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OPTICAL SOLITONS FOR NONLINEAR SCHRÖDINGER EQUATION FORMATTED IN THE ABSENCE OF CHROMATIC DISPERSION THROUGH MODIFIED EXPONENTIAL RATIONAL FUNCTION METHOD AND OTHER DISTINCT SCHEMES

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Abstract. This new work studies a nonlinear Schrödinger equation (NLSE) formatted without chromatic dispersion. New integration algorithms collectively reveal a variety of optical solitons and other exact solutions of distinct physical structures. This study delves into a toolbox of powerful techniques, including various forms of a refined exponential rational function method, to unlock a rich tapestry of solutions for the examined nonlinear Schrödinger equation. Each method's distinct form unveils unique traveling wave solutions alongside the essential parameter constraints governing their existence. Furthermore, under specific parameter conditions, this toolbox yields a treasure trove of novel optical solutions: modulated waves, bright and dark envelope solitons, and periodic and traveling waveforms. These findings illuminate the diverse landscape of solutions for this equation, paving the way for deeper understanding in fields like optical fibers and plasma physics.

Keywords: Schrödinger equation, modified exponential rational function method, soliton solutions
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1. Introduction

Many different types of physics and optics phenomena depend on nonlinear evolution equations. These include the Fokas-Lenells (FL) equation, the nonlinear Schrodinger equation (NLSE), the Maxwell equation for electromagnetic radiation, the Helmholtz equation for thermodynamics, the Korteweg-de Vries equation (KdV) for describing solitary waves (SWs), shocks, and cnoidal waves in plasma, and many more. A crucial part of many nonlinear evolution equations is the balance between dispersion and nonlinearity. This is what makes solitary waves appear in a wide range of systems. Soliton interactions in optical fiber communication have driven much research in recent decades, with researchers trying to harness their unique properties for various purposes [1-6]. Soliton solutions are crucial in nonlinear optics, hydrodynamics, telecommunications, and other domains. The standard form of the NLSE is given by [1-15]. The KdV and Schrödinger families of equations differ in their structure and focus: the KdV-type equations describe solitons (compressive or rarefactive type) propagating at the phase velocity, while Schrödinger-type equations capture modulated envelope solitons (bright-, dark-, gray-type) moving at the group velocity. Our research delves into a novel form of a NLSE, where we utilize powerful techniques to extract various solutions. First, we must point out the standard case of the NLSE, from which the rest of its family members emerge [1-15]

$$iu_t + \frac{1}{2}u_{xx} + |u|^2 u = 0, \tag{1}$$

where $u \equiv u(x,t)$ represents the complex wave function and $i = \sqrt{-1}$. Eq. (1) dissects the intricate dance between light and a unique optical fiber. The second term, often called "group velocity dispersion (GVD)" or "chromatic dispersion (CD)," dictates how different light colors (frequencies) travel at slightly different speeds within the fiber. The final term, "Kerr law nonlinearity," throws a fascinating twist into the mix. It acts like a light-sensitive dimmer switch, adjusting the phase of the light depending on its intensity. This nonlinearity is the secret sauce behind various fascinating effects, as documented in numerous studies [15-20]. Optical solitons, such as bright and dark solitons, demonstrate a captivating duality by exhibiting characteristics of both waves and particles. During the interaction, they show a distinct particle-like behavior resembling billiard balls' collision. However, they have the unique characteristic of maintaining their shape, velocity, and sharing energy without dissipation. These waves' notable characteristics and potential applications have stimulated substantial investigation, as evidenced in Refs. [12-26]. Different optical models can also have other solitons, such as dark solitons, bimodal dark solitons, Gaussian solitons, spatiotemporal solitons, and other unique solitons.

The family of NLSE is a key to unlocking the secrets of light's journey through optical fibers. Its power lies in its uncanny ability to accurately predict the behavior of light pulses, including how they spread, interact with each other in nonlinear ways, and even form rogue waves – phenomena that have sparked intense research in photonics and beyond [21-38]. By precisely modeling these intricate dynamics, the NLSE paves the way for breakthroughs in optical communication, sensing, and manipulation. It's well understood that the interplay between the fiber's dispersion and its non-linear refractive index dictates how light pulses, called solitons, propagate. Remarkably, specific optical systems can amplify these solitons, while others, like birefringent fibers, can give rise to chirped solitons [1-10, 16].

