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DISPERSIVE OPTICAL SOLITONS WITH STOCHASTIC RADHAKRISHNAN-KUNDU-LAKSHMANAN EQUATION IN MAGNETO-OPTIC WAVEGUIDES HAVING POWER LAW NONLINEARITY AND MULTIPLICATIVE WHITE NOISE

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Abstract. In this paper, we introduce a coupled system of the stochastic Radhakrishnan-Kundu-Lakshmanan equation in magneto-optic waveguides for the first time. Power law nonlinearity and multiplicative white noise in the Itô sense are incorporated into the system. The two integration algorithms used are the extended simplest equation approach and the improved Kudryashov's approach. This study uses computer algebra systems to present dark, bright, and singular solitons.

Keywords: optical solitons, white noise, magneto-optics waveguides, Radhakrishnan–Kundu–Lakshmanan equation
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1. Introduction

Nonlinear systems of partial differential equations (PDEs) are widely recognized for their significance in describing various phenomena in the physical sciences, including nonlinear optical fibers, nonlinear waveguides, quantum optics, fluid dynamics, plasma physics, and telecommunications [1-10]. Consequently, researchers' interest in finding explicit soliton solutions for nonlinear PDEs has grown, prompting the investigation of many different techniques [11-25]. Recent advances have focused on dispersive solitons, highly dispersive solitons, pure-cubic solitons, cubic-quartic solitons, dispersion-managed solitons, white noise, magneto-optics waveguides, and many other related topics [26-35]. Magneto-optic waveguides are optical waveguides that use magneto-optical materials to guide light. Magneto-optical materials are substances that exhibit the magneto-optic effect, where an external magnetic field influences their optical properties. These materials can be either ferromagnetic or ferrimagnetic, indicating they have intrinsic magnetic ordering. When exposed to a magnetic field, the alignment of magnetic domains within these materials changes, leading to alterations in their optical behavior. This effect is crucial for manipulating the refractive properties of the material, allowing it to guide light. These waveguides find applications in optical communications, sensors, and systems requiring precise magnetic field analysis. The current study specifically examines dispersive optical solitons, which have been studied in various models such as the Schrödinger-Hirota equation, Fokas-Lenells equation, and others [36-45]. In particular, this paper focuses on dispersive solitons within the context of the well-established Radhakrishnan-Kundu-Lakshmanan (RKL) equation [26-39]. However, in this paper, we present the cubic coupled system of magneto-optics waveguides for the nonlinear RKL equation with power law nonlinearity and multiplicative white noise, which would yield soliton solutions with the Wiener process effect included [46-57]. The enhanced Kudryashov's approach and the extended simplest equation approach are used to obtain solitary wave solutions for this coupled system.

The structure of this article is as follows: Section 2 provides a mathematical analysis. In sections 3 and 4, optical solitons are derived using the enhanced Kudryashov's approach and the extended simplest equation approach, respectively. Lastly, section 5 presents the conclusions drawn from the study.

2. 2. Mathematical Analysis

Consider the dimensionless version of the stochastic RKL equation in polarization-preserving fibers, which includes multiplicative white noise in the Itô sense and a power law nonlinearity of refractive index. The following is an expression for this equation [28]:

$$iq_t + aq_{xx} + b|q|^{2n} q + i\left[\beta q_{xxx} + \alpha\left(|q|^{2n} q\right)_x\right] + \sigma q \frac{dW(t)}{dt} = 0. \tag{1}$$

Here, $q(x,t)$ is the complex-valued function that characterizes the wave profile, while a, b, β, α and σ are real constants and $i=\sqrt{-1}$. The linear temporal evolution is the first term. The constants a and b are the coefficients of chromatic dispersion (CD) and self-phase modulation (SPM), respectively. The constants β and α are the coefficients of third-order dispersion (3OD) and self-steepening (SS) effect terms respectively. White noise is defined as $dW(t)/dt$, where $W(t)$ is the standard Wiener process, while σ is the coefficient of noise strength. Finally, n signifies the full nonlinearity parameter. If $\sigma=0$ Eq. (1) is reduced to the conventional RKL equation [29–36].

In magneto-optic waveguides, Eq. (1) can be divided into two components, each representing a different polarization state of the wave. The first component is described by a complex-valued function, and the second component is represented by another complex-valued function. For the first time, they can be expressed as follows:

$$\begin{aligned} &iu_t + a_1 u_{xx} + \left(b_1|u|^{2n} + c_1|v|^{2n}\right)u + i\left[\beta_1 u_{xxx} + \alpha_1\left(|u|^{2n} u\right)_x\right] + \sigma u \frac{dW(t)}{dt} \\ &= Q_1 v + i\left[\lambda_1 u_x + \mu_1\left(|u|^{2n}\right)_x u + \theta_1 |u|^{2n} u_x\right], \end{aligned} \tag{2}$$

and

$$\begin{aligned} &iv_t + a_2 v_{xx} + \left(b_2|v|^{2n} + c_2|u|^{2n}\right)v + i\left[\beta_2 v_{xxx} + \alpha_2\left(|v|^{2n} v\right)_x\right] + \sigma v \frac{dW(t)}{dt} \\ &= Q_2 u + i\left[\lambda_2 v_x + \mu_2\left(|v|^{2n}\right)_x v + \theta_2 |v|^{2n} v_x\right]. \end{aligned} \tag{3}$$

Here, $u(x,t)$ and $v(x,t)$ represent the complex-valued functions that characterize the wave profiles, while a_j, b_j, c_j, β_j, α_j, Q_j, λ_j, μ_j, θ_j $(j=1,2)$ are real constants. $Q_j\,(j=1,2)$ is the magneto-optic coefficients of waveguide (or Faraday effect coefficients). a_j are the coefficients of CD, while b_j, c_j are coefficients of SPM and cross-phase modulation (XPM), respectively. β_j are the coefficients of 3OD. λ_j are the coefficients of the intermodal dispersion (IMD), α_j are the coefficients of SS effect terms , while μ_j and θ_j are the coefficients of nonlinear dispersion terms.

