Ukrainian Journal of Physical Optics
ISSUE 2
2026

Quiescent Solitons in Magneto–Optic Waveguides Having Kudryashov's First Form of Nonlinear Self-Phase Modulation Structure

Elsayed M. E. Zayed[1], Basel M. M. Saad[2], Ahmed H. Arnous[3,4], Yakup Yildirim[5,6], Anjan Biswas[7,8,9]*, Luminita Moraru[10,11], Carmelia Mariana Balanica Dragomir[12]

[1] Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt
[2] Department of Mathematics, Faculty of Science, Arish University, North Sinai, Egypt
[3] Department of Mathematical Sciences, Saveetha School of Engineering, SIMATS Chennai - 602105, Tamilnadu, India
[4] Research Center of Applied Mathematics, Khazar University, Baku, AZ 1096, Azerbaijan
[5] Department of Computer Engineering, Biruni University, Istanbul–34010, Turkey
[6] Department of Mathematics, Near East University, 99138 Nicosia, Cyprus
[7] Department of Mathematics and Physics, Grambling State University, Grambling, LA 71245–2715, USA
[8] Department of Physics and Electronics, Khazar University, Baku, AZ—1096, Azerbaijan
[9] Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa–0204, South Africa
[10] Faculty of Sciences and Environment, Department of Chemistry, Physics and Environment, Dunarea de Jos University of Galati, 47 Domneasca Street, 800008 Galati, Romania
[11] Department of Physics, School of Science and Technology, Sefako Makgatho Health Sciences University, Medunsa–0204, Pretoria, South Africa
[12] Department of Applied Sciences, Cross—Border Faculty of Humanities, Economics and Engineering, Dunarea de Jos University of Galati, 111 Domneasca Street, Galati—800201, Romania
*Corresponding author: biswas.anjan@gmail.com

Received: 11.02.2025

Abstract. This paper reports the observation of quiescent optical solitons in magneto–optic waveguides. The self–phase modulation structure is based on the one proposed by Kudryashov. Three algorithms have enabled this retrieval: the enhanced direct algebraic method, the extended auxiliary equation approach, and the new mapping scheme. Together, these methods have recovered a full spectrum of quiescent optical solitons. The parameter constraints for their existence are also included. A few numerical simulations demonstrate the analytical results.

Keywords: solitons, magneto–optics, auxiliary algorithm, mapping scheme; algebraic approach
UDC: 535.8; 535.14; 537.86
DOI: 10.3116/16091833/Ukr.J.Phys.Opt.2026.02001

1. Introduction

One of the main sources of quiescent optical solitons is the nonlinearity of chromatic dispersion (CD) [1-6]. Another source involves considering fourth–order dispersion [7-9] rather than CD. In this case, quiescent optical solitons cannot be analytically calculated, but they can be observed numerically within the model. These solitons have been extensively studied in optical fibers using various models and multiple integration schemes based on Lie symmetry [10–12]. The research has also been successfully extended to magneto–optic waveguides with Kudryashov's generalized quintuple–power law and nonlocal nonlinearity, which features nonlinear chromatic dispersion [13]. This paper will examine the evolution of quiescent optical solitons in magneto–optic waveguides with Kudryashov's proposed self–phase modulation (SPM) structure and generalized temporal evolution.

This work employs three integration algorithms for retrieval. They are the enhanced direct algebraic algorithm [14,15], the new mapping method [16], and the extended auxiliary equation approach [17,18]. Collectively, these approaches reveal a full spectrum of optical solitons, as shown in the paper. The parameter constraints for the existence of such stationary optical solitons in magneto–optic waveguides are also provided. The numerical schemes offer a visual representation of these solitons. Details of the three algorithms and their successful application in retrieving soliton solutions to the model are discussed in the rest of the paper.

In parallel with the development of physical models, considerable effort has been devoted to the construction of exact travelling-wave solutions for nonlinear Schrödinger–type equations with nonlinear dispersion. Y. Geng and J. Li considered a nonlinearly dispersive Schrödinger equation and, by means of dynamical systems theory, established the existence of solitary patterns, compactons, kink-type waves, and various periodic structures [19]. Z. Yan, on the other hand, introduced a generalized nonlinear Schrödinger equation with nonlinear dispersion and reported envelope compactons and solitary pattern structures together with conserved quantities, thereby clarifying the influence of nonlinear dispersion on the underlying wave dynamics [20]. This viewpoint was extended to coupled systems, where new exact solution structures were obtained for coupled Klein–Gordon equations and a higher-dimensional generalized coupled NLS model, again emphasising the role of nonlinear dispersion [21].

For perturbed NLS equations, attention has also been given to Kerr-type nonlinearities and systematic solution methods. Z. Zhang et al. conducted a qualitative analysis of a perturbed NLS equation with Kerr law nonlinearity and derived traveling-wave solutions by exploiting the related phase-plane structure [22]. In a follow-up study on the same model, new families of exact solutions were recovered using a modified trigonometric series method [23]. Along a different line, Sirendaoreji developed the auxiliary equation method, where exact solutions of an appropriate first-order nonlinear ordinary differential equation are used to construct traveling-wave solutions for quadratic and cubic nonlinear Klein–Gordon equations; this approach has since become a versatile tool for a broad class of nonlinear evolution equations [24]. Taken together, these works offer a comprehensive mathematical framework for addressing nonlinear dispersion and multi-power nonlinearities in optical soliton models.

More recently, several studies have explored highly dispersive solitons in models that include additional physical effects such as stochastic perturbations, concatenated dynamics, and complex refractive-index structures. M. Ekici and C.A. Sarmaşık analyzed the stochastic concatenation model, combining NLSE-, LPD-, and Sasa–Satsuma-type contributions in the presence of multiplicative white noise, and derived several analytical solutions, including multi-wave, breather, periodic cross-kink, and Peregrine-like rational structures in optical fibers [25]. A. J. M. Jawad and M. J. Abu-AlShaeer obtained highly dispersive optical solitons for NLS-type equations with cubic and cubic–quintic–septic nonlinearities using two distinct analytical methods [26]. At the same time, Jihad and Almuhsan evaluated various dispersion-compensation strategies for impairment mitigation in optical fiber communication systems [27]. Within the same context of highly dispersive dynamics, Y. S. Ozkan and E.Yaşar proposed three efficient schemes for a perturbed stochastic Fokas–Lenells equation, successfully recovering a broad range of highly dispersive optical solitons [28]. In a related but different physical setting, Li et al. constructed bright and dark solitons in a (2+1)-dimensional spin-1 Bose–Einstein condensate, demonstrating how similar nonlinear wave mechanisms emerge in ultra-cold atomic systems [29].

The present work builds on these developments by focusing on quiescent optical solitons in magneto–optic waveguides, governed by a generalized nonlinear Schrödinger equation with Kudryashov's first form of nonlinear self-phase modulation and nonlinear chromatic dispersion. In contrast to the above studies, we recover quiescent solitons for this magneto-optic setting for both linear and generalized temporal evolutions, using a combination of auxiliary-equation, mapping, and algebraic schemes tailored to Kudryashov-type nonlinear refractive-index laws.

