Ukrainian Journal of Physical Optics
SPECIAL ISSUE 5, 2024

Optical Solitons of the Generalized Stochastic Gerdjikov-Ivanov Equation in the Presence of Multiplicative White Noise

Manar S. Ahmed[1], Ahmed H. Arnous[1] & Yakup Yildirim[2,3]

[1]Department of Physics and Engineering Mathematics, Higher Institute of Engineering, El–Shorouk Academy, Cairo, Egypt
[2]Department of Computer Engineering, Biruni University, Istanbul–34010, Turkey
[3]Mathematics Research Center, Near East University, 99138 Nicosia, Cyprus

Received: 17.11.2023

Abstract. The current study focuses on investigating the perturbed Gerdjikov-Ivanov equation, which includes the presence of multiplicative white noise in the Ito sense for the first time. In this study, we utilize two efficient methods for investigating the proposed model, aiming to generate bright, dark, and singular solitons. Also, the emergence and transformation of straddled solitons are available. Straddled solitons are characterized by their unique ability to convert into solitons. Moreover, we obtain Wieirstrass doubly periodic type solutions and exponential solutions. Two techniques are applied for this investigation: the enhanced Kudryashov method and the improved modified, extended tanh-function method. These techniques are utilized to address particular goals of the study, specifically those concerning solitons. These soliton solutions are used as valuable tools for investigating various phenomena in the presence of white noise. Our study addresses the impact of noise on various wave phenomena, particularly focusing on soliton solutions. Our results reveal that white noise primarily affects the phase component of solitons recovered from the governing model. Our paper presents, for the first time, the optical soliton solutions derived from the perturbed Gerdjikov-Ivanov equation under the influence of multiplicative white noise. Our study's novelty lies in applying this effect to a previously unexplored model equation and the unveiling of optical soliton solutions within this new framework. These results represent a significant advancement in understanding wave phenomena in nonlinear optical systems.

Keywords: Wiener process, Kudryashov method, stochastic Gerdjikov–Ivanov equation, noise intensity
UDC: 535.32
DOI: 10.3116/16091833/Ukr.J.Phys.Opt.2024.S1111

1. Introduction

The concept of optical solitons has garnered significant attention in the field of modern communications, mainly due to their potential applications in optical fiber communication. Optical solitons are regarded as an interesting and attractive phenomenon, holding great potential for various technological advancements in this field. In the area of soliton propagation, various mathematical models have been employed to investigate this phenomenon. Among these models, the nonlinear Schrödinger's equation (NLSE) is the most well-known and easily recognized [1-4]. Additionally, other models such as the complex Ginzburg-Landau equation [5-7], Sasa-Satsuma equation [8-10], Lakshmanan-Porsezian-Daniel model [11-15], and several others have also been utilized for this purpose. These models serve as valuable tools for studying the intricate dynamics associated with soliton propagation. The optical soliton perturbation is widely recognized as a crucial element in meeting the fundamental demands of the contemporary telecommunication industry and social media communications.

This study focuses on the analysis of a specific nonlinear evolution equation known as the Gerdjikov-Ivanov (GI) equation [16-19]. In recent years, there has been an increasing research focus on integrating multiplicative white noise into various nonlinear evolution equations. The objective of this study is to examine the influence of noise on the resultant wave phenomena [20-25]. A wide range of researchers have taken part in this field of research, going into the complexities and variations of the topic’s issue.

In this article, we introduce the generalized stochastic GI equation having multiplicative white noise and spatiotemporal dispersion for the first time in the following form:

$$\begin{aligned} & iq_t + aq_{xx} + bq_{xt} + c|q|^4 q + idq^2 q^*{}_x + \sigma(q - ibq_x)W_t(t) \\ & = i\left(\alpha q_x + \lambda\left(|q|^{2n} q\right)_x + \mu\left(|q|^{2n}\right)_x q\right). \end{aligned} \tag{1}$$

The function $q = q(x,t)$ is a complex-valued function representing the wave profile. The symbol i represents the imaginary unit $\sqrt{-1}$. The coefficient of a stands for group velocity dispersion, b is the coefficient of spatiotemporal dispersion, c represents the coefficient of quintic nonlinearity and d is the coefficient of nonlinear dispersion. In this equation, the parameter α represents inter-modal dispersion, while λ takes into account self-steepening effects for short pulses. The coefficient μ accounts for higher-order dispersion, and n is the full nonlinearity parameter. For a multiplicative white noise process, $W(t)$ is a random variable with a mean of 0 and a variance of σ^2, where σ is a constant that denotes the strength of the noise. $W_t(t)$ is a stochastic process where the value of $W_t(t)$ at a given time t is equal to $\Lambda(\eta)\, dW(\eta)$, where $\eta < t$. $\Lambda(\eta)$ is used to represent typical Gaussian white noise, which is commonly referred to as multiplicative white noise in the literature [26].

