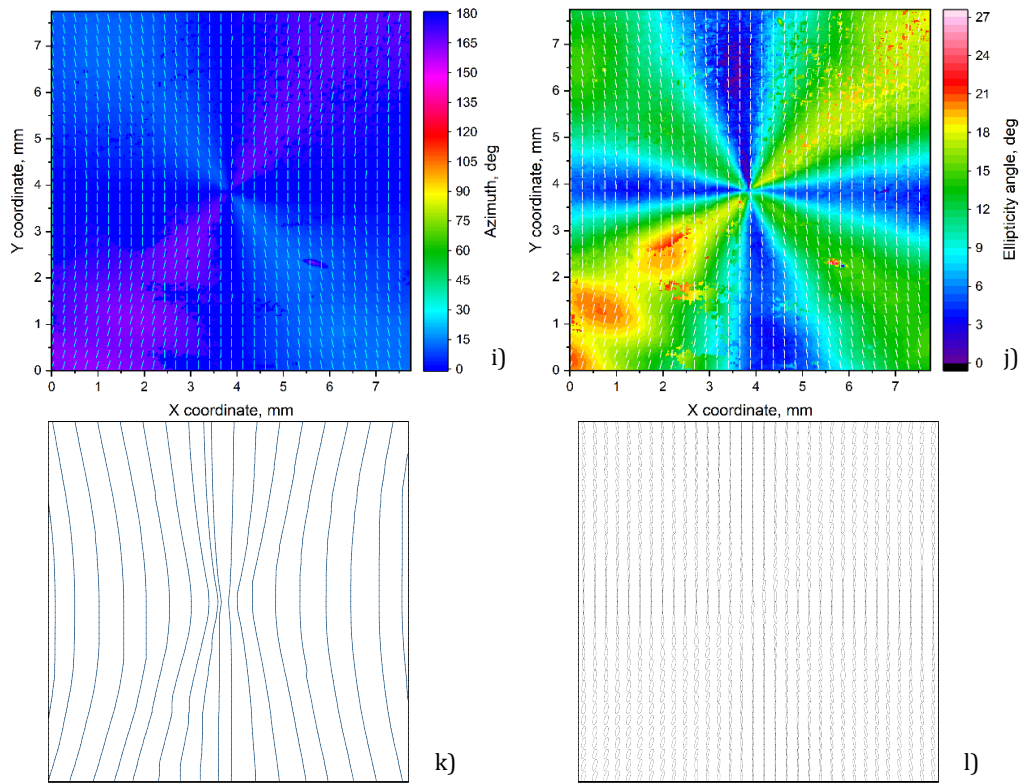


Fig. 8. Maps of the azimuth (a,e,i), ellipticity angle (b,f,j), azimuth streamlines (c,g,k) and polarization ellipses (d,h,l) in the beam cross-section for the light emergent from the C - L θ -cell. The incident linearly polarized light beam has a $P=0$ -degree orientation. $U = 0$ (a,b,c,d); 1.5 (e,f,g,h) and 3.0 V (i,j,k,l). The orientation of the major axes of the polarization ellipses is indicated by the white line segments.

(Fig. 8 g). Such a field-induced transformation of the optical field is in agreement with the theoretical consideration performed in Sec. 3, which shows that the superposition of the optical rotation of the polarization plane and linear birefringence of the circular director field with the point defect is at the origin of the flip-flop invariance of the θ -cell. On increasing the voltage above the threshold value, the nematic director reorients towards the cell normal, thereby lowering the effective birefringence, consequently exiting the Mauguine regime and lowering the optical rotation. At vanishing optical rotation, but still nonzero birefringence the optical field is expected to transform towards the elimination of the difference between the C - L and L - C geometries. The tendency to the similarity of the optical fields for the two (C - L and L - C) geometries supposes to pronounce in the directions of smaller optical rotation, i.e. within the sectors around the Y -axis. It is indeed what is observed, when comparing Figs. 4g and 8g which correspond to the same setting $P=0^\circ$ ($\vec{E}_{in} \parallel \vec{Y}$). Namely, within the sectors around the Y -axis the distribution of the polarization director is radial in both Fig. 4g and 8g. However, naturally, toward the X -axis, where the optical rotation is higher, the difference between the two patterns enhances. While in Fig. 4g one observes the negative jump $\Delta\theta_{disc}^I = \Delta\theta_{disc}^{II} = -\pi/2$ (see Fig. 6a,c for details), in Fig. 8g one finds the positive jump of $\Delta\theta_{disc}^I = \Delta\theta_{disc}^{II} = +\pi/2$ (compare to Fig. 6b,c where the jump is of the same positive sign) on the X -axis. The calculations

