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QUIESCENT OPTICAL SOLITONS FOR FOKAS–LENELLS EQUATION WITH NONLINEAR CHROMATIC DISPERSION HAVING QUADRATIC AND QUADRATIC-QUARTIC FORMS OF SELF-PHASE MODULATION

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Abstract. Quiescent optical soliton solutions within the Fokas-Lenells equation, accounting for nonlinear chromatic dispersion, are being investigated in this study for the first time. Two forms of self-phase modulation, quadratic and quadratic-quartic, are considered, including perturbation terms to introduce added complexity that refers to the inclusion of perturbation terms in the analysis of self-phase modulation. The F-expansion integration method is employed for finding various soliton solutions, including bright, dark, and singular solitons. These solitons are characterized by specific features that are influenced by their behavior.

Keywords: perturbed Fokas–Lenells equation, nonlinear chromatic dispersion, quiescent optical soliton, F-expansion method
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1. Introduction

Quiescent optical soliton solutions, often referred to as stationary solitons, are a particular class of solitons in nonlinear optics [1–5]. These solitons are characterized by their stable, localized waveforms that do not change shape or move as they propagate through a nonlinear optical medium [6–10]. Unlike mobile solitons, which can exhibit a net shift in position over time, quiescent solitons remain stationary and are particularly useful in applications where preserving the waveform's position is critical. Quiescent optical solitons are important in various areas of nonlinear optics and optical communication [11–15]. They can be used in the design of optical communication systems to transmit information without distortion, as the stationary nature of quiescent solitons ensures that they maintain their shape and position during propagation. One of the advantages of quiescent solitons is their inherent stability. Unlike mobile solitons, which can be subject to interactions and perturbations that may alter their shape or position, quiescent solitons tend to be more robust against external influences. Quiescent optical solitons can be described by various nonlinear partial differential equations (PDEs) that govern the propagation of optical pulses in a nonlinear medium [16–18]. The Fokas–Lenells equation (FLE) is an example of a PDE that can describe such solitons [19–27]. The FLE is a two-dimensional integrable system, which means it possesses a rich mathematical structure that allows for analytical solutions [19–23]. Integrable systems have symmetries and conservation laws that make them amenable to mathematical techniques, such as the inverse scattering method. The equation accounts for various nonlinear effects, leading to solitons forming [24–26]. The model considers dispersion effects, which can counteract the self-interaction and cause stationary solitons [27]. The model can be solved using mathematical techniques, such as inverse scattering and F-expansion methods. These methods allow for the determination of analytical solutions and the exploration of the equation's soliton solutions. The F-expansion method is a powerful mathematical technique to find soliton solutions to nonlinear PDEs, such as the FLE [28–30]. It involves making a series expansion of the solution and finding coefficients that satisfy the equation, resulting in various soliton solutions. It has been applied to various physical and mathematical problems, including fluid dynamics, plasma physics, and nonlinear optics. In the context of nonlinear optics, the method can be used to find soliton solutions for equations, which describe the behavior of solitons in some optical systems. These soliton solutions are critical for understanding and designing optical communication systems and other nonlinear optical phenomena. In the current study, the bright, dark, and singular soliton solutions to the FLE are derived by the F-expansion method. Bright solitons are localized waveforms balancing nonlinear self-focusing and dispersion, ensuring stable shapes. They have applications in nonlinear optics and distortion-free optical fiber communication over long distances. Dark solitons are localized reductions in intensity achieved by balancing nonlinear self-defocusing and dispersion. They find applications in nonlinear optics and the manipulation of optical pulses in atomic physics. Singular solitons have singularities in their waveforms, often resulting from a combination of nonlinear, dispersive, and higher-order nonlinear effects. They are less common and are studied for their mathematical properties and extremely localized behaviors in wave phenomena.

2. F–Expansion procedure

Let us take into account the model equation [28–30]:

$$G\left(q,q_x,q_t,q_{xt},q_{xx},\ldots\right)=0, \tag{1}$$

where G is a polynomial $q=q\left(x,t\right)$, q_x,q_t,q_{xt},q_{xx} are the partial derivatives in which the highest order derivatives and nonlinear terms are involved, and x and t represent the spatial and temporal variables, respectively, within the context of the optic wave field $q=q\left(x,t\right)$. Then let's consider the constraints

$$q\left(x,t\right)=U\left(\xi\right),\ \xi=\eta\left(x-\upsilon t\right), \tag{2}$$

where ξ and η take on the roles of the wave variable and wave width, respectively, with υ signifying the wave velocity. It follows that Eq. (1) becomes

$$P\left(U,-\eta\upsilon U',\eta U',\eta^2 U'',\ldots\right)=0. \tag{3}$$

<u>Step–1</u>: In the presence of the condition defined by Eq. (3), the simplified model confirms the solution structure

$$U\left(\xi\right)=\sum_{l=0}^{N}B_l F^l\left(\xi\right)\ , \tag{4}$$

where B_l are real-valued constants, F_l is a new dependent variable, and N is the positive integer balance number.