The subsequent sections of this work are organized as follows: In the second section, a wave transformation structure is employed, incorporating a white noise effect, to facilitate the transformation of the governing model into a nonlinear ordinary differential equation (ODE). In Section 3, a comprehensive account of the employed methodologies is provided. Section 4 of the study addresses the effect of white noise on the governing model in the analysis of solitons. Finally, the conclusion is presented in the final part.

2. Mathematical analysis

In order to solve Eq. (1), the following solution structure is selected:

$$q(x,t) = U(\xi)e^{i(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \phi(\xi))}, \tag{2}$$

where, the wave variable ξ is given by

$$\xi = k(x - vt). \tag{3}$$

Here, the real-valued function $U(\xi)$ represents the amplitude component of the soliton solution, and v is the speed of the soliton, where ω denotes the frequency of the solitons, k stands for wave width, κ corresponds to the wave number, σ represents the noise strength, and $\phi(\xi)$ is a real arbitrary function of ξ. By substituting Eq. (2) into Eq. (1) and decomposing it into its real and imaginary parts, we obtain the following result:

$$\begin{aligned} & k^2(a - bv)U''(\xi) + k^2 U(\xi)(bv - a)\phi'(\xi)^2 + kU(\xi)\phi'(\xi)(2a\kappa + \alpha + b\sigma^2 - b\kappa v - b\omega + v) \\ & + U(\xi)(-a\kappa^2 - \alpha\kappa - (b\kappa - 1)(\sigma^2 - \omega)) + cU(\xi)^5 + dkU(\xi)^3\phi'(\xi) - d\kappa U(\xi)^3 \\ & + U(\xi)^{2n+1}(k\lambda\phi'(\xi) - \kappa\lambda) = 0, \end{aligned} \tag{4}$$

and

$$\begin{aligned}&2k^2(a-bv)\phi'(\xi)U'(\xi)+k^2U(\xi)(a-bv)\phi''(\xi)\\&-kU'(\xi)\left(2a\kappa+\alpha+b\sigma^2-b\kappa v-b\omega+v\right)=0\end{aligned}=0, \tag{5}$$

Eq. (5) can be integrated after multiplying by U to arrive at

$$\phi'=\frac{\begin{pmatrix}2k(n+1)\left(2a\kappa+\alpha+b\sigma^2-b\kappa v-b\omega+v\right)\\-4c_0(n+1)U^{-2}-dk(n+1)U^2+2k(\lambda+2\lambda n+2\mu n)U^{2n}\end{pmatrix}}{4k^2(n+1)(a-bv)}, \tag{6}$$

where c_0 is the integration constant. Now, substituting Eq. (6) into Eq. (4) leads to

$$\begin{aligned}&G_4U(\xi)^{2n+2}+G_5U(\xi)^{2n+4}+G_6U(\xi)^{2n+6}+G_7U(\xi)^{4n+4}\\&+G_3U(\xi)^8+G_2U(\xi)^6+G_1U(\xi)^4+G_0+k^2U(\xi)^3U''(\xi)=0,\end{aligned} \tag{7}$$