performed for the distribution shown in Fig. 8g, result in $\Delta\theta = \Delta\theta_{cont} + \Delta\theta_{discont} = (2 \times \pi/2) + (2 \times \pi/2) = +2\pi$, which after the substitution in Eq. 17 leads to $m = +1$.

Therefore, we are led to conclude, that, at the applied voltage $U = 1.5$ V, despite the residual difference for the two ($C-L$ and $L-C$) geometries in the configurations of the optical fields around the X -axis, which is accompanied by the similarity around the Y -axis, the total polarization order of the singularity in the center of the pattern, appears to be the same $m = +1$ for both geometries. Notice that under the applied voltage $U = 1.5$ V, the initial singularity with the polarization order $m = -1$ existing at $U = 0$ V transforms into the singularity with $m = +1$. Although the transformation of the initial singularity with $m = -1$ into that with $m = +1$ is accompanied by the formation of a discontinuity on the X -axis (Fig. 8g), these two singularities belong to the same topological class, being elements of the same homotopy group, and, thus, there is a continuous topological path between them such that they can be continuously transformed into each other.

Similarly to the previous cases, at the applied voltage of 3.0 V, the polarization state of the exiting wave is close to that of the incident light (Fig. 8i-l), because the nematic director is reoriented quasi-homeotropically with respect to the cell substrates such that the light propagates almost parallel to the optic axis.

For the case of $P = 90^\circ (\vec{E}_{in} \perp \vec{Y})$ the maps of the polarization state are presented in Fig. 9. At zero voltage, the measured map of the polarization director (major axis of the polarization ellipse) contains a singularity with $m = -1$ in the center of the pattern (Fig. 9a,c)