We assume the following forms for the wave profiles for solving Eqs. (2) and (3):

$$u(x,t)=H_1(\xi)e^{i\left[\psi(x,t)+\sigma W(t)-\sigma^2 t\right]}, \tag{4}$$

$$v(x,t)=H_2(\xi)e^{i\left[\psi(x,t)+\sigma W(t)-\sigma^2 t\right]}, \tag{5}$$

and

$$\xi=x-\rho t,\ \psi(x,t)=-\kappa x+\omega t. \tag{6}$$

These components represent the outgoing waves, where ρ, κ and ω are real-valued constants, whilst $\psi(x,t)$, $H_j(\xi)\,(j=1,2)$ represent the real-valued functions. Here, κ represents the wavenumber, ρ represents the velocity, and ω represents the frequency. Also, $\psi(x,t)$ represents the phase component, whereas $H_j(\xi)\,(j=1,2)$ represent the amplitude components. Through the substitution of Eqs. (4) and (5) into (2) and (3), the real parts can be obtained as follows:

$$\begin{aligned}&(3\kappa\beta_1+a_1)H_1''+\left[\kappa(\alpha_1-\theta_1)+b_1\right]H_1^{2n+1}+c_1H_1H_2^{2n}\\&-(\kappa^3\beta_1+\kappa^2a_1+\kappa\lambda_1-\sigma^2+\omega)H_1-Q_1H_2=0,\end{aligned} \tag{7}$$

and

$$\begin{aligned}&(3\kappa\beta_2+a_2)H_2''+\left[\kappa(\alpha_2-\theta_2)+b_2\right]H_2^{2n+1}+c_2H_2H_1^{2n}\\&-(\kappa^3\beta_2+\kappa^2a_2+\kappa\lambda_2-\sigma^2+\omega)H_2-Q_2H_1=0,\end{aligned} \tag{8}$$

while the imaginary parts are derived as follows:

$$\beta_1H_1'''-\left[2n(\mu_1-\alpha_1)+\theta_1-\alpha_1\right]H_1^{2n}H_1'-(3\beta_1\kappa^2+2a_1\kappa+\rho+\lambda_1)H_1'=0, \tag{9}$$

and

$$\beta_2H_2'''-\left[2n(\mu_2-\alpha_2)+\theta_2-\alpha_2\right]H_2^{2n}H_2'-(3\beta_2\kappa^2+2a_2\kappa+\rho+\lambda_2)H_2'=0. \tag{10}$$

Now, let us set

$$H_2(\xi)=\Omega H_1(\xi), \tag{11}$$

where Ω is a nonzero constant, such that $\Omega\neq 1$. Eqs. (7), (8), (9), and (10) can be reduced as:

$$\begin{aligned}&(3\kappa\beta_1+a_1)H_1''+\left[(\alpha_1-\theta_1)\kappa+\Omega^{2n}c_1+b_1\right]H_1^{2n+1}\\&-(\kappa^3\beta_1+\kappa^2a_1+\kappa\lambda_1-\sigma^2+\omega+\Omega Q_1)H_1=0,\end{aligned} \tag{12}$$

$$\begin{aligned}&(3\kappa\beta_2+a_2)H_1''+\left[\Omega^{2n}\left(\kappa(\alpha_2-\theta_2)+b_2\right)+c_2\right]H_1^{2n+1}\\&-\left(\kappa^3\beta_2+\kappa^2a_2+\kappa\lambda_2-\sigma^2+\omega+\frac{Q_2}{\Omega}\right)H_1=0,\end{aligned} \tag{13}$$

$$\beta_1H_1'''-\left[2n(\mu_1-\alpha_1)+\theta_1-\alpha_1\right]H_1^{2n}H_1'-(3\beta_1\kappa^2+2a_1\kappa+\rho+\lambda_1)H_1'=0, \tag{14}$$

and

$$\beta_2H_1'''-\Omega^{2n}\left[2n(\mu_2-\alpha_2)+\theta_2-\alpha_2\right]H_1^{2n}H_1'-(3\beta_2\kappa^2+2a_2\kappa+\rho+\lambda_2)H_1'=0. \tag{15}$$