where

$$\begin{cases}G_0=-\dfrac{c_0^2}{k^2(a-bv)^2},\\[2mm] G_1=\dfrac{k\begin{pmatrix}2v\left(2a\kappa+\alpha+\alpha b\kappa+b(b\kappa-1)(\sigma^2-\omega)\right)\\+4a(\sigma^2-\omega)+\left(\alpha+b(\sigma^2-\omega)\right)^2+v^2(b\kappa-1)^2\end{pmatrix}-6c_0d}{4k(a-bv)^2},\\[2mm] G_2=\dfrac{d\left(\alpha+b(\sigma^2-\omega)+b\kappa v+v\right)}{2(a-bv)^2},\ G_3=\dfrac{16ac-16bcv-5d^2}{16(a-bv)^2},\\[2mm] G_4=\dfrac{c_0n(\lambda+2\mu)}{k(n+1)(a-bv)^2},\ G_5=\dfrac{\lambda\left(\alpha+b(\sigma^2-\omega)+b\kappa v+v\right)}{2(a-bv)^2},\\[2mm] G_6=\dfrac{d\left(\lambda(5n+2)+6\mu n\right)}{4(n+1)(a-bv)^2},\ G_7=\dfrac{-4\lambda\mu n^2-4\mu^2n^2+\lambda^2(2n+1)}{4(n+1)^2(a-bv)^2},\end{cases} \tag{8}$$

provided that $k\neq0$ and $a\neq bv$.

3. A brief summary of integration techniques

Let us consider a governing model represented by the equation

$$G(u,u_x,u_t,u_{xt},u_{xx},\ldots)=0. \tag{9}$$

Here, $u=u(x,t)$ represents a wave profile, and t and x represent the time and space variables, respectively. The given relations

$$u(x,t)=U(\xi),\ \ \xi=k(x-\upsilon t), \tag{10}$$

simplify Eq. (9) to the following nonlinear ODE form:

$$P(U,-k\upsilon U',kU',k^2U'',\ldots)=0. \tag{11}$$

In the given context, the symbol k represents the wave width, ξ represents the new wave variable, and υ represents the wave velocity.

3.1. The enhanced Kudryashov scheme

In this subsection, we present a comprehensive explanation of the basic procedures associated with the enhanced Kudryashov technique. [27].

Step–1: In this research article, we present the explicit solution for the reduced model (11):

$$U(\xi)=\sigma_0+\sum_{i=1}^{N}\left\{\sigma_i R(\xi\ ^{i}+\rho_i\left(\frac{R'(\xi)}{R(\xi)}\right)^{i}\right\}, \tag{12}$$

along with the auxiliary equation

$$R'(\xi)^2=R(\xi)^2(1-\chi R(\xi)^2). \tag{13}$$

The constants σ_0, χ, σ_i, and ρ_i (where $i=1,...,N$) will be provided, with N determined by the balancing procedure in Eq. (11).

Step–2: Eq. (13) gives the soliton waves

$$R(\xi)=\frac{4f}{4f^2e^{\xi}+\chi e^{-\xi}}, \tag{14}$$

where f is constant.

Step–3: By inserting Eq. (12) into Eq. (11) together with Eq. (13), we can derive the requisite constants for Eqs. (10) and (12). In order to incorporate the identified parametric restrictions, they can be substituted into Eq. (14) together with Eq. (12). Consequently, straddled solitons are obtained, which can be further classified as bright, dark, or singular solitons.

3.2. The improved modified extended tanh-function

This subsection presents a thorough overview of the basic procedures with the improved extended tanh-function technique [28].

Step-1: We assume that the solution of Eq. (11) can be expressed in the form:

$$U(\xi)=\alpha_0+\sum_{i=1}^{N}\left\{\alpha_i\theta(\xi)^i+\beta_i\theta(\xi)^{-i}\right\}, \tag{15}$$

where θ satisfies

$$\theta'(\xi)^2=\sum_{l=0}^{4}\tau_l\theta(\xi)^l \tag{16}$$

The equation presented in this study provides a comprehensive range of fundamental solutions [28].

Case-1: $\tau_0=\tau_1=\tau_3=0$.

Two types of solutions are obtained:

$$\theta(\xi)=\sqrt{-\frac{\tau_2}{\tau_4}}\,\mathrm{sech}\left[\sqrt{\tau_2}\,\xi\right],\quad \tau_2>0,\ \tau_4<0, \tag{17}$$

and

$$\theta(\xi)=\sqrt{\frac{\tau_2}{\tau_4}}\,\mathrm{csch}\left[\sqrt{\tau_2}\,\xi\right],\quad \tau_2>0,\ \tau_4>0. \tag{18}$$

The found solutions consist of a bell-shaped solitary wave solution (17) and a singular soliton solution (18).