Through the application of the ancillary equation

$$F'\left(\xi\right)=\sqrt{PF^4\left(\xi\right)+QF^2\left(\xi\right)+R}\ , \tag{5}$$

where P, Q, R are real valued constants, we arrive at the soliton wave profiles

$$F(\xi)=\text{sn}(\xi)=\tanh(\xi),\ P=m^2,\ Q=-(1+m^2),\ R=1,\quad m\to 1^-, \tag{6}$$

$$F(\xi)=\text{ns}(\xi)=\coth(\xi),\ P=1,\ Q=-(1+m^2), R=m^2,\quad m\to 1^-, \tag{7}$$

$$F(\xi)=\text{cn}(\xi)=\text{sech}(\xi),\ P=-m^2,\ Q=2m^2-1,\ R=1-m^2,\ m\to 1^-, \tag{8}$$

$$F(\xi)=\text{ds}(\xi)=\text{csch}(\xi),\ P=1,\ Q=2m^2-1,\ R=-m^2(1-m^2),\ m\to 1^-, \tag{9}$$

$$\begin{gathered}F(\xi)=\text{ns}(\xi)\pm\text{ds}(\xi)=\coth(\xi)\pm\text{csch}(\xi),\\ P=\frac{1}{4},\ \ Q=\frac{m^2-2}{2},\ R=\frac{m^2}{4},\ m\to 1^-,\end{gathered} \tag{10}$$

$$\begin{gathered}F(\xi)=\text{sn}(\xi)\pm i\,\text{cn}(\xi)=\tanh(\xi)\pm i\,\text{sech}(\xi),\\ P=\frac{m^2}{4},\quad Q=\frac{m^2-2}{2},\quad R=\frac{m^2}{4},\quad m\to 1^-,\quad i=\sqrt{-1}\,,\end{gathered} \tag{11}$$

and

$$F(\xi)=\frac{\text{sn}(\xi)}{1\pm\text{dn}(\xi)}=\frac{\tanh(\xi)}{1\pm\text{sech}(\xi)},\ P=\frac{m^2}{4},\ Q=\frac{m^2-2}{2},\ R=\frac{m^2}{4},\ m\to 1^-, \tag{12}$$

where the Jacobi elliptic functions (JEFs) $\text{sn}(\xi)$, $\text{ns}(\xi)$, $\text{cn}(\xi)$, $\text{ds}(\xi)$ and $\text{dn}(\xi)$ are associated with a modulus, $0<m<1$. Furthermore, the constants B_l with l ranging from 0 to N are a product of the balancing approach outlined in Eq. (3).

Step–2: Combining Eqs. (4) and (5) within Eq. (3), we establish a system of equations that leads to the determination of the unknown constants in Eq. (4) through Eq. (12).

3. Quiscent optical solitons

Within this section, the integration method is employed to acquire quiescent optical solitons in conjunction with the model. The subsequent procedures are arranged in the upcoming subsections.

3.1. Quadratic form of SPM

The equation describing the perturbed FLE with both nonlinear chromatic dispersion (CD) and a quadratic self-phase modulation (SPM) for the first time is given by:

$$iq_t+a\left(|q|^n q\right)_{xx}+|q|^2\left(bq+i\sigma q_x\right)+c|q|q=i\left[\alpha q_x+\lambda\left(|q|^2 q\right)_x+\mu\left(|q|^2\right)_x q\right], \tag{13}$$

where $q=q(x,t)$ defines the complex envelope of the electric field, which represents the optical wave. In this context, x denotes the propagation distance along the optical medium, while t represents the time variable. The parameter b stands for the nonlinear coefficient, characterizing the nonlinear self-interaction of the optical field due to the intensity-dependent refractive index of the medium. This nonlinearity arises from the Kerr effect, a phenomenon in which the refractive index changes in response to the intensity of the light. Parameter a signifies the nonlinear CD parameter, and the power-law parameter n introduces nonlinearity to the CD. The parameter σ represents the nonlinear dispersion coefficient, while c stems from the quadratic form of SPM. The first term in the equation, (iq_t), accounts for the temporal evolution of the optical wave as it propagates through the nonlinear medium. The coefficient α is associated with inter-modal dispersion, and λ is related to the self-steepening perturbation term. Finally, μ contributes to the self-frequency shift. The quiescent optical soliton is characterized by the assumed profile:

$$q(x,t)=U(kx)e^{i(\omega t+\theta)}, \tag{14}$$

where both θ and ω are derived from the phase constant and frequency, respectively. The soliton amplitude component, $U(kx)$, is characterized by the soliton wavevector k. By substituting Eq. (14) into Eq. (13), we arrive at the real part

$$ak^2n(n+1)U^n(U')^2+ak^2(n+1)U^{1+n}U''-\omega U^2+bU^4+cU^3=0, \tag{15}$$

and the imaginary part

$$-\alpha kUU'+k(\sigma-3\lambda-2\mu)U^3U'=0. \tag{16}$$

To satisfy the conditions of integrability, one considers

$$n=1, \tag{17}$$

$$\alpha=0, \tag{18}$$

and

$$\sigma-3\lambda-2\mu=0. \tag{19}$$

The governing Eq. (13) is transformed after the implementation of these changes to:

$$iq_t+a(|q|q)_{xx}+|q|^2(bq+i\sigma q_x)+c|q|q=i\left[\lambda\left(|q|^2q\right)_x+\mu\left(|q|^2\right)_x q\right]. \tag{20}$$