This paper analyzes third-order dispersion in the perturbed NLSE (pNLSE), specifically in the absence of chromatic dispersion

$$i\left(u_t + au_{xxx}\right) + b|u|^2 u = i\left(c_1 |u|^2 u_x + c_2 u(|u|^2)_x\right), \tag{2}$$

where $u \equiv u(x,t)$ is a complex wave function that denotes the modulated envelope soliton profile, iu_t indicates the temporal evolution term, iau_{xxx} represents the term of third-order dispersion, $b|u|^2 u$ denotes the term of cubic nonlinearity, whereas the two terms $i\left(c_1 |u|^2 u_x\right)$ and $\left(ic_2 u(|u|^2)_x\right)$ refer to the terms of nonlinear dispersion which arise from perturbation [1-6]. The coefficients (a,b,c_1,c_2) are real values that depend on the physical model under consideration and do not equal zero. Remember that for $c_1 = c_2 = 0$, Eq. (2) reduces to the complex modified KdV or Hirota equation [2-9]. Note that the two terms on the right side of Eq. (2) constitute a couple of Hamiltonian perturbation terms. Schrödinger equations are partial differential equations employed to represent a range of nonlinear phenomena in fields such as quantum physics, nonlinear optics, ocean modulated waves, modulated waves in physics of plasmas, especially rogue waves, and modulated envelope solitons, including bright, dark, gray envelope solitons, and many others.

We aim to uncover traveling wave solutions and optical solitons for the perturbed Schrödinger Eq. (2). To achieve this, we'll leverage the versatility of the modified exponential rational function approach [1-8]. With its diverse and reliable methods, this approach promises a rich harvest of traveling wave solutions. We'll further bolster our search by employing additional proven techniques [9-16] specifically designed to unearth soliton solutions. Through this comprehensive approach, we aim to rigorously derive a variety of modulated envelope solitons for this model, including bright, dark, single, and even exotic combinations. A vital advantage of this multi-pronged attack is the ability to express our solutions in various function forms, including exponential, trigonometric, and hyperbolic, offering valuable insights into the model's behavior.

2. Modified exponential rational (MER) function method: An overview

The recently proposed generalized exponential function method has been demonstrated to be reliable and efficient in analyzing scientific models [1-10]. Nevertheless, in this study, we propose a MER function approach, in which we assume that the solution can be represented in the following manner: $u=\dfrac{Rg(x,t)}{r_0+r_1 f(x,t)}e^{i\Theta}$, where (R,r_0,r_1,k,A) are undetermined parameters and $\Theta=(Ax-kt)$.

The functions $g\equiv g(x,t)$ and $f\equiv f(x,t)$ are not confined to simple expressions, but they can indicate both trigonometric functions and hyperbolic functions. This vast tapestry of possibilities lies at the heart of our exploration, allowing us to weave many traveling wave (TW) solutions with distinct optical soliton structures.

2.1. Using the factor-(I)

Using the powerful MER method, we embark on a journey to unravel the secrets of Eq. (2). Our first step is to postulate a form for the formal solution, expressed in the following form:

$$u=\frac{R\sin(X)}{r_0+r_1\cos(X)}e^{i\Theta}, \tag{3}$$

where all parameters are as previously described. Here, $X=(Ax-Ct)$ where C indicates the group velocity. Putting Eq. (3) into Eq. (2), and subsequently gathering the terms using trigonometric functions, one can solve the resulting equations to obtain the formal outcomes

$$\begin{cases} a=\dfrac{2A}{k^3},\, b=2Ac_2,\, r_0=r_1, \\ c_1=-\dfrac{2AR^2c_2-3kr_1^2}{AR^2},\, C=-5k, \end{cases} \tag{4}$$

where $(A,c_2,r_1,k)=(A,c_2,r_1,k)$, i.e., free parameters and the other parameters are unconstrained. Substituting the obtained results into Eq. (3) gives the following trigonometric solution

$$u=\frac{R\sin(Ax+5kt)}{r_0(1+\cos(Ax+5kt))}e^{i\Theta}$$
$$=\frac{R}{r_0}\tan(\frac{Ax+5kt}{2})e^{i\Theta}, \tag{5}$$

Note that solution (5) is only valid under conditions (4).