Case-2: $\tau_0=\frac{\tau_2{}^2}{4\tau_4},\ \tau_1=\tau_3=0.$

$$\theta(\xi)=\sqrt{-\frac{\tau_2}{2\tau_4}}\,\tanh\left[\sqrt{-\frac{\tau_2}{2}}\,\xi\right],\quad \tau_2\langle 0,\ \tau_4\rangle 0, \tag{19}$$

and

$$\theta(\xi)=\sqrt{-\frac{\tau_2}{2\tau_4}}\coth\left[\sqrt{-\frac{\tau_2}{2}}\xi\right],\quad \tau_2\langle 0,\ \tau_4\rangle 0. \tag{20}$$

The found solutions consist of a kink-shaped solitary wave (19) and a singular soliton solution (20).

Case-3: $\tau_2=\tau_4=0,\ \tau_0\neq 0,\ \tau_1\neq 0,\ \tau_3>0.$

A solution of the Weierstrass elliptic doubly periodic type is achieved:

$$\theta(\xi)=\wp\left(\frac{\sqrt{\tau_3}}{2}\xi, g_2, g_3\right), \tag{21}$$

where $\wp$ is the Weierstrass elliptic function, while $g_2=-(4\tau_1/\tau_3)$ and $g_3=-(4\tau_0/\tau_3)$ are called invariants of the Weierstrass elliptic function.

Case-4: $\tau_0=\frac{\tau_1^2}{4\tau_2},\ \tau_3=\tau_4=0.$

A solution of exponential structure is derived:

$$\theta(\xi)=-\frac{\tau_1}{2\tau_2}+\exp\left[\varepsilon\sqrt{\tau_2}\,\xi\right],\quad \tau_2>0. \tag{22}$$

Case-5: $\tau_1=\tau_3=\tau_4=0.$

$$\theta(\xi)=\pm\sqrt{\frac{\tau_0}{\tau_2}}\sinh\left[\sqrt{-\tau_2}\,\xi\right],\quad \tau_0>0,\ \tau_2>0. \tag{23}$$

Case-6: $\tau_0=\tau_1=0,\ \tau_2,\ \tau_4>0,\ \tau_3\neq\pm 2\sqrt{\tau_2\tau_4}$

Two solitary wave solutions are derived:

$$\theta(\xi)=\frac{-\tau_2\,\mathrm{sech}^2\left[\frac{1}{2}\sqrt{\tau_2}\,\xi\right]}{\pm 2\sqrt{\tau_2\tau_4}\tanh\left[\frac{1}{2}\sqrt{\tau_2}\,\xi\right]+\tau_3}, \tag{24}$$

and

$$\theta(\xi)=\frac{\tau_2\,\mathrm{csch}^2\left[\frac{1}{2}\sqrt{\tau_2}\,\xi\right]}{\pm 2\sqrt{\tau_2\tau_4}\coth\left[\frac{1}{2}\sqrt{\tau_2}\,\xi\right]+\tau_3}. \tag{25}$$

Step-2: The purpose is to determine the positive integer value of N in Eq. (15) through the process of equating the highest order derivatives and the nonlinear terms in Eq. (11).

Step-4: Insert Eq. (15) along with Eq. (16) into Eq. (11). By making this replacement, we have a polynomial in terms of θ. In the context of this polynomial, we combine terms with similar powers and set them equal to zero, resulting in an over-determined system of algebraic equations. This system can be solved using Mathematica software to discover the values of the unknown parameters, namely k, v, α_0, α_i and $\beta_i\ (i=1,2,\ldots)$. As a result, we are able to derive the exact solutions of Eq. (9).

4. Investigating stochastic optical solitons

This section will delve into the various solitons derived from a governing equation. It's worth noting that when n is arbitrary, it's possible to lose some solutions for the equation that includes all the coefficients [29,30]. That's why we'll set n to 1 or 2 to obtain different solitons. First, for n=1, we'll discuss bright, dark, singular, and straddled solitons. Then, for

n=2, we'll focus on bright and straddled solitons. By examining these different types of solitons, we can better understand how the equation behaves under various conditions.