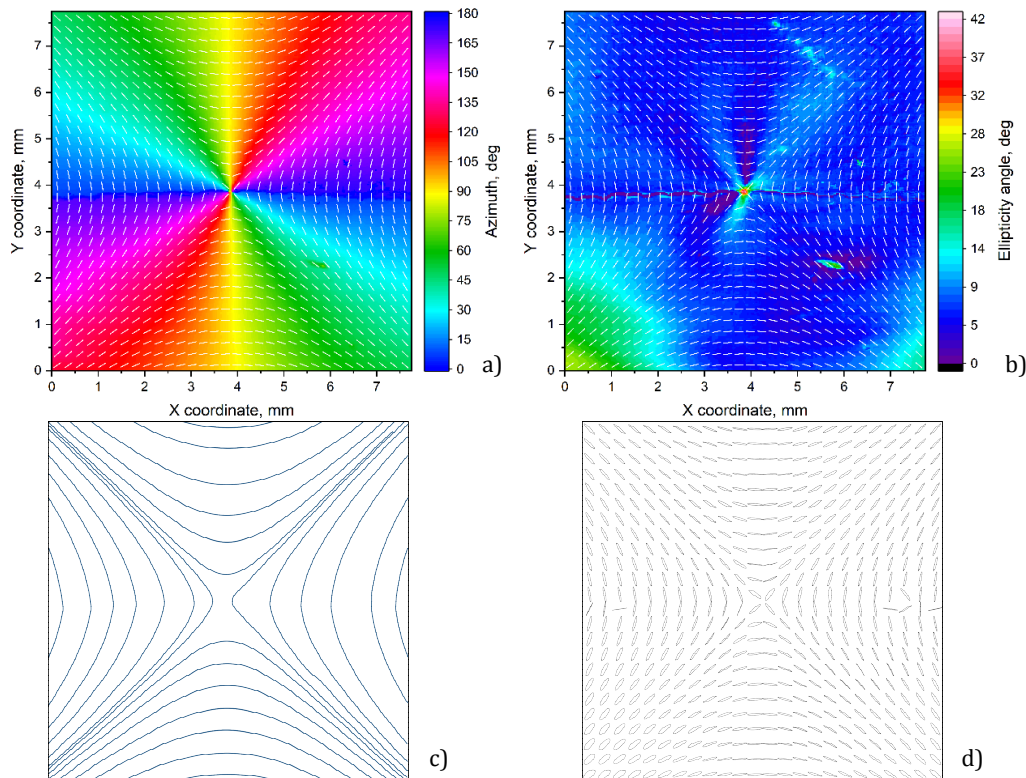


Fig. 9. Maps of the azimuth... (beginning, continued on the next page)

