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Propagation of Broad-Band Optical Pulses in Dispersionless Media

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Abstract. In the present paper, the regimes of propagation of laser pulses in isotropic dispersionless ($\beta\approx0$) media, such as hollow microstructured optical fibers (photonic crystal fibers), are presented. The nonlinear amplitude equation (NAE) is used to describe the evolution of such pulses, which differs from the nonlinear Schrödinger equation by two additional nonparaxial terms. Linear and nonlinear regimes of propagation are considered. In the linear regime, when only diffraction effects dominate the evolution of the laser pulse, its shape is preserved, but the position of the pulse shifts with the distance. This is due to the influence of the nonparaxial term in NAE. In the nonlinear propagation regime, the obtained solution of the NAE describes a dark soliton. It is formed as a result of the balance between the effects of diffraction and the nonlinearity of the medium.

Keywords: photonic crystal fibers, nonlinear amplitude equation, optical solitons, isotropic dispersionless media
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1. Introduction

The study of the linear and nonlinear regime of propagation of ultra-short optical pulses in isotropic dispersionless media such as air [1-3] and some hollow microstructured optical fibers (photonic crystal fibers) [4-6] is a rapidly developing field in modern laser physics. It is known that for air, the dispersion of ultra-short optical pulses is negligibly small ($\beta \approx 0$). Therefore, during the evolution of femtosecond laser pulses at distances of tens and hundreds of diffraction lengths, it does not have a significant effect, and in this case, it can be neglected. The propagation regimes of optical pulses in such media are usually described by the well-known nonlinear Schrödinger equation (NSE) [7,8]. It is derived for narrow-band pulses ($\Delta\lambda \ll \lambda_0$, where $\Delta\lambda$ is the initial bandwidth of the pulse and λ_0 is the carrier wavelength) and characterizes very well their behavior in the nanosecond and picosecond regions. However, within the femtosecond and attosecond regions, where the pulses are broad-band ($\Delta\lambda \approx \lambda_0$), the more general nonlinear amplitude equation (NAE) must be used [9,10]. It correctly describes long as well as ultra-short optical pulses. The NAE differs from the NSE by two nonparaxial terms that characterize the longitudinal diffraction divergence. The nonparaxial diffraction of ultra-short optical pulses has been widely discussed in [11].

The linear propagation regime of laser pulses in isotropic media is well studied. Due to the absence of nonlinear and dispersive effects, the shape and spectrum of the pulse are preserved. In the case of a nonlinear regime, depending on the values of the light radiation intensity, different effects can be observed. One of the most interesting phenomena in waveguide optics is the formation of laser solitons. They belong to a special class of wave packages, where the balance between the effects of dispersion and nonlinearity forms a stable optical pulse [12-17]. Different types of solitons exist depending on the sign of the dispersion: the combination of anomalous dispersion and Kerr-type nonlinearity generates bright optical solitons [17,18], the balance between normal dispersion and nonlinearity leads to the formation of dark laser solitons, where a deep gap in pulse's intensity is observed [17,19]. Usually, solitons can be obtained experimentally in media such as planar waveguides and optical fibers. In recent years, the possibility of a registered soliton regime of propagation of ultra-short high-intensity laser pulses in air [20-22] was studied. The atmospheric air in mid-infrared and long-wavelength infrared regions is transparent [23].

The present paper investigates the evolution of ultra-short broad-band laser pulses governed by NAE in isotropic (linear and nonlinear) dispersionless ($\beta \approx 0$) media. New analytical solutions are found. The solution presents a stable optical pulse with Gaussian form in the linear case. In the nonlinear regime of propagation, a dark soliton is observed. Despite the absence of dispersion, the soliton is formed due to the balance between the diffractive and nonlinear effects of the medium.