Upon utilizing the principle of linear independence on Eqs. (12) and (13), we obtain:

$$\kappa = -\frac{a_1}{3\beta_1} \text{ or } \kappa = -\frac{a_2}{3\beta_2}, \tag{16}$$

and

$$\left.\begin{aligned} &\omega = \sigma^2 - \left(\kappa^3\beta_1 + \kappa^2 a_1 + \kappa\lambda_1 + \Omega Q_1\right) \\ &\text{or } \omega = \sigma^2 - \left(\kappa^3\beta_2 + \kappa^2 a_2 + \kappa\lambda_2 + \frac{Q_2}{\Omega}\right) \end{aligned}\right\}, \tag{17}$$

as well as the parametric restrictions:

$$\left.\begin{aligned} &(\alpha_1 - \theta_1)\kappa + \Omega^{2n} c_1 + b_1 = 0 \\ &\Omega^{2n}\left[\kappa(\alpha_2 - \theta_2) + b_2\right] + c_2 = 0 \end{aligned}\right\}, \tag{18}$$

provided $a_j \neq 0$ and $\beta_j \neq 0 \,(j = 1,2)$. Under the following constraints, the form of Eqs. (14) and (15) is the same:

$$\frac{\beta_1}{\beta_2} = \frac{2n(\mu_1 - \alpha_1) + \theta_1 - \alpha_1}{\Omega^{2n}\left[2n(\mu_2 - \alpha_2) + \theta_2 - \alpha_2\right]} = \frac{3\beta_1\kappa^2 + 2a_1\kappa + \rho + \lambda_1}{3\beta_2\kappa^2 + 2a_2\kappa + \rho + \lambda_2}. \tag{19}$$

From Eqs. (19), the soliton velocity can be determined as:

$$\rho = \frac{\beta_2(2\kappa a_1 + \lambda_1) - \beta_1(2a_2\kappa + \lambda_2)}{\beta_1 - \beta_2}, \tag{20}$$

provided $\beta_1 \neq \beta_2$.

Let us solve Eq. (14) under the constraint conditions (19). To achieve this, Eq. (14) can be integrated with a zero-integration constant

$$\begin{aligned} &\beta_1 H_1'' - \left[\frac{2n(\mu_1 - \alpha_1) + \theta_1 - \alpha_1}{2n+1}\right] H_1^{2n+1} \\ &-\left(3\beta_1\kappa^2 + 2a_1\kappa + \rho + \lambda_1\right) H_1 = 0. \end{aligned} \tag{21}$$

By considering H_1'' and H_1^{2n+1} in Eq. (21), we determine $N = \frac{1}{n}$. Subsequently, by using the transformation:

$$H_1(\xi) = \left[\phi(\xi)\right]^{\frac{1}{n}}, \tag{22}$$

where $\phi(\xi)$ is a new function of ξ, such that $\phi(\xi) > 0$ for $n > 1$, Eq. (21) changes to the following nonlinear ODE:

$$\begin{aligned} &\beta_1(2n+1)\left[n\phi''\phi - (n-1)\phi'^2\right] - n^2(2n+1)\left(3\kappa^2\beta_1 + 2\kappa a_1 + \rho + \lambda_1\right)\phi^2 \\ &+ n^2\left[2n(\alpha_1 - \mu_1) + \alpha_1 - \theta_1\right]\phi^4 = 0. \end{aligned} \tag{23}$$

Next, we will use the extended simplest equation approach and enhanced Kudryashov's approach to constructing the soliton and other exact wave solutions of Eqs. (2) and (3).

3. Enhanced Kudryashov's approach

The formal solution of Eq. (23) is assumed to take the following form by the enhanced Kudryashov's method [28, 35]:

$$\phi(\xi) = \sum_{i=0}^{M} A_i P^i(\xi), \tag{24}$$

where A_i $(i=0,1,2,...,M)$ are constants such that $A_M \neq 0$. The function $P(\xi)$ satisfies the nonlinear ODE:

$$P'^2(\xi)=P^2(\xi)\left[1-\varkappa P^{2s}(\xi)\right]\ln^2 K,\ \ 0<K\neq 1, \tag{25}$$

where $\varkappa$ is a non-zero constant and s is an integer number. Balancing $\phi\phi''$ and ϕ^4 in Eq. (23) gives $2M+2s=4M \Rightarrow M=s$. Next, we discuss the following two cases of s:

(I) Setting $s=1$, the formula (24) becomes:

$$\phi(\xi)=A_0+A_1P(\xi),\ A_1\neq 0, \tag{26}$$

via the auxiliary equation:

$$P'^2(\xi)=P^2(\xi)\left[1-\varkappa P^2(\xi)\right]\ln^2 K. \tag{27}$$

By inserting (26) along with (27) into Eq. (23), the following result can be obtained:

$$\begin{aligned}
&n=n,\ A_0=0,\\
&\rho=\frac{\beta_1\ln^2K-n^2\left(3\kappa^2\beta_1+2a_1\kappa+\lambda_1\right)}{n^2},\\
&A_1=\sqrt{\frac{\beta_1\varkappa(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}},
\end{aligned} \tag{28}$$

provided $\beta_1\varkappa\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]>0.$