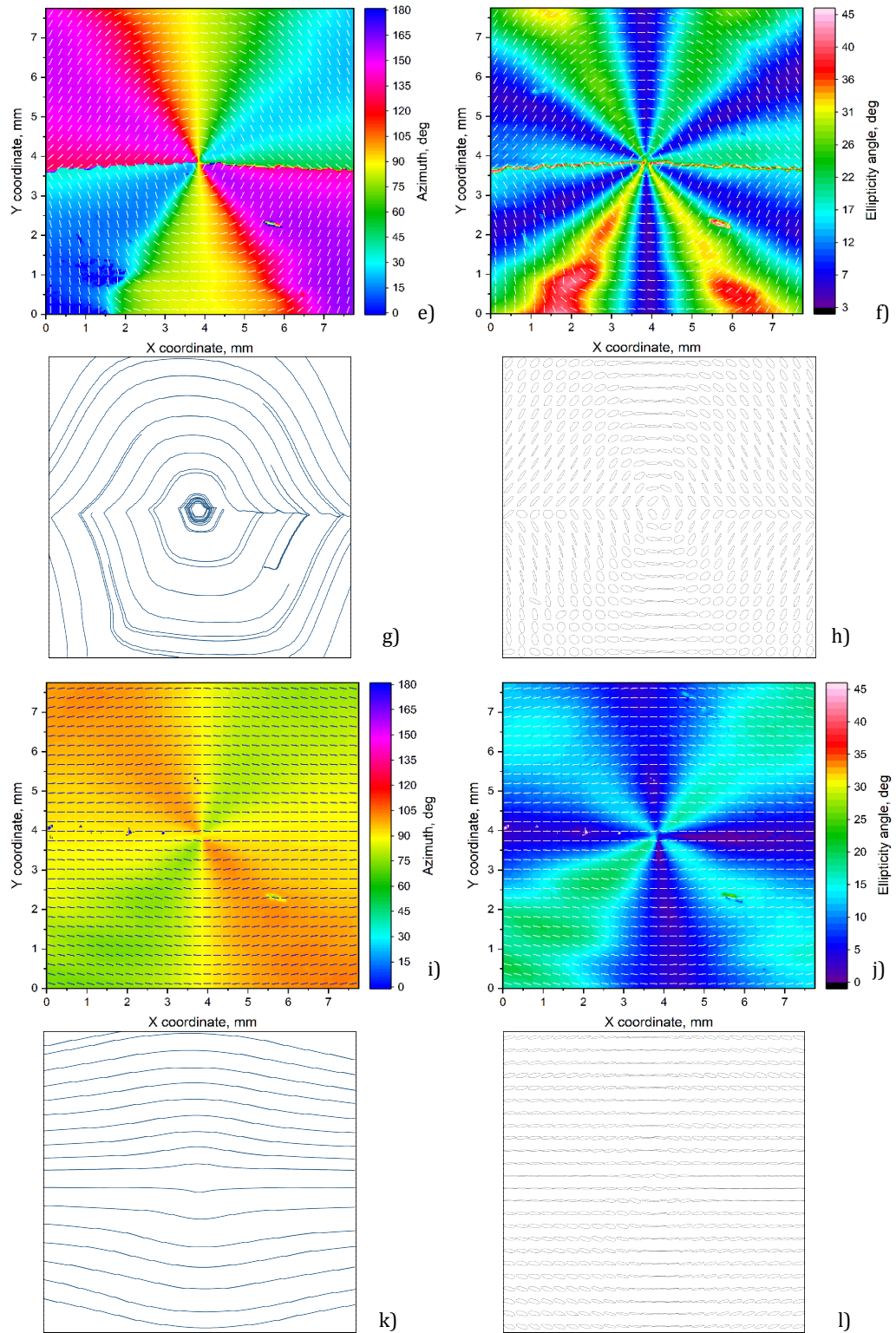


Fig. 9. Maps of the azimuth (a,e,i), ellipticity angle (b,f,j), azimuth streamlines (c,g,k) and polarization ellipses (d,h,l) in the beam cross-section for the light emergent from the C - L θ -cell. The incident linearly polarized light beam has a $P=90$ -degree orientation. $U = 0$ (a,b,c,d); 1.5 (e,f,g,h) and 3.0 V (i,j,k,l). The orientation of the major axes of the polarization ellipses is indicated by the white line segments.

similar to that shown in Fig. 8a,c for the setting $P=0^\circ$ ($\vec{E}_{in} \parallel \vec{Y}$), but with its axes rotated by 45° . It is worth noting that this experimental result agrees with the corresponding theoretically derived pattern schematically shown in Fig. 3c. At the applied voltage of 1.5 V, the initial pattern (Fig. 9 a,c) with $m=-1$ observed at zero voltage transforms into the pattern shown in Fig. 9g such that in the sectors around the Y-axis it is reminiscent of that for an AVB similarly to the pattern (Fig. 5c) obtained for the $L-C$ geometry at the same setting $P=90^\circ$ ($\vec{E}_{in} \perp \vec{Y}$), but with a discontinuity on the X-axis (Fig. 9g) of the same negative $(-\pi/2)$ -jump which is similar to the discontinuity shown in Fig. 6a. The calculations performed for the pattern shown in Fig. 9g, give $\Delta\theta = \Delta\theta_{cont} + \Delta\theta_{discont} = (2 \times 3\pi/2) + 2 \times (-\pi/2) = +2\pi$, which after the substitution in Eq. 17 results in $m=+1$. As for the previous setting ($P=0^\circ$ ($\vec{E}_{in} \parallel \vec{Y}$)), we again notice the tendency to the elimination of the flip-flop noninvariance of the $\square$ -cell under the applied voltage $U=1.5$ V, which reorients the nematic director towards the cell normal and consequently vanishes the optical rotation, which is at the origin of the flip-flop noninvariance.

Further increase of the voltage to $U=3$ V leads to the quasi-homeotropic distribution of the nematic director through the cell such that the light propagates almost along the optic axis and, as a result, the polarization state of the exiting light wave is almost the same as it is for the incident light (Fig. 9i-l) in agreement with other settings and geometry at the same voltage $U=3$ V.

The maps displaying the polarization state of the exiting wave at the 45° -polarization setting of the incident beam are shown in Fig. 10a-l. Similarly to the previous settings ($P=0^\circ$ and 90°) for