Following this, Eq. (15) turns into:

$$2ak^2(U')^2+2ak^2UU''-\omega U+bU^3+cU^2=0. \tag{21}$$

To achieve this, the balance of terms $(U')^2$ or UU'' with U^3 in Eq. (21) results in $N=2$. In this integration process, the solution structure of Eq. (4) is presented in a simplified form as

$$U(\xi)=B_0+B_1F(\xi)+B_2F^2(\xi). \tag{22}$$

The combination of Eq. (22) with Eq. (5) into Eq. (21) leaves us with the equations:

$$20Pak^2B_2^2+bB_2^3=0, \tag{23}$$

$$24Pak^2B_1B_2+3bB_1B_2^2=0, \tag{24}$$

$$4Pak^2B_0B_1+18Qak^2B_1B_2+6bB_0B_1B_2+bB_1^3+2cB_1B_2=0, \tag{25}$$

$$4Rak^2B_0B_2+2Rak^2B_1^2+bB_0^3+cB_0^2-\omega B_0=0, \tag{26}$$

$$2Qak^2B_0B_1+12Rak^2B_1B_2+3bB_0^2B_1+2cB_0B_1-\omega B_1=0, \tag{27}$$

$$\begin{gathered}12Pak^2B_0B_2+6Pak^2B_1^2+16Qak^2B_2^2\\+3bB_0B_2^2+3bB_1^2B_2+cB_2^2=0,\end{gathered} \tag{28}$$

$$\begin{gathered}8Qak^2B_0B_2+4Qak^2B_1^2+12Rak^2B_2^2\\+3bB_0^2B_2+3bB_0B_1^2+2cB_0B_2+cB_1^2-\omega B_2=0.\end{gathered} \tag{29}$$

Upon solving these equations, one uncovers the outcomes:

Result–1:

$$\begin{gathered}k=\pm\sqrt{-\frac{c}{16Qa}},\ \omega=-\frac{15c^2RP}{16Q^2b},\\B_0=0,\ B_1=0,\ B_2=\frac{5Pc}{4Qb}.\end{gathered} \tag{30}$$

The inclusion of Eq. (30) with the help of Eq. (6) into Eq. (22) results in the dark soliton solution:

$$q(x,t)=-\frac{5c}{8b}\tanh^2\left(\sqrt{\frac{c}{32a}}x\right)e^{i\left(-\frac{15c^2}{64b}t+\theta\right)}. \tag{31}$$

Incorporating both Eqs. (30) and (7) into Eq. (22) leads to the singular soliton solution:

$$q(x,t)=-\frac{5c}{8b}\coth^2\left(\sqrt{\frac{c}{32a}}x\right)e^{i\left(-\frac{15c^2}{64b}t+\theta\right)}. \tag{32}$$

When Eq. (30) is employed with the application of Eq. (10) within Eq. (22), the combined singular soliton solution is obtained as

$$q(x,t)=-\frac{5c}{8b}\left\{\coth\left(\sqrt{\frac{c}{8a}}x\right)+\operatorname{csch}\left(\sqrt{\frac{c}{8a}}x\right)\right\}^2 e^{i\left(-\frac{15c^2}{64b}t+\theta\right)}. \tag{33}$$

By inserting Eq. (30) with the utilization of Eq. (11) into Eq. (22), the complexion solution is regained as

$$q(x,t)=-\frac{5c}{8b}\left\{\tanh\left(\sqrt{\frac{c}{8a}}x\right)+i\operatorname{sech}\left(\sqrt{\frac{c}{8a}}x\right)\right\}^2 e^{i\left(-\frac{15c^2}{64b}t+\theta\right)}. \tag{34}$$

When Eq. (30) is incorporated with the help of Eq. (12) within Eq. (22), we retrieve the combo bright–dark soliton solution as

$$q(x,t)=-\frac{5c}{8b}\left\{\frac{\tanh\left(\sqrt{\frac{c}{8a}}x\right)}{1+\operatorname{sech}\left(\sqrt{\frac{c}{8a}}x\right)}\right\}^2 e^{i\left(-\frac{15c^2}{64b}t+\theta\right)}. \tag{35}$$