2.2. Using the factor-(II)

It is noteworthy that we can hypothesize, the solution of Eq. (2) adopts the following form:

$$u=\frac{R\cos(X)}{r_0+r_1\sin(X)}e^{i\Theta}, \tag{6}$$

and, by proceeding as before, we obtain

$$\begin{cases} a=\frac{2k}{A^3}, \quad r_0=r_1, \\ c_1=-\frac{R^2b-3kr_1^2}{AR^2}, \quad c_2=\frac{b}{2A}, \quad C=-5k, \end{cases} \tag{7}$$

where $(A,b,r_1,k)=(A,b,r_1,k)$, i.e., free parameters.

Inserting Eq. (7) into solution (6), we get

$$u=\frac{R\cos(Ax+5kt)}{r_1(1+\sin(Ax+5kt))}e^{i\Theta}, \tag{8}$$

valid under the results obtained in (7).

2.3. Using the factor-(III)

To further analyze the situation for Eq. (2) based on the MER method, the following ansatz is introduced

$$u=\frac{R\sinh(X)}{r_0+r_1\cosh(X)}e^{i\Theta}, \tag{9}$$

where all parameters are as previously described. Using Eq. (9) in Eq. (2) and subsequently solving the resulting equations, we obtain

$$\begin{cases} a=-\frac{2A}{5k^3}, \ r_0=r_1, \\ c_1=-\frac{5R^2b+3kr_1^2}{5AR^2}, \ c_2=\frac{b}{2A}, \ C=\frac{7}{5}k, \end{cases} \tag{10}$$

where $(A,b,r_1,k)=(A,b,r_1,k)$, i.e., free parameters.

Inserting Eq. (10) into solution (9), yields

$$u=\frac{R\sinh(Ax-\frac{7}{5}kt)}{r_0\left(1+\cosh(Ax-\frac{7}{5}kt)\right)}e^{i\Theta}, \tag{11}$$

valid under the results obtained in Eq. (10).

2.4. Using the factor-(IV)

In the same way, we can impose the solution of Eq. (2) in the following ansatz

$$u=\frac{R\,\mathrm{sech}(X)}{r_0+r_1\tanh(X)}e^{i\Theta}, \tag{12}$$

where all parameters are as previously described. The following findings are obtained by substituting solution (12) into Eq. (2) and solving the obtained equations,

$$\begin{cases} b = 2c_2 A, \;\; C = -2aA^3, \;\; k = 2aA^3, \\ c_1 = -\dfrac{2\left(3A^2 a r_0^2 - 3A^2 a r_1^2 + R^2 c_2\right)}{R^2}, \end{cases} \tag{13}$$

where $(A,a,c_2,r_0,r_1) = (A,a,c_2,r_0,r_1)$, i.e., free parameters. The combination between Eq. (13) and solution (12) yields the following exponential rational solution

$$u = \frac{R\,\mathrm{sech}(Ax + 2aA^3 t)}{r_0 + r_1 \tanh(Ax + 2aA^3 t)} e^{i(Ax - 2aA^3 t)}. \tag{14}$$

2.5. Using the factor-(V)

In the same way, we can impose the solution of Eq. (2) in the following ansatz

$$u = \frac{R \tan(X)}{r_0 + r_1 \sec(X)} e^{i\Theta}, \tag{15}$$

where all parameters are as previously described. The following findings are obtained by substituting solution (15) into Eq. (2) and solving the resulting equations

$$\begin{cases} a = \dfrac{2k}{A^3}, b = 2c_2 A, \;\; r_1 = -r_0, \\ c_1 = -\dfrac{2c_2 A R^2 - 3k r_0^2}{A R^2}, C = -5k, \end{cases} \tag{16}$$

where $(A, c_2, r_0) = (A, c_2, r_0)$, i.e., free parameters and other parameters are left unconstrained. By inserting Eq. (16) into solution (15), we get the exponential rational solution which is valid under conditions (16).