4.1. Case-A: $\boldsymbol{n}=1$

In this case Eq. (7) reduces to

$$\begin{aligned}&(G_3+G_6+G_7)U(\xi)^8+(G_2+G_5)U(\xi)^6\\&+(G_1+G_4)U(\xi)^4+G_0+k^2U(\xi)^3U''(\xi)=0.\end{aligned}\tag{26}$$

As the balance leads to N=1/2, we use the transformation of the amplitude $U(\xi)$ by a new function $V(\xi)$:

$$U(\xi)=V(\xi)^{\frac{1}{2}},$$

Eq. (26) becomes

$$A_4V(\xi)^4+A_3V(\xi)^3+A_2V(\xi)^2+A_1-k^2\left(V'(\xi)^2-2V(\xi)V''(\xi)\right)=0,\tag{27}$$

where

$$A_4=4(G_3+G_6+G_7),\ A_3=4(G_2+G_5),\ A_2=4(G_1+G_4),\ A_1=4G_0.\tag{28}$$

Balancing $V(\xi)V''(\xi)$ with $V(\xi)^4$ in Eq. (27) gives $N=1$.

4.1.1. The enhanced Kudryashov procedure. The solution is formulated using the enhanced Kudryashov technique, and it is represented in the subsequent structure:

$$V(\xi)=\sigma_0+\sigma_1R(\xi)+\rho_1\left(\frac{R'(\xi)}{R(\xi)}\right).\tag{29}$$

By substituting Eq. (29) and Eq. (13) into Eq. (27), a system of algebraic equations is obtained. By solving these equations by Mathematica, the following solutions are obtained:

<u>Result-1</u>

$$\begin{aligned}&b_1=0,\quad a_0=-\frac{3A_3}{8A_4},\\&a_1=\pm\frac{\sqrt{6\left(9A_3^2-32A_2A_4\right)\chi}}{8A_4},\\&k=\frac{1}{4}\sqrt{\frac{9A_3^2-32A_2A_4}{2A_4}},\\&A_1=\frac{9A_3^2\left(15A_3^2-64A_2A_4\right)}{4096A_4^3}.\end{aligned}\tag{30}$$

The solution of Eq. (1) can be expressed as follows:

$$q(x,t)=\left\{-\frac{3A_3}{8A_4}\pm\frac{4f\sqrt{6\left(9A_3^2-32A_2A_4\right)\chi}}{8A_4\left(4f^2e^{\frac{1}{4}\sqrt{\frac{9A_3^2-32A_2A_4}{2A_4}}(x-vt)}+\chi e^{-\frac{1}{4}\sqrt{\frac{9A_3^2-32A_2A_4}{2A_4}}(x-vt)}\right)}\right\}^{\frac{1}{2}}\times e^{i\left(-\kappa x+\omega t+\sigma W(t)-\sigma^2t+\phi(\xi)\right)}.\tag{31}$$

Selecting $\chi=\pm4c^2$ respectively allows the recovery of bright and singular soliton solutions:

$$q(x,t)=\left\{-\frac{3A_3}{8A_4}\pm\frac{\sqrt{6\left(9A_3^2-32A_2A_4\right)}}{8A_4}\operatorname{sech}\left[\frac{1}{4}\sqrt{\frac{9A_3^2-32A_2A_4}{2A_4}}(x-vt)\right]\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}, \tag{32}$$

and

$$q(x,t)=\left\{-\frac{3A_3}{8A_4}\pm\frac{\sqrt{-6\left(9A_3^2-32A_2A_4\right)}}{8A_4}\operatorname{csch}\left[\frac{1}{4}\sqrt{\frac{9A_3^2-32A_2A_4}{2A_4}}(x-vt)\right]\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}. \tag{33}$$

In Figs. 1–3, we can see 3D plots, contour plots, and 2D plots of bright optical soliton solution in the presence of multiplicative white noise defined by Eq. (32). Figs. 4–6, on the other hand, illustrate how the dark optical soliton solution described by Eq. (36). The parameters have specific values: $c_0=1.8$, $n=1$, $k=1$, $\kappa=1$, $b=1.1$, $a=1.2$, $v=1.4$, $\omega=1.1$, $\alpha=1.2$, $d=1.3$, $\lambda=1.4$, $\mu=2.1$, $c=2.3$, $W(t)=t$, and $\phi(\xi)=1$.