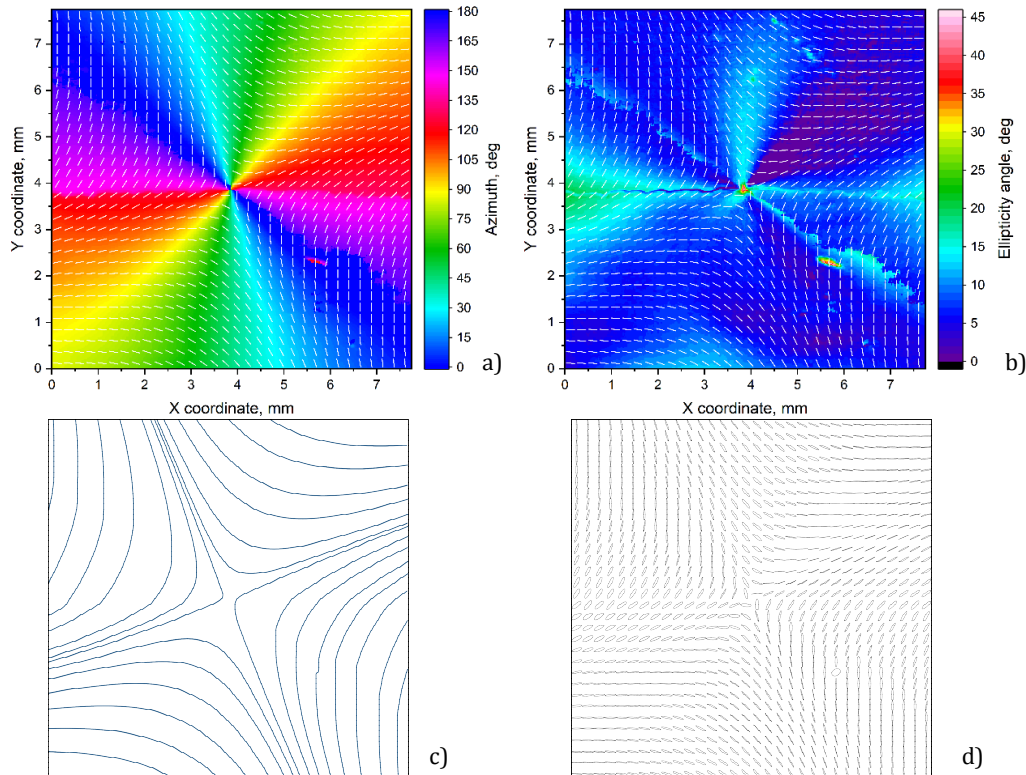


Fig. 10. Maps of the azimuth... (beginning, continued on the next page)