2. Main equation

The standard optical laser pulse is usually linearly polarized in a plane perpendicular to the direction of propagation. In the geometry of the hollow microstructured optical fibers, the 3D+1 nonlinear amplitude equation is reduced to the 1D+1 equation. For this reason, in isotropic nonlinear dispersive media, the NAE in Cartesian coordinate system takes the form:

$$\frac{\partial^2 A'}{\partial z^2} + 2ik_0\left[\frac{\partial A'}{\partial z} + \frac{1}{v}\frac{\partial A'}{\partial t}\right] - k_0 k''\frac{\partial^2 A'}{\partial t^2} - \frac{1}{v^2}\frac{\partial^2 A'}{\partial t^2} + n_2 k_0^2 |A'|^2 A' = 0, \quad (1)$$

where $A'(t,z)$ is the amplitude function, describing the envelope of the laser pulse, $k_0 = 2\pi/\lambda_0$ is its wavenumber, λ_0 is the carrier wavelength, v is the group velocity, k'' is a coefficient, characterizing the group velocity dispersion and n_2 is the nonlinear refractive index.

When studying the evolution of one-dimensional pulses in isotropic nonlinear dispersive media, it is convenient to write the amplitude equation in a "local time" coordinate system. Thus, the following substitutions are made:

$$\tau = \frac{t - \frac{z}{v}}{T_0},\ \xi = \frac{z}{z_0},\ z_0 = vT_0,\ A = \frac{A'}{A_0}, \quad (2)$$

where T_0, z_0 and A_0 are the corresponding initial time duration, the initial length of the pulse and its initial amplitude function.

By using the expression (2), NAE (1) can be presented in dimensionless form. This is quite useful because when solving the equation, it is not needed to check the dimensions after each mathematical operation of differentiation or integration.

Next, the following constants are introduced:

$$\alpha = k_0 z_0,\ \beta = k_0 v^2 k'',\ \gamma = \alpha n_2 |A_0|^2. \quad (3)$$

In the expressions above the constant α characterizes the number of oscillations under the pulse envelope and usually $\alpha \gg 1$. The coefficients β and γ describe respectively the linear dispersion and the nonlinearity of the medium. In this way, by using the substitutions (3) and after a couple of transformations, the one-dimensional NAE, in a "local time" coordinate system, can be presented in a scalar, dimensionless form as follows:

$$i\frac{\partial A}{\partial \xi} + \frac{1}{2\alpha}\left[\frac{\partial^2 A}{\partial \xi^2} - 2\frac{\partial^2 A}{\partial \xi \partial \tau}\right] - \frac{\beta}{2\alpha}\frac{\partial^2 A}{\partial \tau^2} + \gamma |A|^2 A = 0. \quad (4)$$

This is a nonlinear partial differential equation of second order and third degree. It differs significantly from the well-known NSE by the presence of the second term in the brackets. It is important to mention here that physically, this term describes the longitudinal diffraction divergence, which is characteristic of broad-band laser pulses. It is connected with the nonparaxiality in the evolution of such optical pulses. Whether this term is counted depends on the coefficient before the brackets ($1/2\alpha$). Its value is very small for long laser pulses, and the brackets' expression can be neglected. Then, NAE is reduced to NSE. In the case of ultra-short optical pulses, however, this term has considerable value, and therefore, the expression in the brackets must be taken into account. Table 1 presents some values of the coefficient $1/2\alpha$ for laser pulses with different initial time durations (T_0) and carrier wavelength $\lambda = 800nm$.

Table 1. Values of the coefficient $1/2\alpha$ for laser pulses with different initial time durations

T_0	$1/2\alpha$
1 fs	3.1×10^{-1}
5 fs	6.2×10^{-2}
8 fs	3.8×10^{-2}
10 fs	3.0×10^{-2}
50 fs	6.2×10^{-3}
300 fs	1.1×10^{-3}
50 ps	6.2×10^{-6}
50 ns	6.2×10^{-9}