2.6. Using the factor-(VI)

In the same way, the solution of Eq. (2) can be imposed in the following form

$$u = \frac{R\,\mathrm{csch}(Ax - Ct)}{r_0 + r_1 \coth(Ax - Ct)} e^{i\Theta}, \tag{17}$$

where all parameters are as previously described. The following findings are obtained by substituting solution (17) into Eq. (2) and solving the resulting equations

$$\begin{cases} b = 2c_2 A, \\ r_1 = -r_0, \\ c_1 = \dfrac{2\left(3A^2 a r_0^2 - 3A^2 a r_1^2 - R^2 c_2\right)}{R^2}, \\ C = -2aA^3, \;\; k = 2aA^3, \end{cases} \tag{18}$$

where $(A,a,c_2,r_0) = (A,a,c_2,r_0)$ and other parameters are left unconstrained. By inserting Eq. (18) into solution (17), we get the exponential rational solution, which is valid under conditions (18).

3. Bright envelope soliton (BES) solutions

The following ansatz is proposed to obtain a collection of BES solutions to Eq. (2)

$$u = R\,\mathrm{sech}(X) e^{i\Theta}, \tag{19}$$

where the constants (R,A,C,k) are non-zero, $\Theta = (Ax - kt)$, and $X = (Ax - Ct)$.

By inserting Eq. (19) into Eq. (2), analyzing the coefficients of the hyperbolic functions produced, and solving the resulting system, we have

$$\begin{cases} b = 2c_2 A,\ \ C = -2aA^3,\ r_1 = -r_0, \\ c_1 = \dfrac{2(3A^2 a + R^2 c_2)}{R^2},\ \ k = 2aA^3, \end{cases} \tag{20}$$

where $(A, a, c_2, r_0) = (A, a, c_2, r_0)$ and other parameters are left unconstrained. These results demonstrate that the BS solution (19) is applicable within the specified parameters outlined in Eq. (20).

It should be emphasized that the solution of Eq. (2) can be assumed in the following form

$$u = R\sec(X)e^{i\Theta}, \tag{21}$$

by proceeding as implemented earlier, the following solution is obtained

$$u = R\sec(Ax + 4aA^3 t)e^{i(Ax+4aA^3 t)}. \tag{22}$$

4. Dark envelope soliton (DES) solutions

To offer a collection of DES solutions for Eq. (2), we consequently establish

$$u = R\tanh(X)e^{i\Theta}, \tag{23}$$

where the constants (R, A, C, k) have non-zero values.

By inserting Eq. (23) into Eq. (2), analyzing the coefficients of the hyperbolic functions produced, and solving the resulting system, we have

$$\begin{cases} b = \dfrac{A(6A^2 a - R^2 c_1)}{R^2},\ \ C = -5aA^3,\ \ r_1 = -r_0, \\ c_2 = \dfrac{6A^2 a - R^2 c_1}{2R^2},\ \ k = -7A^3 a, \end{cases} \tag{24}$$

where $(A, a, c_1, r_0) = (A, a, c_1, r_0)$ and other parameters are left unconstrained. These findings indicate that the dark soliton solution (23) is applicable within the specified parameters outlined in Eq. (24).

Moreover, following the analysis presented earlier, demonstrating the periodic solution

$$u = R\tan(Ax + aA^3 t)e^{i(Ax-5aA^3 t)}, \tag{25}$$

is possible.

Similarly and based on conditions (24), we may readily obtain the subsequent TW solutions as

$$\begin{cases} u = R\cosh\left(Ax + 2aA^3 t\right)e^{i\left(Ax-2aA^3 t\right)}, \\ u = R\cos\left(Ax + 4aA^3 t\right)e^{i\left(Ax+4aA^3 t\right)}, \\ u = R\operatorname{csch}\left(Ax + \dfrac{R^2\left(c_1 + 2c_2\right)A}{3}t\right)e^{i\left(Ax-\frac{R^2\left(c_1+c_2\right)}{3}\right)t}. \end{cases} \tag{26}$$

5. Exponential solutions

To provide a collection of exponential solutions to Eq. (2), we establish

$$u = R + e^{i\Theta}, \tag{27}$$

where (R, A, k) are non-zero constants.