2. Mathematical analysis

The dimensionless form of the nonlinear Schrödinger equation (NLSE) with Kudryashov's law having nonlinear chromatic dispersion and generalized linear temporal evolution is written as:

$$i\left(q^l\right)_t + a\left(|q|^p q^l\right)_{xx} + \left[\frac{b_1}{|q|^{2n}} + \frac{b_2}{|q|^n} + b_3|q|^n + b_4|q|^{2n}\right]q^l = 0, \qquad (1)$$

The dimensionless form of the NLSE in magneto-optic waveguides with Kudryshov's law having generalized temporal evolutions and nonlinear CD is written as:

$$i\left(q^l\right)_t + a_1\left(|q|^p q^l\right)_{xx} + \left[\begin{array}{l}\frac{b_1}{|q|^{2n}} + \frac{b_2}{|q|^n} + b_3|q|^n + b_4|q|^{2n} \\ + \frac{d_1}{|r|^{2n}} + \frac{d_2}{|r|^n} + d_3|r|^n + d_4|r|^{2n}\end{array}\right]q^l = Q_1 r^l, \qquad (2)$$

and

$$i\left(r^l\right)_t + a_2\left(|r|^p r^l\right)_{xx} + \left[\begin{array}{l}\frac{b_5}{|r|^{2n}} + \frac{b_6}{|r|^n} + b_7|r|^n + b_8|r|^{2n} \\ + \frac{d_5}{|q|^{2n}} + \frac{d_6}{|q|^n} + d_7|q|^n + d_8|q|^{2n}\end{array}\right]r^l = Q_2 q^l, \qquad (3)$$

where $q(x,t)$ and $r(x,t)$ are complex-valued functions that represents the wave profiles with $i = \sqrt{-1}$. The first terms in Eqs. (2) and (3) represent the generalized temporal evolutions with parameter $l \geq 1$. The constants $a_j\,(j=1,2)$ are the coefficients of generalized nonlinear CD with nonlinearity parameter $p \geq 0$. The constants Q_j for $(j=1,2)$ are the coefficients from the magneto–optic waveguides effect. Here, x and t denote the spatial and temporal coordinates, respectively. The functions $q(x,t)$ and $r(x,t)$ represent the slowly varying complex envelopes of the optical fields propagating through the waveguide. The parameters (b_1, b_2, ..., b_8) and (d_1, d_2, ..., d_8) are real constants associated with the nonlinear coefficients of Kudryashov's law, which describe higher-order nonlinear effects such as self-phase and cross-phase modulation. The exponent ($n>0$) determines the nonlinear refractive index order, influencing the intensity dependence of the nonlinearity. The constants Q_1 and

Q_2 correspond to the magneto–optic coupling coefficients that arise from the magneto–optic effect – a phenomenon where an external magnetic field alters the optical properties of the medium, leading to coupling between the two field components q and r. This effect manifests as magnetically induced birefringence or Faraday rotation, enabling energy transfer between the coupled wave modes.

To get quiescent solitons of Eqs. (2) and (3), we consider the wave transformation:

$$q(x,t)=\phi_1(x)e^{i\lambda t},\quad r(x,t)=\phi_2(x)e^{i\lambda t}, \tag{4}$$

where $\phi_j(x)(j=1,2)$ are real-valued functions of x, while λ is a constant representing the frequency. Substituting (4) into Eqs. (2) and (3), we have

$$\begin{aligned}&-l\lambda\phi_1^l+a_1(p+l)\left[(p+l-1)\phi_1^{p+l-2}\phi_1'^2+\phi_1^{p+l-1}\phi_1''\right]\\&+\left[\frac{b_1}{\phi_1^{2n}}+\frac{b_2}{\phi_1^n}+b_3\phi_1^n+b_4\phi_1^{2n}+\frac{d_1}{\phi_2^{2n}}+\frac{d_2}{\phi_2^n}+d_3\phi_2^n+d_4\phi_2^{2n}\right]\phi_1^l=Q_1\phi_2^l,\end{aligned} \tag{5}$$

and

$$\begin{aligned}&-l\lambda\phi_2^l+a_2(p+l)\left[(p+l-1)\phi_2^{p+l-2}\phi_2'^2+\phi_2^{p+l-1}\phi_2''\right]\\&+\left[\frac{b_5}{\phi_2^{2n}}+\frac{b_6}{\phi_2^n}+b_7\phi_2^n+b_8\phi_2^{2n}+\frac{d_5}{\phi_1^{2n}}+\frac{d_6}{\phi_1^n}+d_7\phi_1^n+d_8\phi_1^{2n}\right]\phi_2^l=Q_2\phi_1^l.\end{aligned} \tag{6}$$

Now, let's assume:

$$\phi_2(x)=\eta\phi_1(x), \tag{7}$$

where η is a nonzero constant. Eqs. (5) and (6) become:

$$\begin{aligned}&-\eta^{2n}(l\lambda+\eta^lQ_1)\phi_1^{l+2n}+a_1\eta^{2n}(p+l)(p+l-1)\phi_1'^2\phi_1^{2n+p+l-2}+a_1\eta^{2n}(p+l)\phi_1''\phi_1^{2n+p+l-1}\\&+\eta^{2n}(\eta^nd_3+b_3)\phi_1^{3n+l}+\eta^{2n}(\eta^{2n}d_4+b_4)\phi_1^{4n+l}+\eta^n(d_2+\eta^nb_2)\phi_1^{l+n}+(\eta^{2n}b_1+d_1)\phi_1^l=0,\end{aligned} \tag{8}$$

and

$$\begin{aligned}&-\eta^{2n}(\eta^ll\lambda+Q_2)\phi_1^{l+2n}+a_2\eta^{2n+p+l}(p+l)(p+l-1)\phi_1'^2\phi_1^{2n+p+l-2}\\&+a_2\eta^{2n+p+l}(p+l)\phi_1''\phi_1^{2n+p+l-1}+\eta^{l+2n}(\eta^nb_7+d_7)\phi_1^{3n+l}\\&+\eta^{l+2n}(\eta^{2n}b_8+d_8)\phi_1^{4n+l}+\eta^{l+n}(d_6\eta^n+b_6)\phi_1^{l+n}+\eta^l(d_5\eta^{2n}+b_5)\phi_1^l=0.\end{aligned} \tag{9}$$

Eqs (8) and (9) are equivalent under the constraint conditions:

$$\frac{l\lambda+\eta^lQ_1}{l\lambda\eta^l+Q_2}=\frac{a_1}{a_2\eta^{p+l}}=\frac{\eta^nd_3+b_3}{\eta^{n+l}b_7+\eta^ld_7}=\frac{\eta^{2n}d_4+b_4}{\eta^{2n+l}b_8+\eta^ld_8}=\frac{d_2+\eta^nb_2}{d_6\eta^{l+n}+\eta^lb_6}=\frac{\eta^{2n}b_1+d_1}{d_5\eta^{l+2n}+\eta^lb_5}. \tag{10}$$

In order to solve Eq. (8), we choose $p=n$, then Eq. (8) simplifies to

$$\begin{aligned}&\eta^{2n}(l\lambda+\eta^lQ_1)\phi_1^{2n}+a_1\eta^{2n}(n+l)(n+l-1)\phi_1'^2\phi_1^{3n-2}+a_1\eta^{2n}(n+l)\phi_1''\phi_1^{3n-1}\\&+\eta^{2n}(\eta^nd_3+b_3)\phi_1^{3n}+\eta^{2n}(\eta^{2n}d_4+b_4)\phi_1^{4n}+\eta^n(d_2+\eta^nb_2)\phi_1^n+(\eta^{2n}b_1+d_1)=0.\end{aligned} \tag{11}$$

Balancing $\phi_1''\phi_1^{3n-1}$ with ϕ_1^{4n} in Eq. (11), gives $N=\frac{2}{n}$, where N is the balance number of Eq. (11). Since the balance number is not integer, then we take into consideration the transformation

$$\phi_1(x)=\left[\psi(x)\right]^{\frac{2}{n}}, \tag{12}$$

where $\psi(x)$ is a new function of x. Substituting Eq. (12) into (11), we get

$$\psi^5\psi'' + A_1\psi^4\psi'^2 + B_1\psi^8 + C_1\psi^6 + D_1\psi^4 + E_1\psi^2 + F_1 = 0, \tag{13}$$

where

$$A_1 = \frac{n+2l}{n},\ B_1 = \frac{n(\eta^{2n}d_4 + b_4)}{2a_1(n+l)},\ C_1 = \frac{n(\eta^n d_3 + b_3)}{2a_1(n+l)},$$
$$D_1 = -\frac{n(l\lambda + \eta^l Q_1)}{2a_1(n+l)},\ E_1 = \frac{n(b_2 + d_2\eta^{-n})}{2a_1(n+l)},\ F_1 = \frac{n(b_1 + d_1\eta^{-2n})}{2a_1(n+l)}.$$

In the next three sections, we will solve Eq. (13) by using the following methods.