By combining (28) with the existing solutions of Eq. (27) as reported in [28, 35], we are able to obtain the exact solutions for (26) as:

$$\begin{aligned}
u(x,t)=&\left[\sqrt{\frac{\beta_1\varkappa(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\left(\frac{4C}{4C^2K^{\xi}+\varkappa K^{-\xi}}\right)\right]^{\frac{1}{n}}\\
&\times e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]},
\end{aligned} \tag{29}$$

and

$$\begin{aligned}
v(x,t)=\Omega&\left[\sqrt{\frac{\beta_1\varkappa(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\left(\frac{4C}{4C^2K^{\xi}+\varkappa K^{-\xi}}\right)\right]^{\frac{1}{n}}\text{q}\\
&\times e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]},
\end{aligned} \tag{30}$$

where C is a nonzero constant. Solutions (29) and (30) can be rewritten as the straddled soliton solutions

$$\begin{aligned}
u(x,t)=&\sqrt[2n]{\frac{\beta_1\varkappa(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\\
&\times\left(\frac{4C}{(4C^2+\varkappa)\cosh(\xi\ln K)+(4C^2-\varkappa)\sinh(\xi\ln K)}\right)^{\frac{1}{n}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]},
\end{aligned} \tag{31}$$

and

$$\begin{aligned}
v(x,t)=&\Omega\sqrt[2n]{\frac{\beta_1\varkappa(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\\
&\times\left(\frac{4C}{(4C^2+\varkappa)\cosh(\xi\ln K)+(4C^2-\varkappa)\sinh(\xi\ln K)}\right)^{\frac{1}{n}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}.
\end{aligned} \tag{32}$$

In particular, if $\varkappa = 4C^2$, solutions (31) and (32) can be formulated as the bright soliton solutions

$$u(x,t)=\left[\sqrt{\frac{\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\operatorname{sech}(\xi\ln K)\right]^{\frac{1}{n}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \quad (33)$$

and

$$v(x,t)=\Omega\left[\sqrt{\frac{\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\operatorname{sech}(\xi\ln K)\right]^{\frac{1}{n}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \quad (34)$$

provided $\beta_1\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]>0.$

While $\varkappa = -4C^2$ converts the solutions (31) and (32) to the singular soliton solutions

$$u(x,t)=\left[\sqrt{-\frac{\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\operatorname{csch}(\xi\ln K)\right]^{\frac{1}{n}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \quad (35)$$

and

$$v(x,t)=\Omega\left[\sqrt{-\frac{\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\operatorname{csch}(\xi\ln K)\right]^{\frac{1}{n}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \quad (36)$$

provided $\beta_1\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]<0.$

Fig. 1. shows the numerical simulations of solution (33) in 3D and 2D graphs with $W(t)=\sqrt{t}$ and the following parameter values: $n=K=\beta_1=2,\ \kappa=\rho=\omega=\alpha_1=1,\ \mu_1=0.5$ and $\theta_1=0.25$. Additionally, different values of the coefficient of noise strength σ are shown in graphs (a)-(h).

(II) When $s=2$, Eq. (23) has the formal solution:

$$\phi(\xi)=A_0+A_1P(\xi)+A_2P^2(\xi),\ A_2\neq 0, \quad (37)$$

via the auxiliary equation:

$$P'^2(\xi)=P^2(\xi)\left[1-\varkappa P^4(\xi)\right]\ln^2K. \quad (38)$$

By plugging (37) along with (38) into Eq. (23), the following result can be obtained:

$$\begin{aligned}&\rho=\frac{4\beta_1\ln^2K-n^2\left(3\kappa^2\beta_1+2\kappa a_1+\lambda_1\right)}{n^2},\\&A_0=0,\quad A_1=0,\\&A_2=\sqrt{\frac{4\varkappa\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}},\end{aligned} \quad (39)$$

provided $\varkappa\beta_1\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]>0.$

Inserting (39) together with the documented well-known solutions of Eq. (38) obtained in [28,35] into (37) leaves us with the explicit solutions

$$\begin{aligned}u(x,t)=&\left[\sqrt{\frac{4\varkappa\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\left(\frac{4C}{4C^2K^{2\xi}+\varkappa K^{-2\xi}}\right)\right]^{\frac{1}{n}}\\&\times e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]},\end{aligned} \quad (40)$$