The wave profiles, as characterized in Eq. (31) through Eq. (35), are defined as

$$ac>0. \tag{36}$$

Figs. 1, 2, and 3 offer visual representations of numerical simulations illustrating bright, dark and combo bright–dark soliton solutions, respectively. In Figs. 1(a), 2(a), and 3(a), we observe the impact of nonlinearity when $c=-1$ and $a=-1$. Additionally, Figs. 1(b), 2(b), and 3(b)

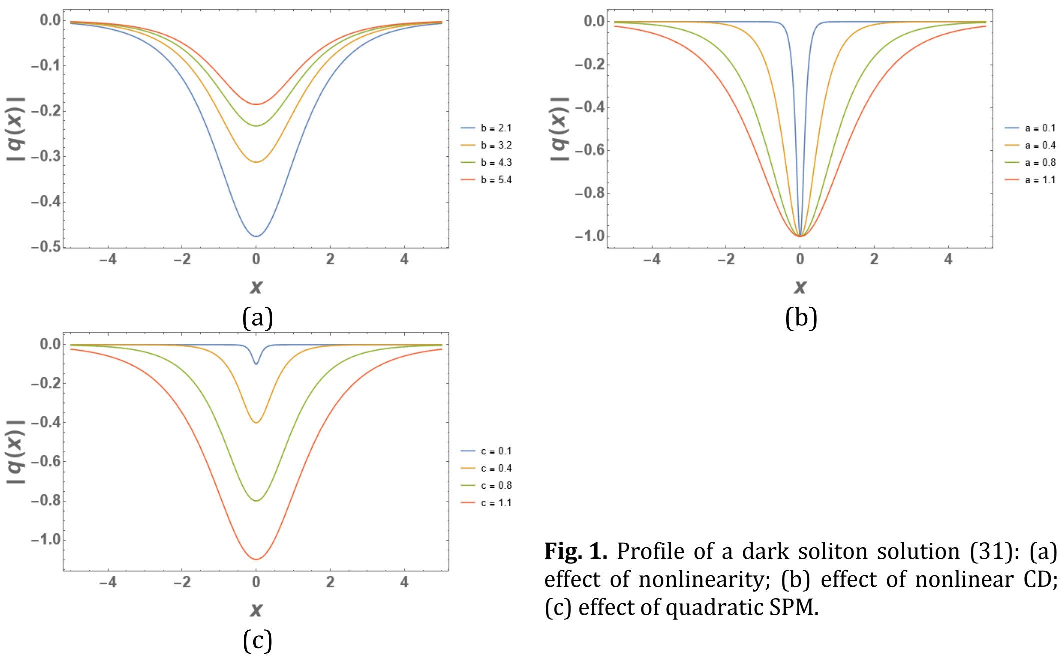


Fig. 1. Profile of a dark soliton solution (31): (a) effect of nonlinearity; (b) effect of nonlinear CD; (c) effect of quadratic SPM.

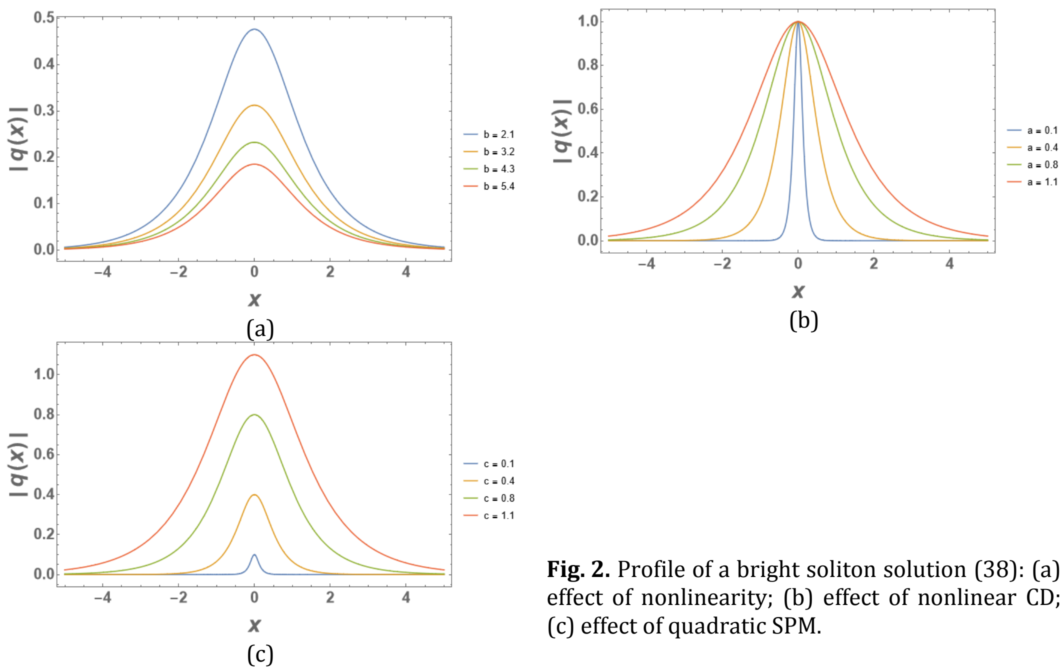


Fig. 2. Profile of a bright soliton solution (38): (a) effect of nonlinearity; (b) effect of nonlinear CD; (c) effect of quadratic SPM.

demonstrate the influence of nonlinear CD when $c=1$ and $b=-1$, while Figs. 1(c), 2(c), and 3(c) depict the effects of quadratic SPM for $a=1$ and $b=-1$.