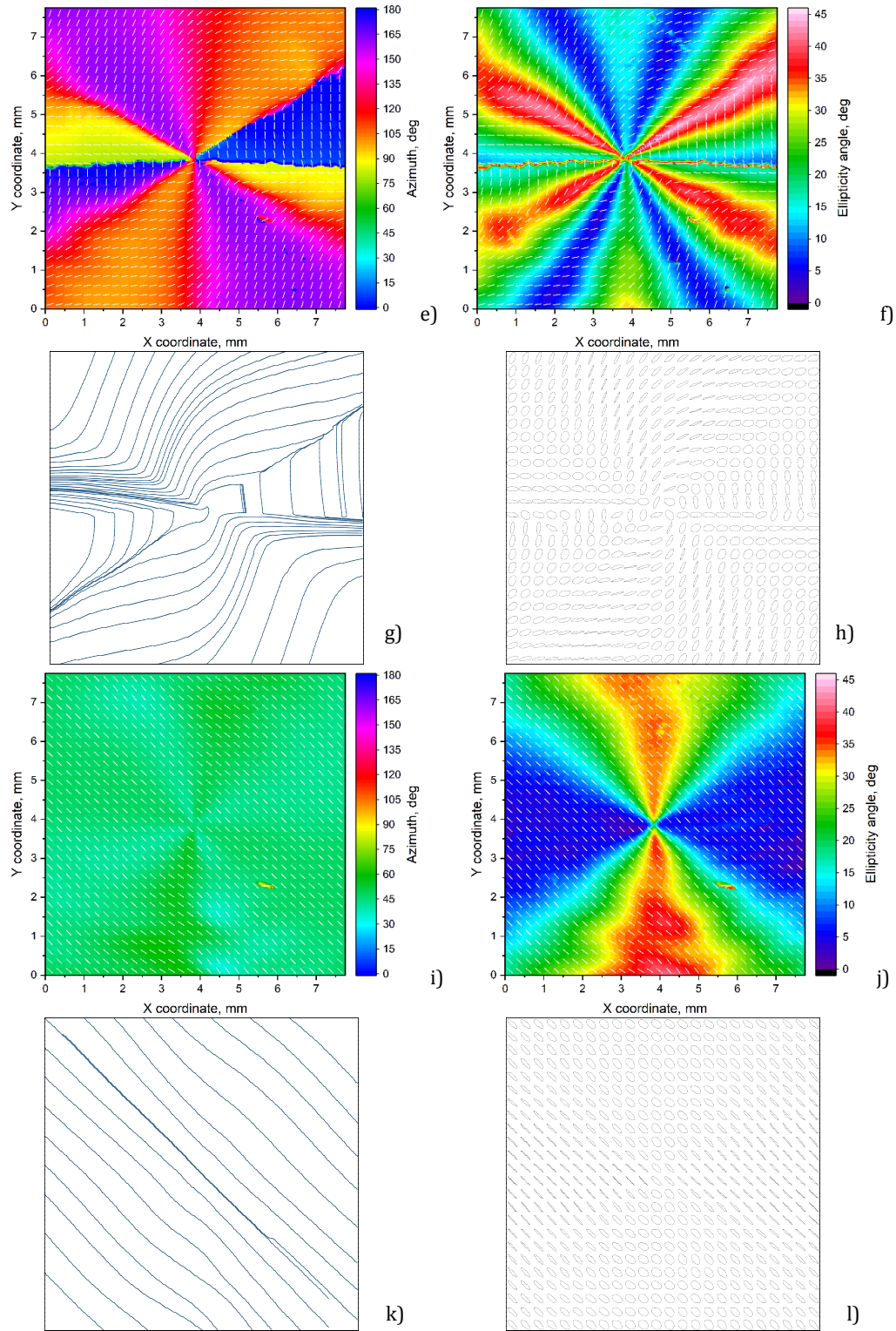


Fig. 10. Maps of the azimuth (a,e,i), ellipticity angle (b,f,j), azimuth streamlines (c,g,k) and polarization ellipses (d,h,l) in the beam cross-section for the light emergent from the C - L θ -cell. The incident linearly polarized light beam has a $P=45$ -degree orientation. $U = 0$ (a,b,c,d); 1.5 (e,f,g,h) and 3.0 V (i,j,k,l). The orientation of the major axes of the polarization ellipses is indicated by the white line segments.

the same C - L geometry, the azimuth map (Fig. 10a,c) displays a hyperbolic-like distribution of the polarization director with the axes of the pattern rotated by 22.5° with respect to the Y -axis and with the singularity of the polarization order $m = -1$. The comparison to the azimuth map corresponding to the same 45° setting in the L - C geometry brings additional experimental evidence for the flip-flop noninvariance of the θ -cell. The application of the voltage $U = 1.5$ V transforms the azimuth map to the configuration shown in Fig. 10e,g, which contains discontinuities in two azimuthal directions and a quite complicated pattern of the polarization director distribution. Additional information is desired to follow the nematic director transformations and the optical field transformations generated by them at intermediate voltages to determine the polarization order of the optical singularity in the center of the pattern; we leave it for future experiments. Further increasing the voltage to $U = 3$ V, as in all cases mentioned above, results in the polarization state of the exiting field being similar to that of the incident field (Fig. 10i-l), in full agreement with the explanations given for other settings and geometry at the same voltage $U = 3$ V.

Finally, we highlight the exact agreement between the experimental and theoretical maps of the polarization azimuth distribution. To verify, one can compare Figs. 3a-d with Figs. 4,5,8,9.