3. Enhanced direct algebraic approach

Balancing $\psi^5\psi''$ and ψ^8 in Eq. (13), then we get $N=1$. Since the balance number obtained from Eq. (11) is non-integer, the transformation in Eq. (12) is employed to reformulate the equation into a form with an integer balance number, thereby enabling the standard balancing algorithm to be used. According to this method, we have the formal solution as [14]:

$$\psi(x) = K_0 + K_1\theta(x) + \frac{L_1}{\theta(x)},\quad L_1^2 + K_1^2 \neq 0, \tag{14}$$

where K_0, K_1 and L_1 are constants to be determined, while $\theta(x)$ satisfies

$$\theta'^2(x) = \sum_{j=0}^{4} \tau_j \theta^j(x) \tag{15}$$

where $\tau_j\,(j=0,1,2,3,4)$ are constants provided that $\tau_4 \neq 0$. Substituting (14) along with (15) into Eq. (12) and equating all the coefficients of $\theta^l(x), (l = 0, \pm1, \pm2, \ldots, \pm8)$ to zero, we obtain a system of algebraic equations, omitted here for simplicity. By solving this system of equations with the assistance of Mathematica or Maple, we obtain the following results:

Case–1: If $\tau_0 = \tau_1 = \tau_3 = 0$, then we have the following results:

$$K_0 = K_0,\quad L_1 = 0,\quad K_1 = K_1,\quad \tau_2 = 4B_1K_0^2,\quad \tau_4 = -2B_1K_1^2.$$

The relations $K_0 = K_0$ and $K_1 = K_1$ simply reflect identities, implying that both K_0 and K_1 are arbitrary free parameters in the solution, along with the following constraint conditions:

$$A_1 = -\frac{3}{2},\qquad C_1 = -4B_1K_0^2,\qquad D_1 = 3B_1K_0^4,\qquad E_1 = F_1 = 0. \tag{16}$$

When $\tau_2 > 0, \tau_4 < 0,$ we have the bright soliton solutions:

$$q(x,t) = \left[K_0 + \sqrt{2}K_0\,\text{sech}\left(2K_0\sqrt{B_1}\,x\right)\right]^{\frac{2}{n}} e^{i\lambda t}, \tag{17}$$

and

$$r(x,t) = \eta\left[K_0 + \sqrt{2}K_0\,\text{sech}\left(2K_0\sqrt{B_1}\,x\right)\right]^{\frac{2}{n}} e^{i\lambda t}, \tag{18}$$

provided $B_1 > 0$. The magneto-optic parameter Q_1 is absent explicitly in Eqs. (17) and (18); however, it contributes indirectly via the relation $D_1 = -\frac{n(l\lambda + \eta^l Q_1)}{2a_1(n+l)}$ which determines B_1. Thus, the soliton profiles remain influenced by the magneto-optic effect.

Case–2: If $\tau_0 = \dfrac{\tau_2^2}{4\tau_4}, \tau_1 = \tau_3 = 0$, then we have the following results:

<u>Result–1</u>:

$$K_1 = 0, \quad \tau_2 = -\frac{2L_1^2\tau_4}{K_0^2},$$

Here, K_0 and L_1 are arbitrary free parameters in the solution, along with the following constraint conditions:

$$A_1 = -\frac{3}{2}, \qquad B_1 = -\frac{L_1^2\tau_4}{2K_0^4}, \qquad C_1 = \frac{2L_1^2\tau_4}{K_0^2}, \qquad D_1 = E_1 = F_1 = 0. \tag{19}$$

When $\tau_2 \langle 0, \tau_4 \rangle 0$, we have the singular soliton solutions:

$$q(x,t) = \left[K_0 \left\{ 1 + \coth\left(\frac{L_1\sqrt{\tau_4}}{K_0} x \right) \right\} \right]^{\frac{2}{n}} e^{i\lambda t}, \tag{20}$$

$$r(x,t) = \eta \left[K_0 \left\{ 1 + \coth\left(\frac{L_1\sqrt{\tau_4}}{K_0} x \right) \right\} \right]^{\frac{2}{n}} e^{i\lambda t}, \tag{21}$$

and the dark soliton solutions

$$q(x,t) = \left[K_0 \left\{ 1 + \tanh\left(\frac{L_1\sqrt{\tau_4}}{K_0} x \right) \right\} \right]^{\frac{2}{n}} e^{i\lambda t}, \tag{22}$$

$$r(x,t) = \eta \left[K_0 \left\{ 1 + \tanh\left(\frac{L_1\sqrt{\tau_4}}{K_0} x \right) \right\} \right]^{\frac{2}{n}} e^{i\lambda t}. \tag{23}$$

In this case, the constraint $D_1 = 0$ together with the general relation $D_1 = -\dfrac{n(l\lambda + \eta^l Q_1)}{2a_1(n+l)}$ implies $l\lambda + \eta^l Q_1 = 0$, so the magneto-optic contribution $\eta^l Q_1$ is exactly cancelled by the term $l\lambda$, and the resulting soliton solutions no longer contain Q_1 explicitly. This occurs in the solutions (20)–(23) as well as in the subsequent solutions (25)–(28).

<u>Result–2</u>:

$$K_1 = \frac{K_0^2}{4L_1}, \; \tau_2 = -\frac{8L_1^2\tau_4}{K_0^2},$$

along with constraint conditions:

$$A_1 = -\frac{3}{2}, \qquad B_1 = -\frac{8L_1^2\tau_4}{K_0^4}, \qquad C_1 = \frac{32L_1^2\tau_4}{K_0^2}, \qquad D_1 = E_1 = F_1 = 0. \tag{24}$$

When $\tau_2 \langle 0, \tau_4 \rangle 0$, we have the dark soliton solutions:

$$q(x,t) = \left[\frac{K_0 \left\{ \tanh\left(\frac{2L_1\sqrt{\tau_4}}{K_0} x \right) + 1 \right\}^2}{2\tanh\left(\frac{2L_1\sqrt{\tau_4}}{K_0} x \right)} \right]^{\frac{2}{n}} e^{i\lambda t}, \tag{25}$$

$$r(x,t)=\eta\left[\frac{K_0\left\{\tanh\left(\frac{2L_1\sqrt{\tau_4}}{K_0}x\right)+1\right\}^2}{2\tanh\left(\frac{2L_1\sqrt{\tau_4}}{K_0}x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}, \quad (26)$$

and the singular soliton solutions

$$q(x,t)=\left[\frac{K_0\left\{\coth\left(\frac{2L_1\sqrt{\tau_4}}{K_0}x\right)+1\right\}^2}{2\coth\left(\frac{2L_1\sqrt{\tau_4}}{K_0}x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}, \quad (27)$$

$$r(x,t)=\eta\left[\frac{K_0\left\{\coth\left(\frac{2L_1\sqrt{\tau_4}}{K_0}x\right)+1\right\}^2}{2\coth\left(\frac{2L_1\sqrt{\tau_4}}{K_0}x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}. \quad (28)$$

Case–3: When $\tau_1=\tau_3=0$, then we have the following results:

(I) If $\tau_0=\frac{m^2(1-m^2)\tau_2^2}{(2m^2-1)^2\tau_4}$. Here m is the elliptic modulus of the Jacobi elliptic functions, with $0<m<1$, then we get

$$L_1=0,\quad \tau_2=-\frac{2K_0^2\tau_4}{K_1^2},$$

along with constraint conditions:

$$A_1=-\frac{3}{2},\quad B_1=-\frac{\tau_4}{2K_1^2},\quad C_1=\frac{2K_0^2\tau_4}{K_1^2},\quad D_1=-\frac{3K_0^4\tau_4(8m^4-8m^2+1)}{2K_1^2(2m^2-1)^2},\quad E_1=F_1=0. \quad (29)$$

Now, the Jacobi elliptic function solutions of Eqs. (2) and (3) are listed as

$$q(x,t)=\left[K_0\left\{1-\frac{\sqrt{2}m}{\sqrt{2m^2-1}}\operatorname{cn}\left(\frac{K_0}{K_1}\sqrt{-\frac{2\tau_4}{2m^2-1}}x\right)\right\}\right]^{\frac{2}{n}}e^{i\lambda t}, \quad (30)$$

and

$$r(x,t)=\eta\left[K_0\left\{1-\frac{\sqrt{2}m}{\sqrt{2m^2-1}}\operatorname{cn}\left(\frac{K_0}{K_1}\sqrt{-\frac{2\tau_4}{2m^2-1}}x\right)\right\}\right]^{\frac{2}{n}}e^{i\lambda t}. \quad (31)$$

Solutions (30) and (31) represent periodic wave solutions expressed in terms of Jacobi elliptic functions, which reduce to bright solitons in the limit $m\to 1^-$

$$q(x,t)=\left[K_0\left\{1-\sqrt{2}\operatorname{sech}\left(\frac{K_0\sqrt{-2\tau_4}}{K_1}x\right)\right\}\right]^{\frac{2}{n}}e^{i\lambda t}, \quad (32)$$

and

$$r(x,t)=\eta\left[K_0\left\{1-\sqrt{2}\operatorname{sech}\left(\frac{K_0\sqrt{-2\tau_4}}{K_1}x\right)\right\}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{33}$$

provided $\tau_4<0$. Since in general $D_1=-\frac{n(l\lambda+\eta^l Q_1)}{2a_1(n+l)}$, equating this with the explicit expression of D_1 in (29) yields a linear relation between λ and Q_1; therefore, the phase parameter λ, and hence the solutions (30)–(33), depend on the magneto-optic parameter Q_1 through this relation.

(II) If $\tau_0=\left((1-m^2)\tau_2^2\right)/\left((2-m^2)^2\tau_4\right)$ and $0<m<1$, then we get

$$K_1=0,\ \tau_4=\frac{2(m^2-1)\tau_2K_0^2}{L_1^2(m^2-2)^2},$$

along with constraint conditions:

$$A_1=-\frac{3}{2},\ \ B_1=\frac{\tau_2}{4K_0^2},\ \ C_1=-\tau_2,\ \ D_1=\frac{3K_0^2\tau_2m^4}{4(m^2-2)^2},\ \ E_1=F_1=0. \tag{34}$$

Now, the Jacobi elliptic function solutions of Eqs. (2) and (3) are defined as:

$$q(x,t)=\left[K_0-\frac{K_0\sqrt{2m^2-2}}{m\sqrt{m^2-2}\,\operatorname{dn}\left(\sqrt{-\frac{\tau_2}{m^2-2}}\,x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{35}$$

and

$$r(x,t)=\eta\left[K_0-\frac{K_0\sqrt{2m^2-2}}{m\sqrt{m^2-2}\,\operatorname{dn}\left(\sqrt{-\frac{\tau_2}{m^2-2}}\,x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{36}$$

provided $\tau_2>0$. Solutions (35) and (36) represent periodic dn-type Jacobi elliptic wave solutions of Eqs. (2) and (3).

(III) If $\tau_0=\frac{m^2\tau_2^2}{(m^2+1)^2\tau_4}$ and $0<m<1$, then we get

<u>Result–1</u>:

$$L_1=0,\ \ \tau_2=-\frac{2K_0^2\tau_4}{K_1^2},$$

along with constraint conditions:

$$A_1=-\frac{3}{2},\ \ B_1=-\frac{\tau_4}{2K_1^2},\ \ C_1=\frac{2K_0^2\tau_4}{K_1^2},\ \ D_1=-\frac{3K_0^4\tau_4(m^4-2m^2+1)}{2K_1^2(m^2+1)^2},\ \ E_1=F_1=0. \tag{37}$$

Now, the Jacobi elliptic function solutions of Eqs. (2) and (3) are enumerated as:

$$q(x,t)=\left[K_0+mK_0\sqrt{\frac{2}{m^2+1}}\operatorname{sn}\left(\frac{K_0}{K_1}\sqrt{\frac{2\tau_4}{m^2+1}}\,x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{38}$$

and

$$r(x,t)=\eta\left[K_0+mK_0\sqrt{\frac{2}{m^2+1}}\operatorname{sn}\left(\frac{K_0}{K_1}\sqrt{\frac{2\tau_4}{m^2+1}}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{39}$$

Solutions (38) and (39) correspond to periodic wave solutions described by the Jacobi elliptic sn-function, which reduce to dark soliton solutions in the limit $m\to 1^-$.

$$q(x,t)=\left[K_0+K_0\tanh\left(\frac{K_0\sqrt{\tau_4}}{K_1}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{40}$$

and

$$r(x,t)=\eta\left[K_0+K_0\tanh\left(\frac{K_0\sqrt{\tau_4}}{K_1}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{41}$$

provided $\tau_4>0$.

Result–2:

$$K_1=\frac{2K_0^2m}{L_1\left(m^2+6m+1\right)},$$
$$\tau_2=-\frac{\left(m^2+6m+1\right)L_1^2\left(m^2+1\right)\tau_4}{2K_0^2m^2},$$

along with constraint conditions:

$$A_1=-\frac{3}{2},\quad B_1=-\frac{\tau_4L_1^2\left(m^2+6m+1\right)^2}{8K_0^4m^2},\quad C_1=\frac{\tau_4L_1^2\left(m^2+6m+1\right)^2}{2K_0^2m^2},$$
$$D_1=-\frac{3\left(m^4-4m^3+6m^2-4m+1\right)L_1^2\tau_4}{8m^2},\quad E_1=F_1=0. \tag{42}$$

Now, the Jacobi elliptic function solutions of Eqs. (2) and (3) are defined as:

$$q(x,t)=\left[\frac{K_0\left\{\sqrt{2}m\operatorname{sn}\left(\frac{L_1\sqrt{2\tau_4\left(m^2+6m+1\right)}}{2K_0m}x\right)^2+\sqrt{m^2+6m+1}\operatorname{sn}\left(\frac{L_1\sqrt{2\tau_4\left(m^2+6m+1\right)}}{2K_0m}x\right)+\sqrt{2}\right\}}{\sqrt{m^2+6m+1}\operatorname{sn}\left(\frac{L_1\sqrt{2\tau_4\left(m^2+6m+1\right)}}{2K_0m}x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{43}$$

and

$$r(x,t)=\eta\left[\frac{K_0\left\{\sqrt{2}m\operatorname{sn}\left(\frac{L_1\sqrt{2\tau_4\left(m^2+6m+1\right)}}{2K_0m}x\right)^2+\sqrt{m^2+6m+1}\operatorname{sn}\left(\frac{L_1\sqrt{2\tau_4\left(m^2+6m+1\right)}}{2K_0m}x\right)+\sqrt{2}\right\}}{\sqrt{m^2+6m+1}\operatorname{sn}\left(\frac{L_1\sqrt{2\tau_4\left(m^2+6m+1\right)}}{2K_0m}x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{44}$$

Solutions (43) and (44) describe periodic Jacobi elliptic wave solutions of the sn-type, which reduce to the same dark solutions as (25) - (26) when the elliptic modulus approaches $m \to 1^-$.