Result–2:

$$\begin{aligned}
k &= \frac{\sqrt{-2a\left(9PR-2Q^2\right)\left(-Q+3\sqrt{-4PR+Q^2}\right)c}}{8a\left(9PR-2Q^2\right)}, \\
\omega &= -\frac{5c}{16b\left(9PR-2Q^2\right)} \\
&\times\left(-\frac{3\left(-Q+3\sqrt{-4PR+Q^2}\right)cPQR}{2\left(9PR-2Q^2\right)}+\frac{\left(-Q+3\sqrt{-4PR+Q^2}\right)cQ^3}{2\left(9PR-2Q^2\right)}+6PRc-Q^2c\right), \\
B_0 &= -\frac{5}{12b}\left(-\frac{Q\left(-Q+3\sqrt{-4PR+Q^2}\right)c}{2\left(9PR-2Q^2\right)}+c\right), \\
B_1 &= 0, \\
B_2 &= \frac{5P}{2b\left(-Q+3\sqrt{-4PR+Q^2}\right)}\left(-\frac{Q\left(-Q+3\sqrt{-4PR+Q^2}\right)c}{2\left(9PR-2Q^2\right)}-c\right).
\end{aligned} \tag{37}$$

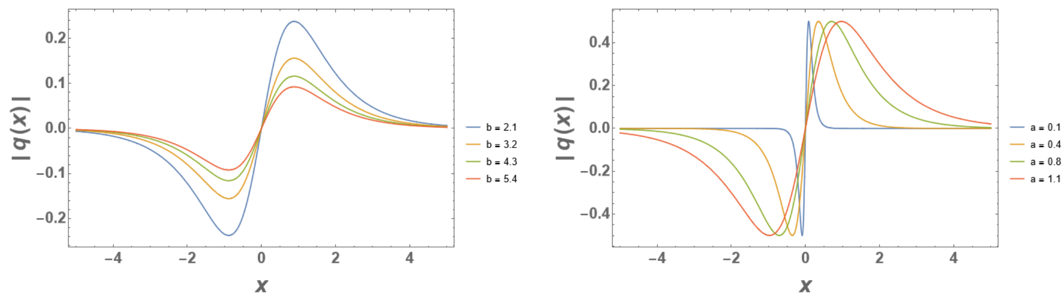
Incorporating both Eqs. (37) and (8) into Eq. (22) leads to the structuring of the bright soliton solution

$$q(x,t)=\left\{-\frac{5c}{8b}+\frac{5c}{8b}\operatorname{sech}^2\left(\sqrt{\frac{c}{32a}}x\right)\right\}e^{i\left(-\frac{15c^2}{64b}t+\theta\right)}. \tag{38}$$

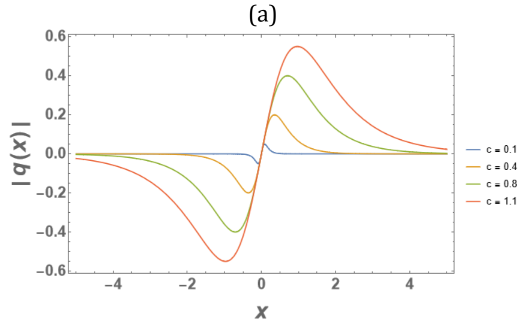
The inclusion of Eq. (37) with the use of Eq. (9) into Eq. (22) results in the structure of the singular soliton solution as

$$q(x,t)=\left\{-\frac{5c}{8b}-\frac{5c}{8b}\operatorname{csch}^2\left(\sqrt{\frac{c}{32a}}x\right)\right\}e^{i\left(-\frac{15c^2}{64b}t+\theta\right)}. \tag{39}$$

The wave profiles, detailed from Eqs. (38) to (39), are revealed by Eq. (36).



(a) (b)



(c)

Fig. 3. Profile of a combo bright–dark soliton solution (35): (a) effect of nonlinearity; (b) effect of nonlinear CD; (c) effect of quadratic SPM.