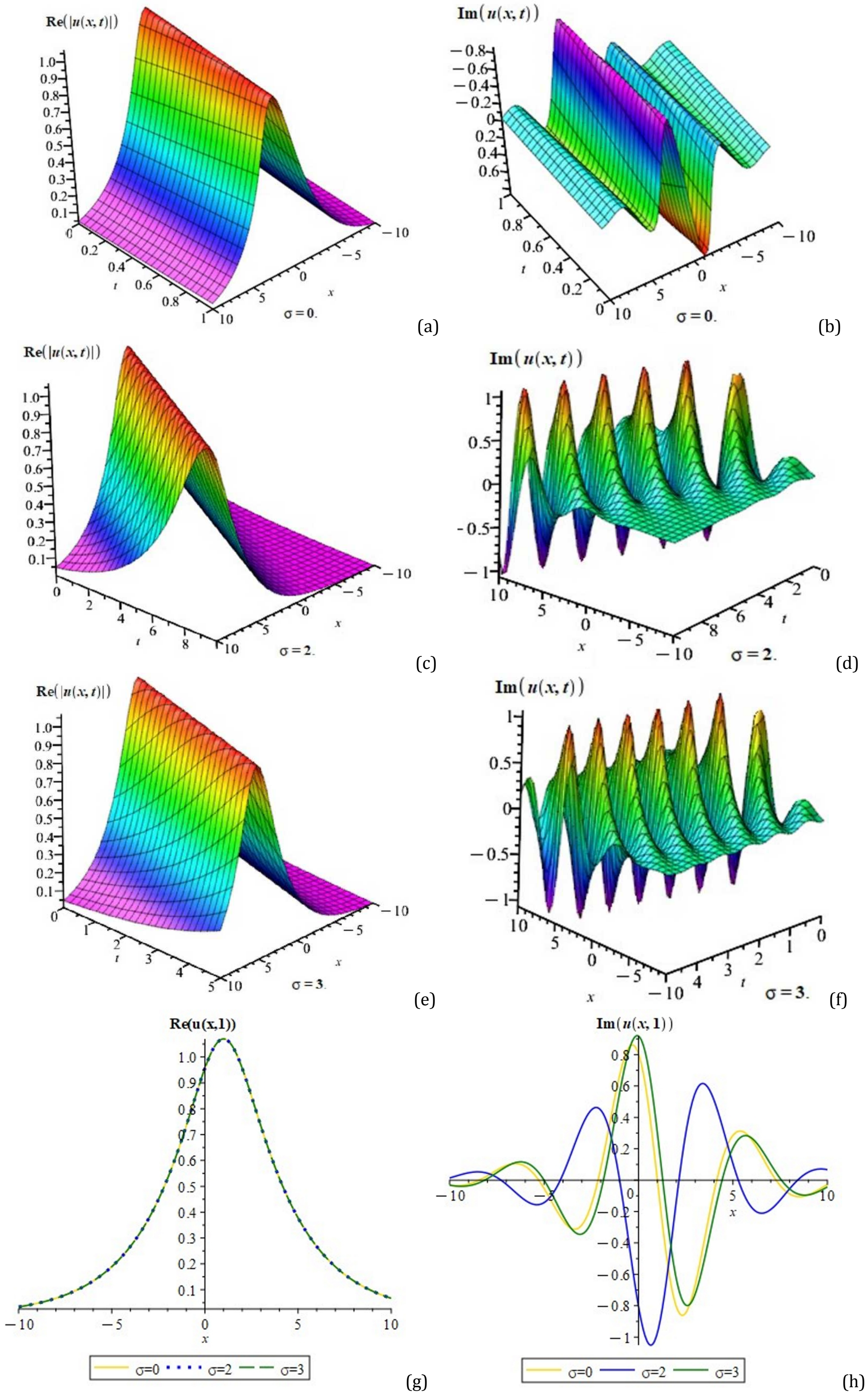

Fig. 1. The profile of the bright soliton solution (33).

and

$$v(x,t)=\Omega\left[\sqrt{\frac{4\varkappa\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\left(\frac{4C}{4C^2K^{2\xi}+\varkappa K^{-2\xi}}\right)\right]^{\frac{1}{n}} \times e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}. \tag{41}$$

Solutions (40) and (41) can be reconstructed as the straddled soliton solutions

$$u(x,t)=\sqrt[2n]{\frac{4\varkappa\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}} \times\left(\frac{4C}{(4C^2+\varkappa)\cosh(2\xi\ln K)+(4C^2-\varkappa)\sinh(2\xi\ln K)}\right)^{\frac{1}{n}} \times e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{42}$$

and

$$v(x,t)=\Omega\sqrt[2n]{\frac{4\varkappa\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}} \times\left(\frac{4C}{(4C^2+\varkappa)\cosh(2\xi\ln K)+(4C^2-\varkappa)\sinh(2\xi\ln K)}\right)^{\frac{1}{n}} \times e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}. \tag{43}$$

In particular, if $\varkappa=4C^2$, solutions (42) and (43) can be formulated as the bright soliton solutions

$$u(x,t)=\left[\sqrt{\frac{4\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\operatorname{sech}(2\xi\ln K)\right]^{\frac{1}{n}} e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{44}$$

and

$$v(x,t)=\Omega\left[\sqrt{\frac{4\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\operatorname{sech}(2\xi\ln K)\right]^{\frac{1}{n}} e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{45}$$

provided $\beta_1\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]>0.$

While $\varkappa=-4C^2$ converts the solutions (42) and (43) to the singular soliton solutions

$$u(x,t)=\left[\sqrt{-\frac{4\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\operatorname{csch}(2\xi\ln K)\right]^{\frac{1}{n}} e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{46}$$

and

$$v(x,t)=\Omega\left[\sqrt{-\frac{4\beta_1(n+1)(2n+1)\ln^2K}{n^2\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]}}\operatorname{csch}(2\xi\ln K)\right]^{\frac{1}{n}} e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{47}$$

provided $\beta_1\left[2n(\alpha_1-\mu_1)+\alpha_1-\theta_1\right]<0.$

In a similar manner, numerous additional solutions of the system (2) and (3) can be found by selecting various values for the parameter s. For the sake of simplicity, these values are not included here.