Case–4: When $\tau_0 = \tau_1 = 0$, then we have the following results

$$L_1 = 0, \quad \tau_2 = -\frac{K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)}{2K_1^2},$$

along with constraint conditions:

$$\begin{aligned} &A_1 = -\frac{3}{2}, \quad B_1 = -\frac{\tau_4}{2K_1^2}, \quad C_1 = \frac{K_0\left(8K_0\tau_4 - 3K_1\tau_3\right)}{4K_1^2}, \\ &D_1 = -\frac{3K_0^3\left(2K_0\tau_4 - K_1\tau_3\right)}{4K_1^2}, \quad E_1 = F_1 = 0. \end{aligned} \tag{45}$$

When $\tau_2 > 0, \tau_4 > 0$ and $K_0\left(4K_0\tau_4 - 3K_1\tau_3\right) < 0$, we have the soliton solutions

$$q(x,t) = \left[K_0 + \frac{K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)\operatorname{sech}\left(\frac{1}{4}\sqrt{-\frac{2K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)}{K_1^2}}\,x\right)^2}{2\sqrt{-2K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)\tau_4}\tanh\left(\frac{1}{4}\sqrt{-\frac{2K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)}{K_1^2}}\,x\right) + 2K_1\tau_3}\right]^{\frac{2}{n}} e^{i\lambda t}, \tag{46}$$

$$r(x,t) = \eta\left[K_0 + \frac{K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)\operatorname{sech}\left(\frac{1}{4}\sqrt{-\frac{2K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)}{K_1^2}}\,x\right)^2}{2\sqrt{-2K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)\tau_4}\tanh\left(\frac{1}{4}\sqrt{-\frac{2K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)}{K_1^2}}\,x\right) + 2K_1\tau_3}\right]^{\frac{2}{n}} e^{i\lambda t}, \tag{47}$$

and

$$q(x,t) = \left[K_0 - \frac{K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)\operatorname{csch}\left(\frac{1}{4}\sqrt{-\frac{2K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)}{K_1^2}}\,x\right)^2}{2\sqrt{-2K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)\tau_4}\coth\left(\frac{1}{4}\sqrt{-\frac{2K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)}{K_1^2}}\,x\right) + 2K_1\tau_3}\right]^{\frac{2}{n}} e^{i\lambda t}, \tag{48}$$

$$r(x,t) = \eta\left[K_0 - \frac{K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)\operatorname{csch}\left(\frac{1}{4}\sqrt{-\frac{2K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)}{K_1^2}}\,x\right)^2}{2\sqrt{-2K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)\tau_4}\coth\left(\frac{1}{4}\sqrt{-\frac{2K_0\left(4K_0\tau_4 - 3K_1\tau_3\right)}{K_1^2}}\,x\right) + 2K_1\tau_3}\right]^{\frac{2}{n}} e^{i\lambda t}. \tag{49}$$

Solutions (46)–(49) describe mixed bright–dark soliton solutions, where the sech component generates a bright-type profile, and the tanh/ coth component produces a dark-type modulation.

(II) When $\tau_2 > 0, \tau_3 \neq 0$ and $K_0(4K_0\tau_4 - 3K_1\tau_3) < 0$, we have the soliton solutions

$$q(x,t) = \left[K_0 + \frac{K_0K_1(4K_0\tau_4 - 3K_1\tau_3)\tau_3 \operatorname{sech}\left(\frac{1}{4}\sqrt{-\frac{2K_0(4K_0\tau_4 - 3K_1\tau_3)}{K_1^2}}x\right)^2}{2K_1^2\tau_3^2 + K_0(4K_0\tau_4 - 3K_1\tau_3)\tau_4\left\{1 - \tanh\left(\frac{1}{4}\sqrt{-\frac{2K_0(4K_0\tau_4 - 3K_1\tau_3)}{K_1^2}}x\right)\right\}^2}\right]^{\frac{2}{n}} e^{i\lambda t}, \tag{50}$$

$$r(x,t) = \eta\left[K_0 + \frac{K_0K_1(4K_0\tau_4 - 3K_1\tau_3)\tau_3 \operatorname{sech}\left(\frac{1}{4}\sqrt{-\frac{2K_0(4K_0\tau_4 - 3K_1\tau_3)}{K_1^2}}x\right)^2}{2K_1^2\tau_3^2 + K_0(4K_0\tau_4 - 3K_1\tau_3)\tau_4\left\{1 - \tanh\left(\frac{1}{4}\sqrt{-\frac{2K_0(4K_0\tau_4 - 3K_1\tau_3)}{K_1^2}}x\right)\right\}^2}\right]^{\frac{2}{n}} e^{i\lambda t}, \tag{51}$$

and

$$q(x,t) = \left[K_0 - \frac{K_0K_1(4K_0\tau_4 - 3K_1\tau_3)\tau_3 \operatorname{csch}\left(\frac{1}{4}\sqrt{-\frac{2K_0(4K_0\tau_4 - 3K_1\tau_3)}{K_1^2}}x\right)^2}{2K_1^2\tau_3^2 + K_0(4K_0\tau_4 - 3K_1\tau_3)\tau_4\left\{1 - \coth\left(\frac{1}{4}\sqrt{-\frac{2K_0(4K_0\tau_4 - 3K_1\tau_3)}{K_1^2}}x\right)\right\}^2}\right]^{\frac{2}{n}} e^{i\lambda t}, \tag{52}$$

$$r(x,t) = \eta\left[K_0 - \frac{K_0K_1(4K_0\tau_4 - 3K_1\tau_3)\tau_3 \operatorname{csch}\left(\frac{1}{4}\sqrt{-\frac{2K_0(4K_0\tau_4 - 3K_1\tau_3)}{K_1^2}}x\right)^2}{2K_1^2\tau_3^2 + K_0(4K_0\tau_4 - 3K_1\tau_3)\tau_4\left\{1 - \coth\left(\frac{1}{4}\sqrt{-\frac{2K_0(4K_0\tau_4 - 3K_1\tau_3)}{K_1^2}}x\right)\right\}^2}\right]^{\frac{2}{n}} e^{i\lambda t}. \tag{53}$$

Solutions (50)–(53) describe straddled bright–dark soliton solutions, where the sech term generates a bright component while the tanh or coth term produces a dark or anti-dark component.

4. New mapping method

Balancing $\psi^5\psi''$ and ψ^8 in Eq. (13), then we get $N = 1$. According to this method [16], we have the formal solution as:

$$\psi(x) = L_0 + L_1 f(x) + L_2 f^2(x), \quad L_2 \neq 0, \tag{54}$$

while $f(x)$ satisfies the following first order ordinary differential equation (ODE):

$$f'^2(x) = r + pf^2(x) + \frac{1}{2}qf^4(x) + \frac{1}{3}sf^6(x), \quad s \neq 0, \tag{55}$$

where L_0, L_1, L_2, r, p, q, and s are arbitrary constants. Substituting (54) along with (55) into Eq. (12) and equating all the coefficients of $f^k(x)[f'(x)]^j, (k = 0,1,2,\ldots,16, j = 0,1)$, to zero, we obtain a system of algebraic equations, omitted here for simplicity. Here, the symbols r and q appearing in Eq. (55) denote constant parameters of the auxiliary ordinary differential equation and should not be confused with the functions $r(x,t)$ and $q(x,t)$ introduced earlier in Eqs. (2) and (3).

By solving this system of equations with the assistance of Mathematica or Maple, we obtain the following results:

Case–1: If $s = \frac{3q^2}{16p}, r = \frac{16p^2}{27q}$, then we have the following results $L_1 = 0, \ p = \frac{3L_0 q}{4L_2}$, along with constraint conditions:

$$A_1 = -\frac{3}{2}, \quad B_1 = -\frac{q}{6L_0 L_2}, \quad C_1 = -\frac{L_0 q}{2L_2}, \quad D_1 = E_1 = F_1 = 0, \tag{56}$$

where L_0 and L_2 are arbitrary constants.