By inserting Eq. (27) into Eq. (2), analyzing the coefficients of the exponential functions produced, and solving the resulting solution, we derive

$$\begin{cases} r_1 = -r_0,\ R = \pm 1, \\ c_1 = -\dfrac{2b}{A}, c_2 = \dfrac{b}{A},\ k = -A^3 a, \end{cases} \tag{28}$$

where $(A,a,b,r_0) = (A,a,b,r_0)$ and other parameters are left unconstrained. This demonstrates that the exponential solution (27) given by

$$u = \pm 1 + e^{i(Ax + A^3 at)}, \tag{29}$$

is valid under conditions (28).

6. Second set of exponential solutions

Similarly, to provide alternative exponential solutions to Eq. (2), we establish

$$u = \frac{1}{R + e^{i\Theta}}, \tag{30}$$

where (R,A,k) are non-zero constants.

By inserting Eq. (30) into Eq. (2), analyzing the coefficients of the resulting exponential functions, and solving the resulting system of equations, we obtain

$$\begin{cases} a = \dfrac{c_1 + 2c_2}{6A^2},\ b = -Ac_2, \\ R = \pm 1,\ k = -\dfrac{A(c_1 + 2c_2)}{6}, \end{cases} \tag{31}$$

where $(A,c_1,c_2) = (A,c_1,c_2)$ and other parameters are left unconstrained. This demonstrates the validity of the exponential solution (30) within the specified parameters (31).

7. Periodic solutions

This section will focus on proving the existence of periodic solutions for Eq. (2). We achieve this by introducing the following ansatz:

$$u = R\sec(Ax - Ct)e^{i\Theta}, \tag{32}$$

where (R,A,C,k) are constants.

By inserting Eq. (32) into Eq. (2) and analyzing the coefficients of the trigonometric functions in the resulting expression, we ultimately obtain

$$\begin{cases} b = \dfrac{A(6A^2 a - R^2 c_1)}{R^2},\ c_2 = \dfrac{6A^2 a - R^2 c_1}{2R^2}, \\ C = -4A^3 a,\ k = -4A^3 a, \end{cases} \tag{33}$$

where $(A,a,c_1) = (A,a,c_1)$ and other parameters are left free. Accordingly, the following periodic solutions are obtained

$$u = R\sec(Ax + 4A^3 at)e^{i(Ax + 4A^3 at)}, \tag{34}$$

Note that solutions (34) are only valid under conditions (33).

It should be highlighted that we can additionally demonstrate the singular solution

$$u = R\csc(Ax + 4A^3 at)e^{i(Ax + 4A^3 at)}, \tag{35}$$

is also valid under conditions (33).

8. Variety of other TW solutions

The following schemes are considered to provide a range of solitonic solutions for Eq. (2).

8.1. The combined sec and tan factor

Here, we suppose the solution is in the following form

$$u=\left(R\sec(Ax-Ct)+R_1\tan(Ax-Ct)\right)e^{i\Theta}, \quad ë \tag{36}$$

where $\Theta\left(Ax-kt\right)$ and $\left(R,R_1,A,C,k\right)$ are non-zero constants. By inserting Eq. (36) into Eq. (2), and following the previously stated analysis, we obtain

$$\begin{cases} c_1=\dfrac{3A^3a-2R^2b}{2AR^2}, \quad c=\dfrac{b}{2A}, \\ C=-\dfrac{5aA^3}{2}, \quad k=\dfrac{aA^3}{2}, \quad R_1=R, \end{cases} \tag{37}$$

where $\left(A,a,b,R\right)=\left(A,a,b,R\right)$ and substituting Eq. (37) into solution (36) gives a new TW solution.