4. The extended simplest equation approach

The extended simplest equation approach [28, 37, 40-42] supposes the formal solution of Eq. (23) as follows

$$P(\xi)=B_0+B_1\left[\frac{Z'(\xi)}{Z(\xi)}\right]+C_0\left[\frac{1}{Z(\xi)}\right],\ B_1^2+C_0^2\neq 0, \tag{48}$$

and the function $Z(\xi)$ satisfies the following second-order linear ODE:

$$Z''(\xi)+\delta Z(\xi)=\upsilon, \tag{49}$$

where B_0, B_1, C_0, δ and υ are constants. Now, according to the extended simplest equation approach, there are three cases of results that are discussed as follows:

Case-1. If $\delta<0$, we substitute Eq. (48) into Eq. (23) and utilize Eq. (49) along with the following relation

$$\left(\frac{Z'(\xi)}{Z(\xi)}\right)^2=\Delta_1\left(\frac{1}{Z(\xi)}\right)^2-\delta+\frac{2\upsilon}{Z(\xi)}, \tag{50}$$

where $\Delta_1=\delta\left(A_1^2-A_2^2\right)-\upsilon^2/\delta$, while A_1 and A_2 are constants. Thus, we get the following results:

<u>Result 1.</u>

$$\begin{aligned}&\rho=-\frac{n^2\left(3\beta_1\kappa^2+2a_1\kappa+\lambda_1\right)+\delta\beta_1}{n^2},\\&\upsilon=0,\ B_0=0,\ B_1=0,\\&C_0=\sqrt{\frac{\beta_1\delta(n+1)(2n+1)\left(A_1^2-A_2^2\right)}{n^2\left[2n(\mu_1-\alpha_1)-\alpha_1+\theta_1\right]}},\end{aligned} \tag{51}$$

provided $\beta_1\left(A_1^2-A_2^2\right)\left[2n(\mu_1-\alpha_1)-\alpha_1+\theta_1\right]<0$. Consequently, we obtain the following straddled soliton solutions:

$$\begin{aligned}u(x,t)=&\sqrt[2n]{\frac{\beta_1\delta(n+1)(2n+1)\left(A_1^2-A_2^2\right)}{n^2\left[2n(\mu_1-\alpha_1)-\alpha_1+\theta_1\right]}}\\&\times\left(\frac{1}{A_1\cosh\left(\xi\sqrt{-\delta}\right)+A_2\sinh\left(\xi\sqrt{-\delta}\right)}\right)^{\frac{1}{n}}\\&\times e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]},\end{aligned} \tag{52}$$

and

$$\begin{aligned}v(x,t)=&\Omega\sqrt[2n]{\frac{\beta_1\delta(n+1)(2n+1)\left(A_1^2-A_2^2\right)}{n^2\left[2n(\mu_1-\alpha_1)-\alpha_1+\theta_1\right]}}\\&\times\left(\frac{1}{A_1\cosh\left(\xi\sqrt{-\delta}\right)+A_2\sinh\left(\xi\sqrt{-\delta}\right)}\right)^{\frac{1}{n}}\\&\times e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}.\end{aligned} \tag{53}$$

In particular, if we take $A_1=0$, and $A_2\neq 0$ in (52) and (53), we have the singular soliton solutions

$$u(x,t)=\left[\sqrt{-\frac{\beta_1\delta(n+1)(2n+1)}{n^2\left[2n(\mu_1-\alpha_1)-\alpha_1+\theta_1\right]}}\operatorname{csch}\left(\xi\sqrt{-\delta}\right)\right]^{\frac{1}{n}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \quad (54)$$

and

$$v(x,t)=\Omega\left[\sqrt{-\frac{\beta_1\delta(n+1)(2n+1)}{n^2\left[2n(\mu_1-\alpha_1)-\alpha_1+\theta_1\right]}}\operatorname{csch}\left(\xi\sqrt{-\delta}\right)\right]^{\frac{1}{n}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \quad (55)$$

provided $\beta_1\left[2n(\mu_1-\alpha_1)-\alpha_1+\theta_1\right]>0$, while if we take $A_1\neq 0$, and $A_2=0$ in (52) and (53), we arrive at the bright soliton solutions

$$u(x,t)=\left[\sqrt{\frac{\beta_1\delta(n+1)(2n+1)}{n^2\left[2n(\mu_1-\alpha_1)-\alpha_1+\theta_1\right]}}\operatorname{sech}\left(\xi\sqrt{-\delta}\right)\right]^{\frac{1}{n}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \quad (56)$$

and

$$v(x,t)=\Omega\left[\sqrt{\frac{\beta_1\delta(n+1)(2n+1)}{n^2\left[2n(\mu_1-\alpha_1)-\alpha_1+\theta_1\right]}}\operatorname{sech}\left(\xi\sqrt{-\delta}\right)\right]^{\frac{1}{n}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \quad (57)$$

provided $\beta_1\left[2n(\mu_1-\alpha_1)-\alpha_1+\theta_1\right]<0$.

Result 2.

$$\begin{gathered}n=2,\ \rho=-3\beta_1\kappa^2+\frac{1}{4}\delta\beta_1-2a_1\kappa-\lambda_1,\ \upsilon=2\delta\sqrt{A_1^2-A_2^2},\\ B_0=\sqrt{\frac{5\delta\beta_1}{4(5\alpha_1-4\mu_1-\theta_1)}},\ B_1=0,\ C_0=-\sqrt{\frac{45\delta\beta_1\left(A_1^2-A_2^2\right)}{4(5\alpha_1-4\mu_1-\theta_1)}},\end{gathered} \quad (58)$$

provided $\beta_1(5\alpha_1-4\mu_1-\theta_1)<0$ and $A_1^2-A_2^2>0$. Consequently, the following straddled soliton solutions are thus obtained:

$$\begin{aligned}u(x,t)=&\sqrt[4]{\frac{5\delta\beta_1}{4(5\alpha_1-4\mu_1-\theta_1)}}\\ &\times\left(1-\frac{3\sqrt{\left(A_1^2-A_2^2\right)}}{A_1\cosh\left(\xi\sqrt{-\delta}\right)+A_2\sinh\left(\xi\sqrt{-\delta}\right)+2\sqrt{A_1^2-A_2^2}}\right)^{\frac{1}{2}}\\ &\times e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]},\end{aligned} \quad (59)$$

and

$$\begin{aligned}v(x,t)=&\Omega\sqrt[4]{\frac{5\delta\beta_1}{4(5\alpha_1-4\mu_1-\theta_1)}}\\ &\times\left(1-\frac{3\sqrt{\left(A_1^2-A_2^2\right)}}{A_1\cosh\left(\xi\sqrt{-\delta}\right)+A_2\sinh\left(\xi\sqrt{-\delta}\right)+2\sqrt{A_1^2-A_2^2}}\right)^{\frac{1}{2}}\\ &\times e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}.\end{aligned} \quad (60)$$

In particular, if we take $A_1\neq 0$, and $A_2=0$ in (59) and (60), we arrive at the combo bright soliton solutions

$$u(x,t)=\sqrt[4]{\frac{5\delta\beta_1}{4(5\alpha_1-4\mu_1-\theta_1)}}\left(1-\frac{3\operatorname{sech}(\xi\sqrt{-\delta})}{1+2\operatorname{sech}(\xi\sqrt{-\delta})}\right)^{\frac{1}{2}}e^{i[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t]}, \tag{61}$$

and

$$v(x,t)=\Omega\sqrt[4]{\frac{5\delta\beta_1}{4(5\alpha_1-4\mu_1-\theta_1)}}\left(1-\frac{3\operatorname{sech}(\xi\sqrt{-\delta})}{1+2\operatorname{sech}(\xi\sqrt{-\delta})}\right)^{\frac{1}{2}}e^{i[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t]}. \tag{62}$$

Result 3.

$$\begin{gathered}n=1,\ \rho=-3\beta_1\kappa^2+2\delta\beta_1-2a_1\kappa-\lambda_1,\ \upsilon=0,\ B_0=0,\\ B_1=\sqrt{-\frac{6\beta_1}{3\alpha_1-2\mu_1-\theta_1}},\ C_0=0,\end{gathered} \tag{63}$$

provided $\beta_1(3\alpha_1-2\mu_1-\theta_1)<0$. Consequently, straddled soliton solutions are obtained:

$$\begin{aligned}u(x,t)=&\sqrt{\frac{6\delta\beta_1}{3\alpha_1-2\mu_1-\theta_1}}\\ &\times\left(\frac{A_1\sinh(\xi\sqrt{-\delta})+A_2\cosh(\xi\sqrt{-\delta})}{A_1\cosh(\xi\sqrt{-\delta})+A_2\sinh(\xi\sqrt{-\delta})}\right)e^{i[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t]},\end{aligned} \tag{64}$$

and

$$\begin{aligned}v(x,t)=&\Omega\sqrt{\frac{6\delta\beta_1}{3\alpha_1-2\mu_1-\theta_1}}\\ &\times\left(\frac{A_1\sinh(\xi\sqrt{-\delta})+A_2\cosh(\xi\sqrt{-\delta})}{A_1\cosh(\xi\sqrt{-\delta})+A_2\sinh(\xi\sqrt{-\delta})}\right)e^{i[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t]}.\end{aligned} \tag{65}$$

In particular, if we take $A_1\neq 0$, and $A_2=0$ in (64) and (65), we arrive at the dark soliton solutions

$$u(x,t)=\sqrt{\frac{6\delta\beta_1}{3\alpha_1-2\mu_1-\theta_1}}\tanh(\xi\sqrt{-\delta})e^{i[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t]}, \tag{66}$$

and

$$v(x,t)=\Omega\sqrt{\frac{6\delta\beta_1}{3\alpha_1-2\mu_1-\theta_1}}\tanh(\xi\sqrt{-\delta})e^{i[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t]}, \tag{67}$$

while if we take $A_1=0$, $A_2\neq 0$ in (64) and (65), we arrive at the singular soliton solutions

$$u(x,t)=\sqrt{\frac{6\delta\beta_1}{3\alpha_1-2\mu_1-\theta_1}}\coth(\xi\sqrt{-\delta})e^{i[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t]}, \tag{68}$$

and

$$v(x,t)=\Omega\sqrt{\frac{6\delta\beta_1}{3\alpha_1-2\mu_1-\theta_1}}\coth(\xi\sqrt{-\delta})e^{i[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t]}. \tag{69}$$

Fig. 2. shows the numerical simulations of solution (67) in 3D and 2D graphs with $W(t)=\sqrt{t}$ and the following parameter values: $\theta_1=\mu_1=\kappa=\omega=1,\ \alpha_1=\rho=-1,\ \beta_1=\Omega=2$ and $\delta=-2$. Additionally, different values of the coefficient of noise strength σ are shown in graphs (a)-(h).