From (54), (56), and $f_1(x) - f_2(x)$ of step 4 of the new mapping approach obtained in [8], we get the bright soliton solutions:

$$q(x,t) = \left[\frac{3L_0}{4\cosh^2\left(\frac{\epsilon}{2}\sqrt{-\frac{L_0 q}{L_2}}\,x\right) - 1}\right]^{\frac{2}{n}} e^{i\lambda t}, \tag{57}$$

and

$$r(x,t) = \eta\left[\frac{3L_0}{4\cosh^2\left(\frac{\epsilon}{2}\sqrt{-\frac{L_0 q}{L_2}}\,x\right) - 1}\right]^{\frac{2}{n}} e^{i\lambda t}. \tag{58}$$

We also have the bright soliton solutions:

$$q(x,t) = \left[-\frac{3L_0}{4\cosh^2\left(\frac{\epsilon}{2}\sqrt{-\frac{L_0 q}{L_2}}\,x\right) - 3}\right]^{\frac{2}{n}} e^{i\lambda t}, \tag{59}$$

and

$$r(x,t) = \eta\left[-\frac{3L_0}{4\cosh^2\left(\frac{\epsilon}{2}\sqrt{-\frac{L_0 q}{L_2}}\,x\right) - 3}\right]^{\frac{2}{n}} e^{i\lambda t}, \tag{60}$$

provided $qL_0L_2<0$ and $\epsilon=\pm1$. In this case, the constraint $D_1=0$ together with the general relation $D_1=-\frac{n(l\lambda+\eta^l Q_1)}{2a_1(n+l)}$ implies that the resulting soliton solutions no longer contain Q_1 explicitly. This occurs in solutions (57)–(60).

Case–2: If $s=\frac{3q^2}{16p}, r=0$, then we have the following results:

$$L_1=0,\quad p=\frac{L_0 q}{2L_2},$$

along with constraint conditions:

$$A_1=-\frac{3}{2},\quad B_1=-\frac{q}{4L_0L_2},\quad C_1=-\frac{L_0 q}{2L_2},\quad D_1=\frac{3L_0^3 q}{4L_2},\quad E_1=F_1=0, \tag{61}$$

where L_0 and L_2 are arbitrary constants. From (54), (61), and $f_5(x)-f_6(x)$ of step 4 of the new mapping approach obtained in [8], we get the dark soliton solutions:

$$q(x,t)=\left[-L_0\tanh\left(\epsilon\sqrt{\frac{L_0 q}{2L_2}}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{62}$$

and

$$r(x,t)=\eta\left[-L_0\tanh\left(\epsilon\sqrt{\frac{L_0 q}{2L_2}}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{63}$$

We also have the singular soliton solutions:

$$q(x,t)=\left[-L_0\coth\left(\epsilon\sqrt{\frac{L_0 q}{2L_2}}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{64}$$

and

$$r(x,t)=\eta\left[-L_0\coth\left(\epsilon\sqrt{\frac{L_0 q}{2L_2}}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{65}$$

provided $qL_0L_2>0$ and $\epsilon=\pm1$.

Case–3: If $r=0$, then we have the following results:

$$L_1=0,\quad p=-\frac{L_0(8L_0 s-9L_2 q)}{12L_2^2},$$

along with constraint conditions:

$$A_1=-\frac{3}{2},\quad B_1=-\frac{2s}{3L_2^2},\quad C_1=\frac{(16L_0 s-9L_2 q)L_0}{6L_2^2},\quad D_1=\frac{3L_0^3L_2 q-4L_0^4 s}{2L_2^2},\quad E_1=F_1=0, \tag{66}$$

where L_0 and L_2 are arbitrary constants. From (54), (66), and $f_7(x)-f_8(x)$ of step 4 of the new mapping approach obtained in [8], we get the soliton solutions. These solutions correspond to bright kink-type solitons, generated by a sech^3-profile modulated by a tanh function.

$$q(x,t)=$$

$$\left[L_0+\frac{3qL_0L_2(8L_0s-9L_2q)\mathrm{sech}^2\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)}{18L_2^2q^2+2sL_0(8L_0s-9L_2q)\left\{1+\epsilon\tanh^2\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{67}$$

and

$$r(x,t)=$$

$$\eta\left[L_0+\frac{3qL_0L_2(8L_0s-9L_2q)\mathrm{sech}^2\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)}{18L_2^2q^2+2sL_0(8L_0s-9L_2q)\left\{1+\epsilon\tanh^2\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{68}$$

We also have the soliton solutions. These solutions correspond to bright anti-kink solitons, produced by a sech^3-profile combined with a coth function.

$$q(x,t)=$$

$$\left[L_0-\frac{3qL_0L_2(8L_0s-9L_2q)\mathrm{csch}^2\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)}{18L_2^2q^2+2sL_0(8L_0s-9L_2q)\left\{1-\epsilon\coth^2\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{69}$$

and

$$r(x,t)=$$

$$\eta\left[L_0-\frac{3qL_0L_2(8L_0s-9L_2q)\mathrm{csch}^2\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)}{18L_2^2q^2+2sL_0(8L_0s-9L_2q)\left\{1-\epsilon\coth^2\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{70}$$

provided $L_0(8L_0s-9L_2q)<0$ and $\epsilon=\pm1$.

From (54), (66), and $f_9(x)-f_{10}(x)$ of step 4 of the new mapping approach obtained in [8], we get soliton solutions. These solutions correspond to bright kink–type solitons, constructed from a combination of sech^3 and tanh profiles.

$$q(x,t)=$$

$$\left[L_0+\frac{L_0(8L_0s-9L_2q)\mathrm{sech}^2\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)}{2L_2\left\{3q+2\epsilon\sqrt{-\frac{L_0s(8L_0s-9L_2q)}{L_2^2}}\tanh\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{71}$$

and

$$r(x,t)=$$
$$\eta\left[L_0+\frac{L_0(8L_0s-9L_2q)\operatorname{sech}^2\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)}{2L_2\left\{3q+2\epsilon\sqrt{-\frac{L_0s(8L_0s-9L_2q)}{L_2^2}}\tanh\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{72}$$

We also have the soliton solutions: These solutions correspond to bright anti-kink solitons, represented by the mixed sech^3–coth structure.

$$q(x,t)=$$
$$\left[L_0-\frac{L_0(8L_0s-9L_2q)\operatorname{csch}^2\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)}{2L_2\left\{3q+2\epsilon\sqrt{-\frac{L_0s(8L_0s-9L_2q)}{L_2^2}}\coth\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{73}$$

and

$$r(x,t)=$$
$$\eta\left[L_0-\frac{L_0(8L_0s-9L_2q)\operatorname{csch}^2\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)}{2L_2\left\{3q+2\epsilon\sqrt{-\frac{L_0s(8L_0s-9L_2q)}{L_2^2}}\coth\left(\frac{1}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{74}$$

ovided $L_0(8L_0s-9L_2q)\langle 0,s\rangle 0$ and $\epsilon=\pm 1$.