In the present paper, we will investigate the propagation of broad-band optical pulses in a waveguide medium with negligible dispersion, such as hollow microstructured optical fibers (photonic crystal fibers). In this case, $\beta \approx 0$ and Eq. (4) take the form:

$$2i\alpha\frac{\partial A}{\partial \xi} + \frac{\partial^2 A}{\partial \xi^2} - 2\frac{\partial^2 A}{\partial \xi \partial \tau} + 2\alpha\gamma |A|^2 A = 0. \quad (5)$$

3. Linear regime of propagation of broad-band optical pulses in a dispersionless medium

We will first study the linear propagation regime of laser pulses in a microstructured hollow fiber. In this case, $\gamma \approx 0$. It is known that under these conditions, dispersive and nonlinear effects do not affect the evolution of light pulses. Thus, the Eq. (5) takes the form:

$$2i\alpha\frac{\partial A}{\partial \xi} + \frac{\partial^2 A}{\partial \xi^2} - 2\frac{\partial^2 A}{\partial \xi \partial \tau} = 0. \quad (6)$$

This is a partial differential equation of second order and third degree for the amplitude function $A(\xi,\tau)$. Its solution can easily be found by using the Fourier transform:

$$A(\xi,\tau)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{A}(\xi,\omega)e^{-i\omega\tau}d\omega, \tag{7}$$

where ω is the carrier frequency of the optical pulse. By substituting the expression above in Eq. (6) and after a couple of transformations, we obtain:

$$\frac{d^2\tilde{A}}{d\xi^2}+2i(\alpha+\omega)\frac{d\tilde{A}}{d\xi}=0. \tag{8}$$

The Eq. (8) is a linear homogeneous ordinary differential equation of the second order. We search for a solution to this equation of the kind:

$$\tilde{A}(\xi,\omega)=e^{p\xi},\ p=const. \tag{9}$$

We substitute expression (9) in Eq. (8) and after short calculations, we obtain the following characteristic equation:

$$p^2+2i(\alpha+\omega)p=0. \tag{10}$$

Considering the roots of the equation above: $p_1=0$ and $p_2=-2i(\alpha+\omega)$, we find the following partial solution of the Eq. (10):

$$\tilde{A}(\xi,\omega)=\tilde{A}(0,\omega)e^{-2i(\alpha+\omega)\xi}, \tag{11}$$

where $\tilde{A}(0,\omega)$ is the Fourier-form of the initial pulse for ξ =0. We assume that the amplitude function of the initial laser pulse is presented by a Gaussian function:

$$A(0,\tau)=e^{-\frac{\tau^2}{2}},\ \tilde{A}(0,\omega)=\sqrt{2\pi}e^{-\frac{\omega^2}{2}}. \tag{12}$$

To obtain the solution of the main equation (6), we go back through all the substitutions and assumptions made so far. We replace the partial solution (11) and the expressions (12) in the integral (7), and after a couple of transformations, we find the following solution:

$$A(\xi,\tau)=e^{-2i\alpha\xi}e^{-\frac{1}{2}(\tau+2\xi)^2}. \tag{13}$$

The intensity of the optical pulse, with amplitude function described by expression (13), has a Gaussian form:

$$|A(\xi,\tau)|^2=e^{-(\tau+2\xi)^2}. \tag{14}$$

In Fig. 1. graphs of the intensity profile of the obtained expression (14) are presented. As can be seen, the shape of the pulse is preserved, but its position changes with increasing the variable ξ. This effect is due to the specific second term with coefficient $1/2\alpha$, in Eq. (4), which describes the nonparaxial evolution of the laser pulse. To understand the physics of the observed process, it is necessary to take into account the fact that the optical pulses always have 3D+1 dimensions. The one-dimensional task only gives a picture along the *z*-axis (*x*=0 and *y*=0). The group velocity of the pulse is related to its center of gravity. During the hemispherical (nonparaxial) $\lambda^{(3)}$ diffraction, the component along the *z*-axis (*x*=0 and *y*=0) is shifted forward in the Galilean coordinate system and in the opposite direction in the "local time" coordinate system, while the parts of the pulse remote from the *z*-axis lag behind. The center of gravity, as well as the group velocity of the pulse in $\lambda^{(3)}$ diffraction, do not change. Still, in the case of the one-dimensional task (on the *z*-axis, *x*=0, and *y*=0), only a change in the position of the one-dimensional pulse is observed.