3.2. Quadratic–Quartic form of SPM

The perturbed FLE, including nonlinear CD and a quadratic-quartic SPM for the first time, is expressed as:

$$\begin{aligned} &iq_t + a\left(|q|^n q\right)_{xx} + |q|^2\left(bq + i\sigma q_x\right) + c|q|q + d|q|^3 q \\ &= i\left[\alpha q_x + \lambda\left(|q|^2 q\right)_x + \mu\left(|q|^2\right)_x q\right], \end{aligned} \tag{40}$$

where d is associated with the quadratic-quartic SPM. When Eq. (14) is applied to Eq. (40), we arrive at the real part

$$\begin{aligned} &ak^2 n(n+1)U^n\left(U'\right)^2 + ak^2(n+1)U^{1+n}U'' \\ &-\omega U^2 + bU^4 + cU^3 + dU^5 = 0, \end{aligned} \tag{41}$$

and the imaginary part Eq. (16). In order to ensure integrability, it is necessary to meet the criteria provided in Eq. (17) through Eq. (19). Following the implementation of these criteria, the governing Eq. (40) is modified:

$$\begin{aligned} &iq_t + a\left(|q|q\right)_{xx} + |q|^2\left(bq + i\sigma q_x\right) + c|q|q + d|q|^3 q \\ &= i\left[\lambda\left(|q|^2 q\right)_x + \mu\left(|q|^2\right)_x q\right]. \end{aligned} \tag{42}$$

In this process, Eq. (41) is simplified to:

$$2ak^2\left(U'\right)^2 + 2ak^2 UU'' - \omega U + bU^3 + cU^2 + dU^4 = 0. \tag{43}$$

With the aim of achieving this, the terms $\left(U'\right)^2$ or UU'' with U^4 are balanced in Eq. (21) which leads to $N = 1$. Through this integration technique, the solution structure of Eq. (4) is simplified to:

$$U(\xi) = B_0 + B_1 F(\xi). \tag{44}$$

By incorporating Eq. (44) along with Eq. (5) into Eq. (43), we obtain the simplest equations:

$$6Pak^2B_1^2 + dB_1^4 = 0, \tag{45}$$

$$4Pak^2B_0B_1 + 4dB_0B_1^3 + bB_1^3 = 0, \tag{46}$$

$$4Qak^2B_1^2 + 6dB_0^2B_1^2 + 3bB_0B_1^2 + cB_1^2 = 0, \tag{47}$$

$$2Qak^2B_0B_1 + 4dB_0^3B_1 + 3bB_0^2B_1 + 2cB_0B_1 - \omega B_1 = 0, \tag{48}$$

$$2Rak^2B_1^2 + dB_0^4 + bB_0^3 + cB_0^2 - \omega B_0 = 0. \tag{49}$$

By addressing these equations, the results can be extracted:

$$\begin{gathered}
b = \pm\frac{10}{3d}\sqrt{-\frac{-3Qc\sqrt{-4PR+Q^2} - 36cPR + 3cQ^2}{144PRd - 20Q^2d}}, \\
k = \pm\frac{\sqrt{a\left(36PR - 5Q^2\right)\left(2Q + 3\sqrt{-4PR+Q^2}\right)c}}{2a\left(36PR - 5Q^2\right)}, \\
B_0 = \pm\sqrt{-\frac{-3Qc\sqrt{-4PR+Q^2} - 36cPR + 3cQ^2}{144PRd - 20Q^2d}}, \\
B_1 = \pm\sqrt{-\frac{9Pc\sqrt{-4PR+Q^2} + 6PcQ}{72PRd - 10Q^2d}}, \\
\omega = \pm\frac{1}{2Q + 3\sqrt{-4PR+Q^2}}\sqrt{-\frac{-3Qc\sqrt{-4PR+Q^2} - 36cPR + 3cQ^2}{144PRd - 20Q^2d}} \\
\times\left(\frac{18PR\left(2Q + 3\sqrt{-4PR+Q^2}\right)c}{36PR - 5Q^2} - \frac{13Q^2\left(2Q + 3\sqrt{-4PR+Q^2}\right)c}{2\left(36PR - 5Q^2\right)} + Qc\right).
\end{gathered} \tag{50}$$

Combining Eqs. (50) with the assistance of Eq. (8) into Eq. (44) results in the bright soliton solution:

$$q(x,t) = \left\{\pm\sqrt{\frac{3c}{10d}} \pm \sqrt{\frac{3c}{10d}}\operatorname{sech}\left(\pm\sqrt{\frac{c}{20a}}x\right)\right\}e^{i\left(\pm\frac{3c}{10}\sqrt{\frac{3c}{10d}}t + \theta\right)}. \tag{51}$$

By incorporating Eq. (50) with the help of Eq. (9) into Eq. (44), the complexion solution is obtained:

$$q(x,t) = \left\{\pm\sqrt{\frac{3c}{10d}} \pm i\sqrt{\frac{3c}{10d}}\operatorname{csch}\left(\pm\sqrt{\frac{c}{20a}}x\right)\right\}e^{i\left(\pm\frac{3c}{10}\sqrt{\frac{3c}{10d}}t + \theta\right)}. \tag{52}$$