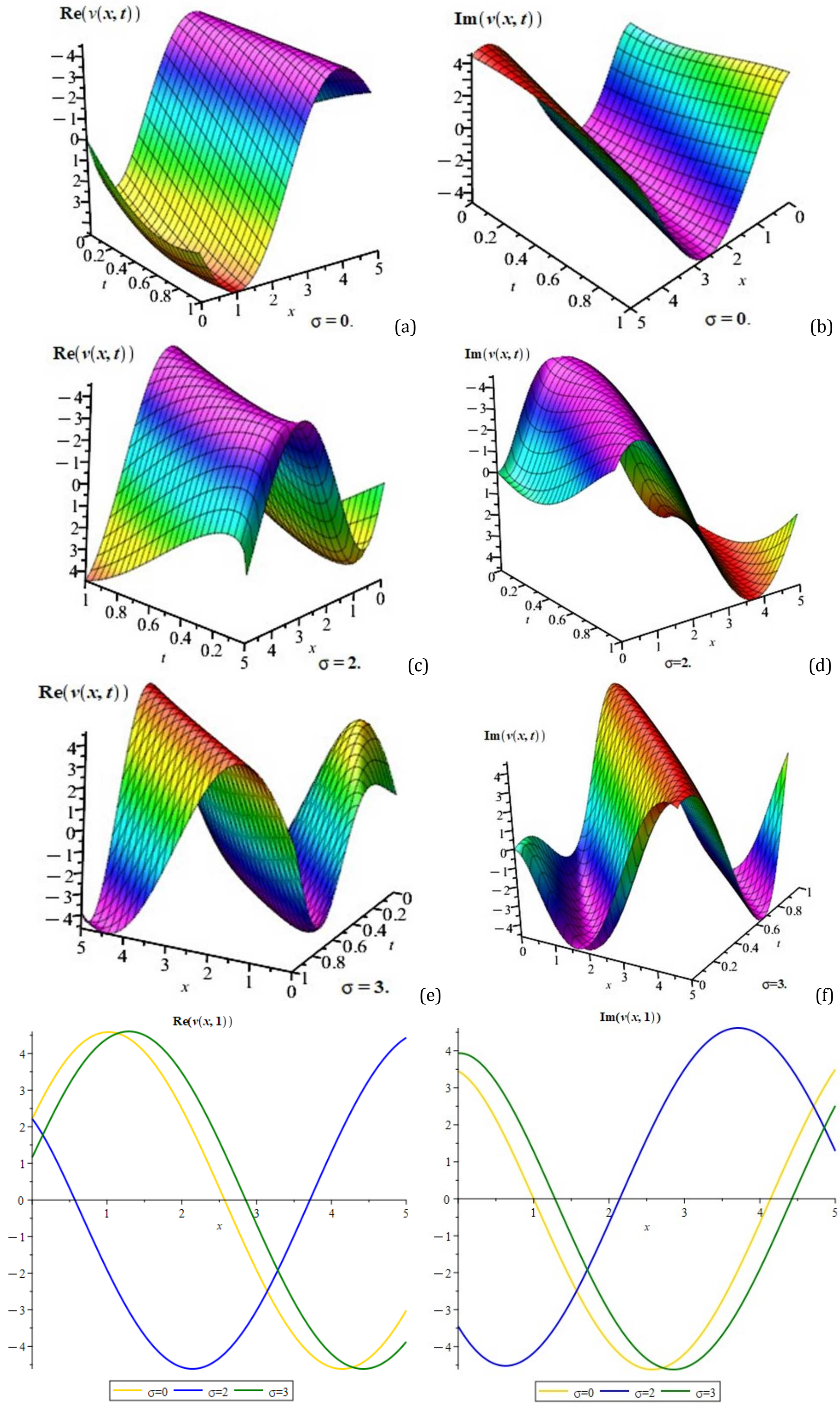

Fig. 2. The profile of the dark soliton solution (67).

Remark. When $\delta > 0$, we have the periodic solutions, while when $\delta = 0$, we have the rational solutions. The latter do not play any role in the optics theory and are omitted here.

5. Conclusions

In magneto-optic waveguides featuring multiplicative white noise in the Itô sense and power law nonlinearity, two mathematical techniques have been used in this article: the extended simplest equation approach and the enhanced Kudryashov's approach to obtain the solitons and other explicit wave solutions of the stochastic RKL equation. Solutions for singular, bright, and dark solitons are reported for the first time. The current work also outlines the specific constraints to obtain solutions for solitons. In this study, we have developed a new model in nonlinear optics, which sets the results apart from those found in other publications. Numerical simulations of solutions (33) and (67) were shown in Fig. 1 and 2, using two- and three-dimensional plots, respectively, at different noise coefficient levels and without noise. These figures show that the surface looks less smooth when there is no noise but smoother after minor transitional behaviors when the noise level rises. This suggests that multiplicative noise influences the solutions and maintains their stability. In the end, this research finds that soliton solutions are significantly impacted by the noise effect, especially the noise's intensity.

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Анотація. *У цій статті вперше представлена зв'язана система стохастичних рівнянь Радхакришнана-Кунду-Лакшманана для магнітооптичних хвилеводів. У систему* *введено степеневу нелінійність і мультиплікативний білий шум у розумінні Іто. Для інтегрування системи використовуються два алгоритми: розширений метод найпростішого рівняння та удосконалений підхід Кудряшова. З використанням систем комп'ютерних розрахунків у цій роботі отримані розв'язки для темних, світлих та сингулярних солітонів.*

Ключові слова*: оптичні солінони, білий шум, магнітооптичні хвилеводи, рівняння Радхакрішнана–Кунду–Лакшманана*