Result-2

$$\begin{gathered} a_0=-\frac{3A_3}{8A_4}, a_1=0, b_1=\pm\frac{\sqrt{27A_3^2-96A_2A_4}}{8A_4}, \\ k=\frac{1}{8}\sqrt{\frac{32A_2A_4-9A_3^2}{A_4}}, A_1=-\frac{3\left(3A_3^2-16A_2A_4\right)^2}{1024A_4^3}. \end{gathered} \tag{34}$$

The solution of Eq. (1) can be expressed as follows:

$$q(x,t)=\left\{-\frac{3A_3}{8A_4}\pm\frac{\sqrt{27A_3^2-96A_2A_4}}{8A_4}\left(\frac{\chi-4c^2e^{\frac{1}{4}\sqrt{\frac{32A_2A_4-9A_3^2}{A_4}}(x-vt)}}{4c^2e^{\frac{1}{4}\sqrt{\frac{32A_2A_4-9A_3^2}{A_4}}(x-vt)}+\chi}\right)\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}. \tag{35}$$

By selecting $\chi=\pm 4c^2$, one can respectively recover dark and singular solitons in the context of the model:

$$q(x,t)=\left\{-\frac{3A_3}{8A_4}\pm\frac{\sqrt{27A_3^2-96A_2A_4}}{8A_4}\tanh\left[\frac{1}{8}\sqrt{\frac{32A_2A_4-9A_3^2}{A_4}}(x-vt)\right]\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}, \tag{36}$$

and

$$q(x,t)=\left\{-\frac{3A_3}{8A_4}\pm\frac{\sqrt{27A_3^2-96A_2A_4}}{8A_4}\coth\left[\frac{1}{8}\sqrt{\frac{32A_2A_4-9A_3^2}{A_4}}(x-vt)\right]\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}, \tag{37}$$

4.1.2. The improved extended tanh function technique. In accordance with the improved extended tanh function technique, the solution is expressed in the following structure:

$$V(\xi)=\alpha_0+\alpha_1\theta(\xi)+\frac{\beta_1}{\theta(\xi)}. \tag{38}$$

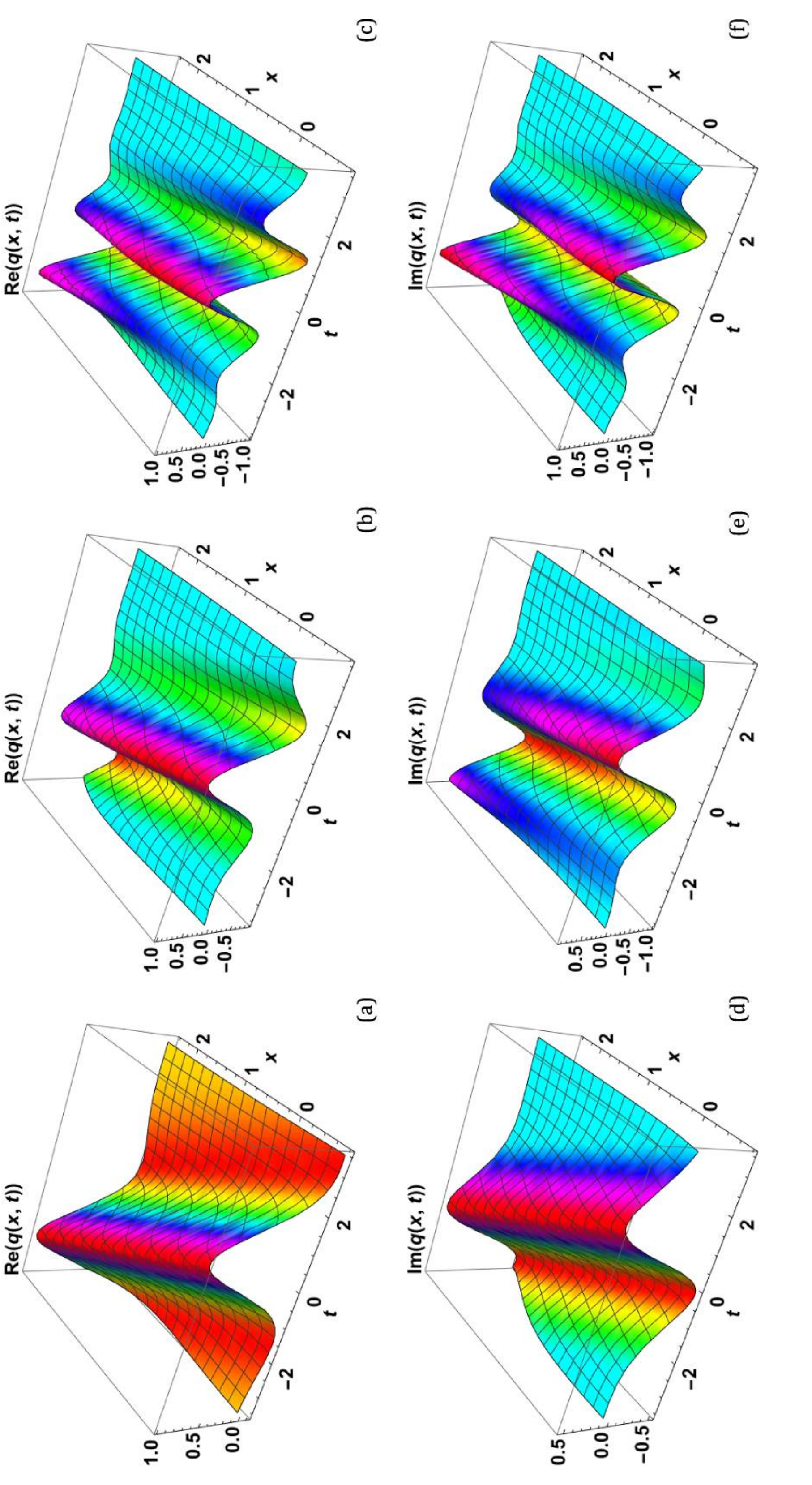

Fig. 1. 3D plot of a bright optical soliton solution in the presence of multiplicative white noise: (a) - $\sigma = 0$, (b) - $\sigma = 1.9$, (c) - $\sigma = 2.9$, (d) - $\sigma = 0$, (e) - $\sigma = 1.9$, (f) - $\sigma = 2.9$.