From (54), (66), and $f_{13}(x)-f_{14}(x)$ of step 4 of the new mapping approach obtained in [8], we get the bright soliton solutions:

$$q(x,t)=$$
$$\left[L_0-\frac{L_0(8L_0s-9L_2q)}{L_2\left\{\begin{array}{l}2\sqrt{\frac{4L_0(8L_0s-9L_2q)s}{L_2^2}+9q^2}\cosh^2\left(\frac{\epsilon}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)\\-\sqrt{\frac{4L_0(8L_0s-9L_2q)s}{L_2^2}+9q^2}-3q\end{array}\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{75}$$

and

$$r(x,t)=$$
$$\eta\left[L_0-\frac{L_0(8L_0s-9L_2q)}{L_2\left\{\begin{array}{l}2\sqrt{\frac{4L_0(8L_0s-9L_2q)s}{L_2^2}+9q^2}\cosh^2\left(\frac{\epsilon}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)\\-\sqrt{\frac{4L_0(8L_0s-9L_2q)s}{L_2^2}+9q^2}-3q\end{array}\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{76}$$

We also have the singular soliton solutions:

$$q(x,t)=\left[L_0-\frac{L_0(8L_0s-9L_2q)}{L_2\left\{\begin{array}{l}2\sqrt{\frac{4L_0(8L_0s-9L_2q)s}{L_2^2}+9q^2}\sinh^2\left(\frac{\epsilon}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)\\+\sqrt{\frac{4L_0(8L_0s-9L_2q)s}{L_2^2}+9q^2}-3q\end{array}\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{77}$$

and

$$r(x,t)=\eta\left[L_0-\frac{L_0(8L_0s-9L_2q)}{L_2\left\{\begin{array}{l}2\sqrt{\frac{4L_0(8L_0s-9L_2q)s}{L_2^2}+9q^2}\sinh^2\left(\frac{\epsilon}{6}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)\\+\sqrt{\frac{4L_0(8L_0s-9L_2q)s}{L_2^2}+9q^2}-3q\end{array}\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{78}$$

provided $L_0(8L_0s-9L_2q)<0, s<0$ and $\epsilon=\pm1$.

From (54), (66) and $f_{17}(x)$, and $f_{20}(x)$ of step 4 of the new mapping approach obtained in [8], we get the bright soliton solutions:

$$q(x,t)=\left[L_0-\frac{L_0(8L_0s-9L_2q)}{L_2\left\{\begin{array}{l}\epsilon\sqrt{\frac{4L_0(8L_0s-9L_2q)s}{L_2^2}+9q^2}\\\cosh\left(\frac{1}{3}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)-3q\end{array}\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{79}$$

and

$$r(x,t)=\eta\left[L_0-\frac{L_0(8L_0s-9L_2q)}{L_2\left\{\begin{array}{l}\epsilon\sqrt{\frac{4L_0(8L_0s-9L_2q)s}{L_2^2}+9q^2}\\\cosh\left(\frac{1}{3}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)-3q\end{array}\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{80}$$

We also have the singular soliton solutions:

$$q(x,t)=\left[L_0-\frac{L_0(8L_0s-9L_2q)}{L_2\left\{\begin{array}{l}\epsilon\sqrt{\frac{-4L_0(8L_0s-9L_2q)s}{L_2^2}-9q^2}\\\sinh\left(\frac{1}{3}\sqrt{-\frac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)-3q\end{array}\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{81}$$

and

$$r(x,t)=\eta\left[L_0-\frac{L_0(8L_0s-9L_2q)}{L_2\left\{\begin{array}{l}\epsilon\sqrt{\dfrac{-4L_0(8L_0s-9L_2q)s}{L_2^2}-9q^2}\\ \sinh\left(\dfrac{1}{3}\sqrt{-\dfrac{3L_0(8L_0s-9L_2q)}{L_2^2}}x\right)-3q\end{array}\right\}}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{82}$$

provided $L_0(8L_0s-9L_2q)<0$ and $\epsilon=\pm1$.

5. The extended auxiliary equation algorithm

Balancing $\psi^5\psi''$ and ψ^8 in Eq. (13), then we get $N=1$. According to this method, we have the formal solution as [17]:

$$\psi(x)=L_0+L_1F(x)+L_2F^2(x),\quad L_2\neq 0, \tag{83}$$

where L_0, L_1 and L_2 are constants, while $F(x)$ satisfies:

$$F'^2(x)=C_0+C_2F^2(x)+C_4F^4(x)+C_6F^6(x),\quad C_6\neq 0, \tag{84}$$

where $C_j\,(j=0,2,4,6)$ are constants. Eq. (84) holds

$$F(x)=\frac{1}{2}\left[-\frac{C_4}{C_6}(1\pm f(x))\right]^{\frac{1}{2}}. \tag{85}$$

Here, the term $f(x)$ denotes the functional form obtained from solving the auxiliary polynomial differential equation (84). According to [18], $f(x)$ can take various forms expressed through the Jacobi elliptic functions sn, cn, dn, and their reciprocals, or their trigonometric/hyperbolic degeneracies. The notation $f(x)$ is therefore used as a compact representation of these possible solutions.

Substituting (83) along with (84) into Eq. (12) and equating all the coefficients of $F^k(x)\left[F'(x)\right]^j,(k=0,1,2,\ldots,16,j=0,1)$, to zero, then we obtain a system of algebraic equations which are omitted here for simplicity. By solving this system of equations with the assistance of Mathematica or Maple, we obtain the following results:

$$L_1=0,\ C_4=\frac{4L_0C_6}{L_2},$$

along with constraint conditions:

$$\begin{gathered}A_1=-\frac{1}{2},\quad B_1=-\frac{6C_6}{L_2^2},\quad C_1=\frac{12L_0^2C_6-2L_2^2C_2}{L_2^2},\\ D_1=-\frac{2L_0\left(3L_0^3C_6-L_0L_2^2C_2+L_2^3C_0\right)}{L_2^2},\quad E_1=F_1=0,\end{gathered} \tag{86}$$

where L_0 and L_2 are arbitrary constants.

Family-1. If $C_0=\dfrac{C_4^3(m^2-1)}{32C_6^2m^2}, C_2=\dfrac{C_4^2(5m^2-1)}{16C_6m^2}, C_6>0$ and $0<m<1$, then

$$q(x,t)=\left[-L_0\,\mathrm{sn}\left(\frac{2L_0\sqrt{C_6}}{L_2m}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{87}$$

$$r(x,t)=\eta\left[-L_0\,\mathrm{sn}\left(\frac{2L_0\sqrt{C_6}}{L_2 m}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{88}$$

and

$$q(x,t)=\left[-\frac{L_0}{m}\mathrm{ns}\left(\frac{2L_0\sqrt{C_6}}{L_2 m}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{89}$$

$$r(x,t)=\eta\left[-\frac{L_0}{m}\mathrm{ns}\left(\frac{2L_0\sqrt{C_6}}{L_2 m}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{90}$$

When $m\to 1^-$, then we get dark soliton solutions

$$q(x,t)=\left[-L_0\tanh\left(\frac{2L_0\sqrt{C_6}}{L_2}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{91}$$

$$r(x,t)=\eta\left[-L_0\tanh\left(\frac{2L_0\sqrt{C_6}}{L_2}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{92}$$

and the singular soliton solutions

$$q(x,t)=\left[-L_0\coth\left(\frac{2L_0\sqrt{C_6}}{L_2}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{93}$$

$$r(x,t)=\eta\left[-L_0\coth\left(\frac{2L_0\sqrt{C_6}}{L_2}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{94}$$

Family-2. If $C_0=\dfrac{C_4^3(1-m^2)}{32C_6^2}, C_2=\dfrac{C_4^2(5-m^2)}{16C_6}, C_6>0$ and $0<m<1$, then

$$q(x,t)=\left[-mL_0\,\mathrm{sn}\left(\frac{2L_0\sqrt{C_6}}{L_2}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{95}$$

$$r(x,t)=\eta\left[-mL_0\,\mathrm{sn}\left(\frac{2L_0\sqrt{C_6}}{L_2}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{96}$$

and

$$q(x,t)=\left[-L_0\,\mathrm{ns}\left(\frac{2L_0\sqrt{C_6}}{L_2}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{97}$$

$$r(x,t)=\eta\left[-L_0\,\mathrm{ns}\left(\frac{2L_0\sqrt{C_6}}{L_2}x\right)\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{98}$$

When $m\to 1^-$, then (95) and (96) reveal the same dark soliton solutions as (91) and (92), while (97) and (98) reveal the same singular soliton solutions as (93) and (94).