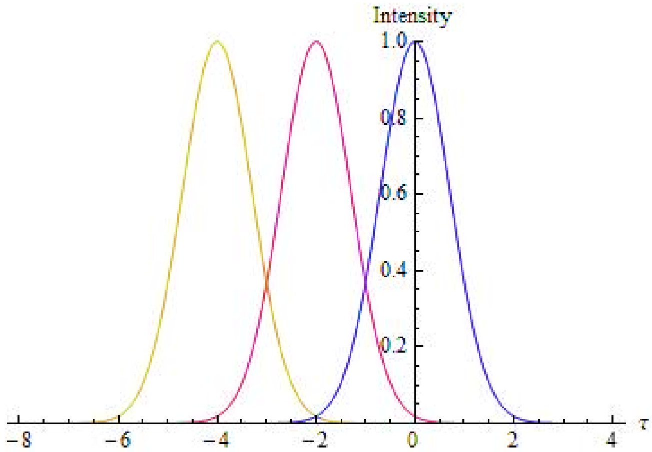

Fig. 1. Intensity profile of ultra-short optical pulse with Gaussian form at different distances: blue line ξ =0, red line ξ =1, and yellow line ξ =2. The result is obtained by using the solution (14).

4. Nonlinear regime of propagation of broad-band optical pulses in a dispersionless medium

In our next step, we will consider the evolution of ultra-short optical pulse in nonlinear Kerr-type media with negligibly small dispersion ($\beta \approx 0$), such as hollow microstructured fibers. Its behavior is described by the main Eq. (5), which is a partial differential equation of second order and third degree.

We search for a solution to this equation of the kind:

$$A(\xi,\tau) = \Phi(x) e^{ia\tau + ib\xi}, \quad x = \tau + u\xi. \tag{15}$$

where a , b , and u are constants, about to be defined, and $\Phi(x)$ is a real amplitude function.

We substitute the expression above in Eq. (5). Thus, we obtain the following complex ordinary differential equation of second order and third degree:

$$\begin{aligned} &2i\alpha u\Phi' - 2\alpha b\Phi + u^2\Phi'' + 2ibu\Phi' - b^2\Phi - 2u\Phi'' \\ &-2ib\Phi' - 2iau\Phi' + 2ab\Phi + 2\alpha\gamma\Phi^3 = 0. \end{aligned} \tag{16}$$

In Eq. (16), (') and (") represent the first and second derivatives with respect to x .

The mathematical model requires to separate the real and imaginary parts on both sides of the Eq. (16). In this way, we obtain the following system of two ordinary differential equations:

$$Re: \ \Phi''(u^2 - 2u) - \Phi(2\alpha b + b^2 - 2ab) + 2\alpha\gamma\Phi^3 = 0, \tag{17}$$

$$Im: \ 2\Phi'(\alpha u + bu - b - au) = 0. \tag{18}$$

For an arbitrary value of the argument *x*, the following condition must be satisfied $\Phi'(x) \neq 0$.

Thus, from the Eq. (18), we can find the relation between the constants:

$$u = \frac{b}{\alpha + b - a} = const. \tag{19}$$

Our next step is to consider Eq. (17), obtained by equalizing the real parts of Eq. (16). It is convenient to write it in the form:

$$\Phi'' - \Phi\left[\frac{b^2 + 2b(\alpha - a)}{u^2 - 2u}\right] + 2\left[\frac{\alpha\gamma}{u^2 - 2u}\right]\Phi^3 = 0. \tag{20}$$

Now, we will find an expression for the denominator in the Eq. (20):

$$u^2 - 2u = -\frac{\left(b^2 - 2b\left(a-\alpha\right)\right)}{\left(b-\left(a-\alpha\right)\right)^2} < 0. \tag{21}$$