The wave forms depicted in Eqs. (51) and (52) are given by

$$ac > 0, \quad cd > 0. \tag{53}$$

4. Conclusions

In this study, a comprehensive exploration of quiescent optical soliton solutions within the framework of the FLE, accounting for the influence of nonlinear CD in optical systems, was undertaken for the first time. Two distinct forms of SPM, namely the quadratic and quadratic-quartic variations, were investigated, along with the introduction of perturbation terms, to unveil the complexities inherent in the behavior of optical solitons. The choice of the F-expansion integration method as the analytical tool proved invaluable in deciphering the intricate dynamics within this optical system. This method confirmed the existence of various soliton solutions, and their categorization into three distinct types – bright, dark, and singular solitons – was made possible. With their localized, amplitude-modulated waveforms, bright solitons represent a robust category of solitons well-suited for signal transmission and manipulation. On the other hand, dark solitons demonstrate the ability to confine regions of reduced intensity within their profiles, making them significant in scenarios where controlled signal attenuation is required. The exceptional singular solitons, with their unique characteristics, further expand our understanding of the diverse behaviors exhibited by optical pulses. From a practical standpoint, profound implications for optical communication systems and nonlinear optical devices are carried out by this research. The ability to harness and manipulate solitons is essential in optimizing signal transmission and information processing in these systems, and the contributions made by our findings to this endeavor are significant.

References

1. Al-Ghafri, K. S., Sankar, M., Krishnan, E. V., Khan, S., & Biswas, A. (2023). Chirped gap solitons in fiber Bragg gratings with polynomial law of nonlinear refractive index. *Journal of the European Optical Society*, *19*(1).
2. Al-Ghafri, K. S., Krishnan, E. V., & Biswas, A. (2022). Cubic–quartic optical soliton perturbation and modulation instability analysis in polarization-controlled fibers for Fokas–Lenells equation. *Journal of the European Optical Society-Rapid Publications*, *18*(2), 9.
3. Topkara, E., Milovic, D., Sarma, A., Majid, F., & Biswas, A. (2009). A study of optical solitons with Kerr and power law nonlinearities by He's variational principle. *Journal of the European Optical Society-Rapid Publications*, *4*.
4. González-Gaxiola, O., Biswas, A., & Belic, M. R. (2019). Optical soliton perturbation of Fokas-Lenells equation by the Laplace-Adomian decomposition algorithm. *Journal of the European Optical Society-Rapid Publications*, *15*, 1-9.
5. Arnous, A. H., Biswas, A., Kara, A. H., Yildirim, Y., Moraru, L., Iticescu, C., Moldovanu, S., & Alghamdi, A. A. (2023). Optical solitons and conservation laws for the concatenation model with spatio-temporal dispersion (internet traffic regulation). *Journal of the European Optical Society-Rapid Publications*, *19*(2), 35.
6. Zayed, E. M., El-Horbaty, M., Alngar, M. E., Shohib, R. M., Biswas, A., Yildirim, Y., Moraru, L., Iticescu, C., Bibicu, D., Georgescu, P. L., & Asiri, A. (2023). Dynamical system of optical soliton parameters by variational principle (super-Gaussian and super-sech pulses). *Journal of the European Optical Society-Rapid Publications*, *19*(2), 38.
7. Biswas, A. (2020). Optical soliton cooling with polynomial law of nonlinear refractive index. *Journal of Optics*, *49*(4), 580-583.
8. Wang, S. (2023). Novel soliton solutions of CNLSEs with Hirota bilinear method. *Journal of Optics*, 1-6.
9. Han, T., Li, Z., Li, C., & Zhao, L. (2023). Bifurcations, stationary optical solitons and exact solutions for complex Ginzburg–Landau equation with nonlinear chromatic dispersion in non-Kerr law media. *Journal of Optics*, *52*(2), 831-844.
10. Nandy, S., & Lakshminarayanan, V. (2015). Adomian decomposition of scalar and coupled nonlinear Schrödinger equations and dark and bright solitary wave solutions. *Journal of Optics*, *44*, 397-404.
11. Chen, W., Shen, M., Kong, Q., & Wang, Q. (2015). The interaction of dark solitons with competing nonlocal cubic nonlinearities. *Journal of Optics*, *44*, 271-280.
12. Xu, S. L., Petrović, N., & Belić, M. R. (2015). Two-dimensional dark solitons in diffusive nonlocal nonlinear media. *Journal of Optics*, *44*, 172-177.
13. Dowluru, R. K., & Bhima, P. R. (2011). Influences of third-order dispersion on linear birefringent optical soliton transmission systems. *Journal of Optics*, *40*, 132-142.
14. Singh, M., Sharma, A. K., & Kaler, R. S. (2011). Investigations on optical timing jitter in dispersion managed higher order soliton system. *Journal of Optics*, *40*, 1-7.
15. Janyani, V. (2008). Formation and Propagation-Dynamics of Primary and Secondary Soliton-Like Pulses in Bulk Nonlinear Media. *Journal of Optics*, *37*, 1-8.
16. Hasegawa, A. (2004). Application of Optical Solitons for Information Transfer in Fibers—A Tutorial Review. *Journal of Optics*, *33*(3), 145-156.
17. Mahalingam, A., Uthayakumar, A., & Anandhi, P. (2013). Dispersion and nonlinearity managed multisoliton propagation in an erbium doped inhomogeneous fiber with gain/loss. *Journal of Optics*, *42*, 182-188.
18. Jawad, A. J. M., & Abu-AlShaeer, M. J. (2023). Highly dispersive optical solitons with cubic law and cubic-quinticseptic law nonlinearities by two methods. *Al-Rafidain J. Eng. Sci.*, *1*(1), 1-8.