Family-3. If $C_0=\frac{C_4^3}{32m^2C_6^2}, C_2=\frac{C_4^2\left(4m^2+1\right)}{16C_6m^2}, C_6<0$ and $0<m<1$, then

$$q(x,t)=\left[-L_0\operatorname{cn}\left(\frac{2L_0\sqrt{-C_6}\,x}{L_2m}\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{99}$$

$$r(x,t)=\eta\left[-L_0\operatorname{cn}\left(\frac{2L_0\sqrt{-C_6}\,x}{L_2m}\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{100}$$

and

$$q(x,t)=\left[-\frac{L_0\sqrt{1-m^2}\operatorname{sn}\left(\frac{2L_0}{L_2m}\sqrt{-C_6}\,x\right)}{\operatorname{dn}\left(\frac{2L_0}{L_2m}\sqrt{-C_6}\,x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{101}$$

$$r(x,t)=\eta\left[-\frac{L_0\sqrt{1-m^2}\operatorname{sn}\left(\frac{2L_0}{L_2m}\sqrt{-C_6}\,x\right)}{\operatorname{dn}\left(\frac{2L_0}{L_2m}\sqrt{-C_6}\,x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{102}$$

When $m\to1^-$, then (99) and (100) leave bright soliton solutions. The solutions in (101) and (102) describe periodic elliptic-function solitons, specifically of the sn/dn–type, representing periodic bright-type wave trains.

$$q(x,t)=\left[-L_0\operatorname{sech}\left(\frac{2L_0\sqrt{-C_6}\,x}{L_2}\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{103}$$

$$r(x,t)=\eta\left[-L_0\operatorname{sech}\left(\frac{2L_0\sqrt{-C_6}\,x}{L_2}\right)\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{104}$$

Family-4. If $C_0=\frac{C_4^3m^2}{32C_6^2\left(m^2-1\right)}, C_2=\frac{C_4^2\left(5m^2-4\right)}{16C_6\left(m^2-1\right)}, C_6<0$ and $0<m<1$, then

$$q(x,t)=\left[-\frac{L_0}{\sqrt{1-m^2}}\operatorname{dn}\left(\frac{2L_0}{L_2}\sqrt{\frac{C_6}{\left(m^2-1\right)}}\,x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{105}$$

$$r(x,t)=\eta\left[-\frac{L_0}{\sqrt{1-m^2}}\operatorname{dn}\left(\frac{2L_0}{L_2}\sqrt{\frac{C_6}{\left(m^2-1\right)}}\,x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{106}$$

and

$$q(x,t)=\left[-L_0\operatorname{nd}\left(\frac{2L_0}{L_2}\sqrt{-\frac{C_6}{\left(m^2-1\right)}}\,x\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{107}$$

$$r(x,t)=\eta\left[-L_0\operatorname{nd}\left(\frac{2L_0}{L_2}\sqrt{-\frac{C_6}{\left(m^2-1\right)}}\,x\right)\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{108}$$

The solutions in (105)–(108) correspond to periodic cnoidal-type solitons, governed by the dn and nd elliptic functions; they form localized periodic wave solutions rather than isolated solitons.

Family-5. If $C_0=\frac{C_4^3}{32C_6^2\left(1-m^2\right)}, C_2=\frac{C_4^2\left(4m^2-5\right)}{16C_6\left(m^2-1\right)}, C_6>0$ and $0<m<1$, then

$$q(x,t)=\left[-\frac{L_0}{\operatorname{cn}\left(\frac{2L_0}{L_2}\sqrt{\frac{C_6}{1-m^2}}x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{109}$$

$$r(x,t)=\eta\left[-\frac{L_0}{\operatorname{cn}\left(\frac{2L_0}{L_2}\sqrt{\frac{C_6}{1-m^2}}x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{110}$$

and

$$q(x,t)=\left[-\frac{L_0\operatorname{dn}\left(\frac{2L_0}{L_2}\sqrt{-\frac{C_6}{\left(m^2-1\right)}}x\right)}{\sqrt{1-m^2}\operatorname{cn}\left(\frac{2L_0}{L_2}\sqrt{-\frac{C_6}{\left(m^2-1\right)}}x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{111}$$

$$r(x,t)=\eta\left[-\frac{L_0\operatorname{dn}\left(\frac{2L_0}{L_2}\sqrt{-\frac{C_6}{\left(m^2-1\right)}}x\right)}{\sqrt{1-m^2}\operatorname{cn}\left(\frac{2L_0}{L_2}\sqrt{-\frac{C_6}{\left(m^2-1\right)}}x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{112}$$

The solutions in (109)–(112) correspond to periodic cnoidal-type solitons (elliptic-function solitons), specifically cn/dn-modulated periodic waves.

Family-6. If $C_0=\frac{C_4^3m^2}{32C^2}, C_2=\frac{C_4^2\left(m^2+4\right)}{16C_6}, C_6<0$ and $0<m<1$, then

$$q(x,t)=\left[-L_0\operatorname{dn}\left(\frac{2L_0\sqrt{-C_6}\,x}{L_2}\right)\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{113}$$

$$r(x,t)=\eta\left[-L_0\operatorname{dn}\left(\frac{2L_0\sqrt{-C_6}\,x}{L_2}\right)\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{114}$$

and

$$q(x,t)=\left[-\frac{L_0\sqrt{1-m^2}}{\operatorname{dn}\left(\frac{2L_0\sqrt{-C6}}{L_2}x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}, \tag{115}$$

$$r(x,t)=\eta\left[-\frac{L_0\sqrt{1-m^2}}{\mathrm{dn}\left(\dfrac{2L_0\sqrt{-C6}}{L_2}x\right)}\right]^{\frac{2}{n}}e^{i\lambda t}. \tag{116}$$

When $m\to 1^-$, then (113) and (114) leave us the same bright soliton solutions as (103) and (104). The solutions in (115) and (116) correspond to bright solitons, obtained as the $m\to 1^-$, limit of the associated elliptic-function solutions.

6. Conclusions

The current paper retrieved and classified quiescent soliton solutions that emerged from magneto–optic waveguides, considering Kudryashov's form of SPM and generalized temporal evolution. Three integration algorithms showed a full spectrum of quiescent optical solitons. The results arose after using Jacobi's elliptic functions when the ellipticity modulus of the cnoidal waves approached unity. The results are solid, indicating a promising future for this research. For example, besides magneto–optic waveguides, these methods can also be applied to other optoelectronic devices, such as optical couplers, optical metamaterials, polarization-maintaining optical fibers, dispersion–flattened fibers, and gap solitons in Bragg gratings. These findings will be published later after further validation and alignment with existing work.

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Zayed, E. M. E., Saad, B. M. M., Arnous, A. H., Yildirim Ya., Biswas, A., Moraru, L., Dragomir, C. M. B. (2026). Quiescent Solitons in Magneto–Optic Waveguides Having Kudryashov's First Form of Nonlinear Self-Phase Modulation Structure**.** *Ukrainian Journal of Physical Optics*, *27*(2), 02001–02022. doi: 10.3116/16091833/Ukr.J.Phys.Opt.2026.02001

Анотація. *Ця стаття описує отримання спокійних оптичних солітонів у магнітооптичних хвилеводах. Структури само фазової модуляції, що використовуються, були запропоновані Кудряшовим. Три алгоритми інтегрування зробили можливим отримання цих розв'язків. Вдосконалений прямий алгебраїчний метод, розширений підхід до допоміжних рівнянь та нова схема відображення разом дозволили відновити повний спектр спокійних оптичних солітонів. Також наведені параметричні обмеження для існування таких солітонів. Декілька чисельних симуляцій ілюструють аналітичний результат.*

Ключові слова: *солітони, магнітооптика, допоміжний алгоритм, схема відображення; алгебраїчний підхід*