Under this condition, the Eq. (20) can be presented as follows:

$$\Phi'' + 2B\Phi - 2\Gamma\Phi^3 = 0, \tag{22}$$

where

$$2B = \left(b-\left(a-\alpha\right)\right)^2 > 0, \tag{23}$$

$$\Gamma = \alpha\gamma \frac{(b-(a-\alpha))^2}{(b^2 - 2b(a-\alpha))} > 0, \text{when} \quad b > 2(a-\alpha). \tag{24}$$

It is well-known that the solution of this equation is the function:

$$\Phi\left(x\right) = \sqrt{\frac{B}{\Gamma}} th\left(x\sqrt{\Gamma} + C\right), \tag{25}$$

where C is the integration constant.

As a result of the substitutions and assumptions made so far, the analytical solution of Eq. (5) takes the form:

$$A\left(\xi,\tau\right) = \sqrt{\frac{B}{\Gamma}} th\left(x\sqrt{\Gamma} + C\right) e^{ia\tau + ib\xi}, \quad x = \tau + u\xi, \tag{26}$$

where

$$a = \alpha + \sqrt{2B}\sqrt{1 - \frac{\alpha\gamma}{\Gamma}}, \tag{27}$$

$$b = \sqrt{2B}\left[1 + \sqrt{1 - \frac{\alpha\gamma}{\Gamma}}\right], \tag{28}$$

$$u = 1 + \sqrt{1 - \frac{\alpha\gamma}{\Gamma}}. \tag{29}$$

The obtained solution (26) presents a dark soliton, which is formed as a result of the balance between the effects of longitudinal diffraction and the Kerr-type nonlinearity. The coefficient $\sqrt{B/\Gamma}$ in (26) has the meaning of amplitude of the obtained dark soliton. This is an analytical solution of the main Eq. (5) when the dispersion is nearly zero ($\beta \approx 0$). The found constants (27), (28) and (29) give the relation between the parameters of the optical pulse and the medium. Thus, to satisfy the second inequality in (24) it is necessary:

$$\sqrt{1 - \frac{\alpha\gamma}{\Gamma}} < 1. \tag{30}$$

Therefore, the condition for the formation of a dark soliton is $\alpha\gamma \leq \Gamma$. When we substitute (3) in this expression, we obtain:

$$\frac{L_{diff}}{L_{NL}} \leq \Gamma, \tag{31}$$

where $L_{diff} = k_0 z_0^2$ and $L_{NL} = \frac{1}{k_0 n_2 \left|A_0\right|^2}$.

With L_{diff} and L_{NL} are presented respectively the longitudinal diffractive length and nonlinear length. These are the distances in the waveguide medium at which the corresponding effects become significant. As we already mentioned before, the obtained dark soliton depends on the balance between these two phenomena. It is known that, under certain conditions, the nonlinearity of the medium leads to pulse compression, while diffraction expands spatially the pulse.

As a next step, we will determine the conditions necessary for the observation of a fundamental dark soliton. Let's assume that $\alpha\gamma = \Gamma = 1$ and $B = 1$. By using the expressions for the constants (27), (28), and (29), we obtain that $a = \alpha$, $b = \sqrt{2}$ and $u = 1$. In this case, the soliton solution of NAE (5) takes the form:

$$A(\xi,\tau) = th(\tau + \xi)e^{i\alpha\tau + i\sqrt{2}\xi}. \qquad (32)$$

In Fig. 2. it is shown a graph of the intensity profile of the obtained soliton solution (32) for different values of the variable ξ. It is observed the typical intensity gap characteristic of the fundamental dark laser solitons. It is clearly seen that the pulse retains its shape, but it shifts in position with the distance traveled. This effect is due to the two nonparaxial terms in NAE (4).