19. Triki, H., Zhou, Q., Biswas, A., Liu, W., Yildirim, Y., Alshehri, H. M., & Belic, M. R. (2022). Localized pulses in optical fibers governed by perturbed Fokas–Lenells equation. *Physics Letters A*, *421*, 127782.
20. Triki, H., Zhou, Q., Liu, W., Biswas, A., Moraru, L., Yildirim, Y., Alshehri, H. M., & Belic, M. R. (2022). Chirped optical soliton propagation in birefringent fibers modeled by coupled Fokas-Lenells system. *Chaos, Solitons & Fractals*, *155*, 111751.
21. Triki, H., & Wazwaz, A. M. (2017). Combined optical solitary waves of the Fokas—Lenells equation. *Waves in Random and Complex Media*, *27*(4), 587-593.
22. Triki, H., & Wazwaz, A. M. (2017). New types of chirped soliton solutions for the Fokas–Lenells equation. *International Journal of Numerical Methods for Heat & Fluid Flow*, *27*(7), 1596-1601.
23. Bansal, A., Kara, A. H., Biswas, A., Khan, S., Zhou, Q., & Moshokoa, S. P. (2019). Optical solitons and conservation laws with polarization–mode dispersion for coupled Fokas–Lenells equation using group invariance. *Chaos, Solitons & Fractals*, *120*, 245-249.
24. Li, Z., & Huang, C. (2023). Bifurcation, phase portrait, chaotic pattern and optical soliton solutions of the conformable Fokas–Lenells model in optical fibers. *Chaos, Solitons & Fractals*, *169*, 113237.
25. Wang, B. H., Wang, Y. Y., Dai, C. Q., & Chen, Y. X. (2020). Dynamical characteristic of analytical fractional solitons for the space-time fractional Fokas-Lenells equation. *Alexandria Engineering Journal*, *59*(6), 4699-4707.
26. Wang, Z., He, L., Qin, Z., Grimshaw, R., & Mu, G. (2019). High-order rogue waves and their dynamics of the Fokas–Lenells equation revisited: a variable separation technique. *Nonlinear Dynamics*, *98*, 2067-2077.
27. Li, Z., Fan, W., & Miao, F. (2023). Chaotic pattern, phase portrait, sensitivity and optical soliton solutions of coupled conformable fractional Fokas-Lenells equation with spatio-temporal dispersion in birefringent fibers. *Results in Physics*, *47*, 106386.
28. Yildirim, Y., Biswas, A., Moraru, L., & Alghamdi, A. A. (2023). Quiescent optical solitons for the concatenation model with nonlinear chromatic dispersion. *Mathematics*, *11*(7), 1709.
29. Yildirim, Y., & Mirzazadeh, M. (2020). Optical pulses with Kundu-Mukherjee-Naskar model in fiber communication systems. *Chinese Journal of Physics*, *64*, 183-193.
30. Yildirim, Y., Biswas, A., Kara, A. H., Ekici, M., Alzahrani, A. K., & Belic, M. R. (2021). Cubic–quartic optical soliton perturbation and conservation laws with generalized Kudryashov's form of refractive index. *Journal of Optics*, *50*, 354-360.

Yakup Yildirim. (2024). Quiescent Optical Solitons for Fokas–Lenells Equation with Nonlinear Chromatic Dispersion Having Quadratic and Quadratic-Quartic Forms of Self-Phase Modulation. *Ukrainian Journal of Physical Optics*, *25*(5), S1039 – S1048.
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Анотація. *У цьому дослідженні вперше розглядаються розв'язки стаціонарних оптичних солітонів в межах рівняння Фокаса-Ленеллса з урахуванням нелінійної хроматичної дисперсії. Розглянуто дві форми самомодуляції фази - квадратична та квадратично-квартична, з включенням членів збурення для введення додаткової складності, яка стосується включення членів збурення в аналізі самомодуляції фази. Використовується метод інтегрування F-розкладу для знаходження різних розв'язків солітонів, включаючи яскраві, темні та сингулярні солітони. Ці солітони характеризуються певними особливостями, які визначають їхню поведінку.*

Ключові слова*: збурене рівняння Фокаса–Ленеллса, нелінійна хроматична дисперсія, стаціонарний оптичний солітон, метод F-розкладу.*