5. Conclusions

In the present work, we have studied theoretically and experimentally the polarization state of the singular optical field of the beam exiting the nematic θ -cell, assembled from the substrates with the linear (L) and circular (C) surface alignments for the nematic director. Our theoretical Jones matrix formalism consideration predicted that it does matter from which L - or C -substrate the light enters the cell. We term the flip-flop noninvariance of the optical field generated by the θ -cell to describe its spatial nonreciprocity with respect to the direction of light propagation. To verify this prediction, we have measured the azimuth and ellipticity maps for the spatial distributions, respectively, of the orientation of the major axis (polarization director) and ellipticity of the polarization ellipse in two L - C and C - L geometries, when the light enters the cell through the L - and C -substrates, respectively. The measurements are performed at three different settings ($0^\circ, 45^\circ, 90^\circ$) for the linear polarization of the normally incident laser beam with respect to the alignment direction of the L -substrate. The experimental measurements confirm the theoretically predicted flip-flop noninvariance of the singular optical field generated with the θ -cell. Namely, for the L - C geometry we register the distribution of the polarization director with the singularity of the polarization order (topological strength) $m = +1$ in the center of the pattern with the radial and circular configuration of the polarization director (RVB and AVB beams, respectively) at the orthogonal settings of the incident light polarization. Contrary, for the C - L geometry, the azimuth map displays a hyperbolic-like distribution of the polarization director, which corresponds to the polarization order of $m = -1$ for the singularity at the center of the pattern.

At the applied voltage $U = 1.5$ V, well above the threshold ($U_{th} = 0.75$ V), the nematic director reorients toward the cell normal, and the optical field behind the cell undergoes a drastic transformation, forming a discontinuity in the distribution of the polarization director. We have developed a special procedure to account for jumps at these discontinuities when calculating the polarization order for such patterns. We find that for both L - C and C - L geometries, under the applied voltage, the optical field of the exiting beam transforms to the configurations with

$m = +1$ for most settings except for the case of the L - C geometry, when the incident light polarization is parallel to the direction of the nematic director alignment at the L -substrate; for this case we find $m = +2$. With the applied voltage, the configurations of the polarization director in the C - L geometry, measured in the directions of the lowest optical rotation, transform towards patterns similar to those observed in the L - C geometry and, thus, towards vanishing the difference between the two geometries. Taking into account that the applied voltage reorients the nematic director toward the cell normal and consequently lowers the optical rotation, we conclude that such a tendency is in agreement with our theoretically drawn conclusion that optical rotation coexisting with the linear birefringence is at the origin of the flip-flop noninvariance of the singular optical field generated behind the θ -cell.

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Анотація. Теоретично та експериментально досліджені сингулярні оптичні поля, що генеруються за допомогою так званої θ -комірки, сформованої з підкладок із лінійним (L) та круговим (C) орієнтуванням поверхонь і нематичного директора. Дослідження проведені з використанням формалізму Джонса та експериментального вимірювання мап азимуту й еліптичності вихідного світлового променя. Теоретичний розгляд передбачає топологічно різні розподіли великої осі еліпса поляризації (директора поляризації) для протилежних напрямків поширення світлового променя (від L– до C–підкладки та назад, L–C та C–L геометрії, відповідно). Ця просторова невзаємність відносно напрямку поширення світла була названа фліп-флоп-неінваріантністю оптичного поля. Виявлено, що у зв'язку з фліп-флоп-неінваріантністю порядок поляризації (топологічна сила) оптичної сингулярності в центрі діаграми становить $m=+1$ для L–C, та $m=-1$ для C–L геометрії. Зроблено висновок, що просторово розподілене оптичне обертання, що співіснує з лінійним двопроменезаломленням, є основою неінваріантності сингулярного оптичного поля, що генерується θ -коміркою. Теоретичні висновки були експериментально підтверджені для трьох орієнтацій падаючого лінійно поляризованого світла (0° , 45° , 90°) відносно напрямку орієнтування L-підкладки. Досліджено топологічні перетворення результуючих оптичних полів під дією електричного поля, прикладеного вздовж нормалі до підкладок. Виявлено, що зумовлена електричним полем переорієнтація нематичного директора в напрямку нормалі комірки супроводжується утворенням розривів у розподілах директора поляризації. Розроблено спеціальну процедуру для врахування стрибків під час розрахунків порядку поляризації. Експериментально встановлено, що прикладене електричне поле зменшує обертання поляризації світла і, як наслідок, усуває різницю між оптичними полями променя, що спостерігаються в двох геометріях (L–C та C–L).

Ключові слова: векторний пучок, порядок поляризації, нематична θ -комірка, електричне поле, картографування поляризації, топологічний дефект, обробка зображень, цифрова обробка даних