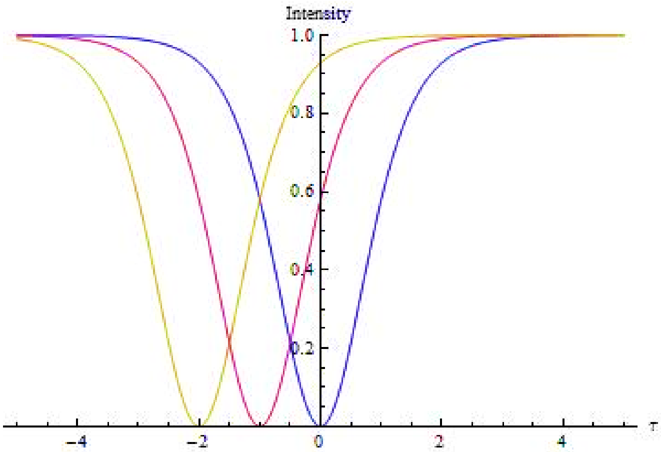

Fig. 2. Intensity profile of ultra-short dark optical soliton at different distances: blue line ξ =0, red line ξ =1, and yellow line ξ =2. The result is obtained by using the solution (32).

This type of solitons can be observed in hollow microstructured fibers, in which the dispersion has a negligibly small value.

5. Conclusion

The present paper investigates the evolution of ultra-short broad-band light pulses in linear and nonlinear regimes. It is considered isotropic dispersionless ($\beta \approx 0$) media, such as hollow microstructured optical fibers (photonic crystal fibers). The nonlinear amplitude equation describes the behavior of laser pulses in such waveguides. It differs from the nonlinear Schrödinger equation by two additional nonparaxial terms. They have an important role in the evolution of ultra-short optical pulses and describe their longitudinal diffraction divergence. The main Eq. (5) is presented in the scalar, dimensionless form in the "local time" coordinate system.

First, NAE (6) is solved for the linear case. The found solution (13) presents a stable optical pulse with Gaussian form. Fig. 1. shows the intensity profile of ultra-short optical pulse at different distances: ξ =0, ξ =1, and ξ =2. It is clearly seen that its shape is preserved, but the position of the pulse shifts with the distance. This effect is due to the additional nonparaxial terms in Eq. (4) that describe the nonparaxial evolution of the laser pulse.

In the second case, the nonlinear propagation regime of optical pulses in the frame of NAE (5) is considered. An analytical solution (26), which describes a dark soliton, is obtained. Despite the absence of dispersion ($\beta \approx 0$), the soliton could be formed due to the balance between the effects of diffraction and the nonlinearity of the medium. In Fig. 2, the intensity profile of a fundamental dark soliton is presented by the solution (32). The typical intensity gap, characteristic of the dark laser solitons, is seen. The optical pulse keeps its shape and parameters, but its position changes with the distance.

The presented results are important for a better understanding the processes observed during the evolution of ultra-short broad-band laser pulses in different dispersionless ($\beta \approx 0$) media, such as air or hollow photonic crystal fibers. They can find applications in atmospheric observation systems, telecommunication systems, optical sensors, and many others.

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Анотація. *У цій роботі представлені режими поширення лазерних імпульсів в ізотропних бездисперсійних (β≈0) середовищах, таких як порожнисті мікроструктуровані оптичні волокна (фотонно-кристалічні волокна). Для опису еволюції таких імпульсів використовується нелінійне амплітудне рівняння (НАР), яке відрізняється від нелінійного рівняння Шредінгера двома додатковими членами, що відповідають непараксіальності. Розглянуто лінійний і нелінійний режими поширення. У лінійному режимі, коли в еволюції лазерного імпульсу домінують тільки дифракційні ефекти, його форма зберігається, але положення імпульсу зміщується з відстанню. Це пов'язано з впливом непараксіального члена в НАР. У режимі нелінійного розповсюдження отриманий розв'язок НАР описує темний солітон. Він утворюється в результаті балансу між ефектами дифракції та нелінійністю середовища.*

Ключові слова: *фотонні кристалічні волокна, нелінійне амплітудне рівняння, оптичні солітони, ізотропне бездисперсійне середовище*