8.2. The combined sin and cos factor

Here, we suppose the solution is in the following form

$$u=\left(R\sin(Ax-Ct)+R_1\cos(Ax-Ct)\right)e^{i\Theta}, \tag{38}$$

where $\left(R,R_1,A,C,k\right)$ are non-zero constants. By inserting Eq. (38) into Eq. (2), and following the previously provided methodology, we obtain

$$\begin{cases} b=-Ac_1, \;\; c_2=-\dfrac{1}{2}c_1, \\ C=-4A^3a, \;\; k=-4A^3a, \\ R_1=R, \end{cases} \tag{39}$$

where $\left(A,a,c_1,R\right)=\left(A,a,c_1,R\right)$ and substituting system (39) into solution (38) gives a new TW solution.

8.3. The combined **tan** *and* **cot** *factor*

Here, we suppose the solution is in the following form

$$u=\left(R\tan(Ax-Ct)+R_1\cot(Ax-Ct)\right)e^{i\Theta}, \tag{40}$$

where $\left(R,R_1,A,C,k\right)$ are non-zero constants. By inserting Eq. (40) into Eq. (2) and following the previously stated analysis, we obtain the following three sets of solutions:

Set-I

$$\begin{cases} c_1=\dfrac{6A^3a-R^2b}{AR^2}, c_2=\dfrac{b}{2A}, \\ C=-7A^3a, k=-13A^3a, \; R_1=R, \end{cases} \tag{41}$$

where $\left(A,a,b,R\right)=\left(A,a,b,R\right)$.

Set-II

$$\begin{cases} c_1=\dfrac{6A^3a-R^2b}{AR^2}, c_2=\dfrac{b}{2A}, \\ C=5A^3a, k=23A^3a, \; R_1=-R, \end{cases} \tag{43}$$

where $\left(A,a,b,R\right)=\left(A,a,b,R\right)$.

By substituting systems (41) and (42) into solution (40), new TW solutions can be obtained.

9. Conclusion

The following points encapsulate the significant findings of our research:

- Using integration algorithms and modified exponential rational function methods, this paper found a wide range of solutions to the nonlinear Schrödinger equation (NLSE) with third-order dispersion and in the absence of chromatic dispersion.
- Novel approaches have been applied to extract a rich collection of new and compatible wave solutions for a specific NLSE.
- Wide ranges of solutions to the proposed problem have been derived using integration algorithms and modified exponential rational function methods.
- The investigation unveils various modulated envelope soliton solutions, including bright and dark envelope solitons, alongside other new waveforms for the studied equation.
- Employing innovative techniques, the authors uncover diverse solitonic and wave-like solutions with exponential, trigonometric, and hyperbolic features.
- The work rigorously derives the conditions for the existence of modulated structures within the obtained solution family, paving the way for further exploration.
- This study presents valuable solutions and establishes a framework for future investigations into related nonlinear Schrödinger equations.

The solutions we obtained are expected to explain many ambiguities around some nonlinear phenomena that arise in various plasma physics systems, in addition to optical fibers and many other media.

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Анотація. *Це нове дослідження вивчає нелінійне рівняння Шредінгера (NLSE), сформоване без врахування хроматичної дисперсії. Нові алгоритми інтегрування розкривають різноманіття оптичних солітонів та інших точних розв'язків різних фізичних структур. Це дослідження використовує арсенал потужних методів, включаючи різні форми модифікованого методу експоненціальних раціональних функцій, для розкриття багатого розмаїття розв'язків для досліджуваного нелінійного рівняння Шредінгера. Кожна конкретна методика розкриває унікальні розв'язки для біжучої хвилі разом із основними обмежуючими параметрами, які визначають їхнє існування. Крім того, за певних параметрів цей інструментарій дає набір нових оптичних рішень: модульовані хвилі, яскраві та темні солітони огинаючої, а також періодичні та біжучі форми хвиль. Одержані дані розкривають різноманіття розв'язків для цього рівняння, відкриваючи шлях для глибшого розуміння в таких галузях, як оптичні волокна та фізика плазми.*

Ключові слова*: рівняння Шредінгера, модифікований метод експоненціальноої раціональної функції, солітонні розв'язки.*