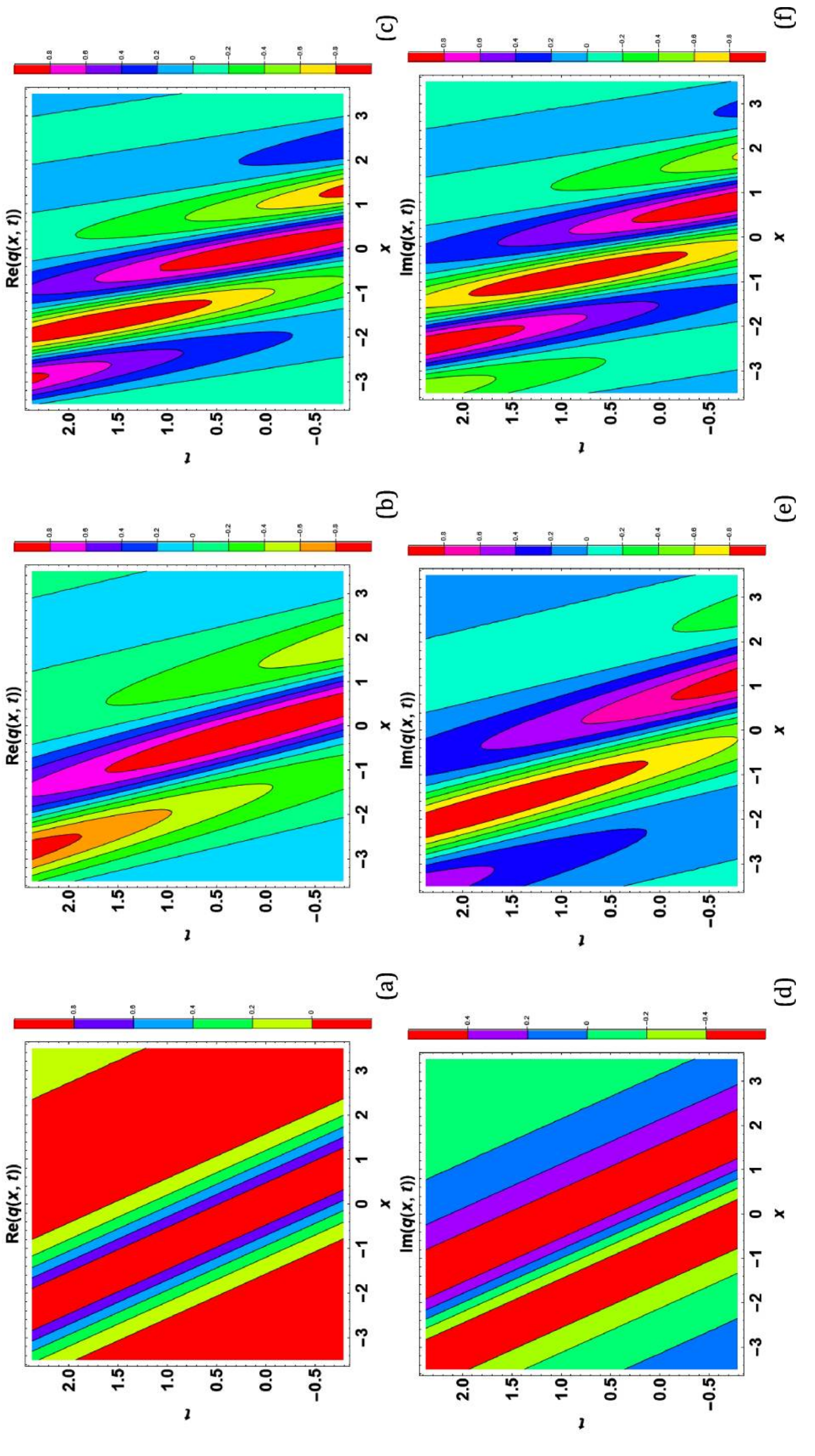

Fig. 2. Contour of a bright optical soliton solution in the presence of multiplicative white noise:

(a) - $\sigma = 0$, (b) - $\sigma = 1.9$, (c) - $\sigma = 2.9$, (d) - $\sigma = 0$, (e) - $\sigma = 1.9$, (f) - $\sigma = 2.9$.

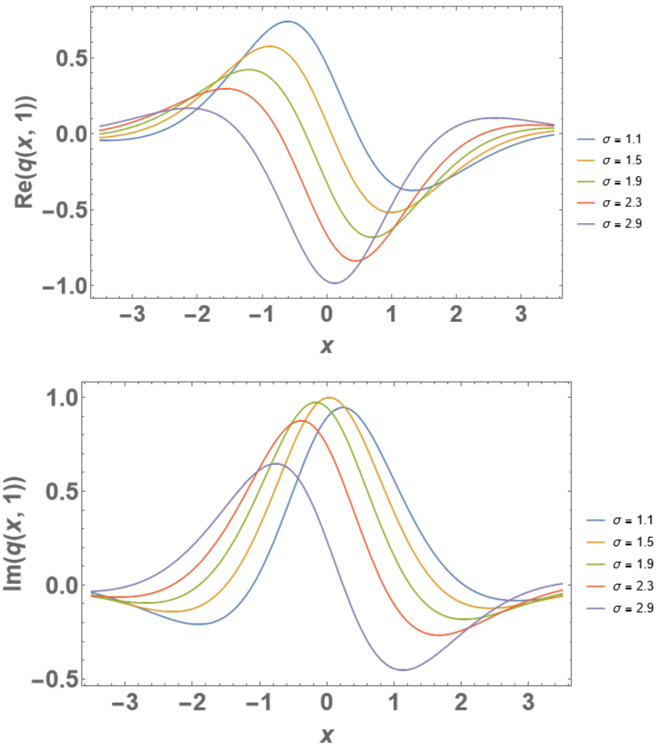

Fig. 3. 2D plot of a bright optical soliton solution in the presence of multiplicative white noise.

By substituting Eq. (38) and Eq. (16) into Eq. (27), a system of algebraic equations is obtained. By solving these equations by Mathematica, the following results are obtained:

Case-1: If we set $\tau_0 = \tau_1 = \tau_3 = 0$, we obtain

$$\alpha_0 = -\frac{3A_3}{8A_4}, \quad \beta_1 = 0, \quad A_1 = -\frac{9\left(64A_2A_3^2A_4 - 15A_3^4\right)}{4096A_4^3}, \\ \tau_2 = \frac{9A_3^2 - 32A_2A_4}{32A_4k^2}, \quad \tau_4 = -\frac{\alpha_1^2 A_4}{3k^2}. \tag{39}$$

One can respectively represent the bright and the singular soliton solutions of Eq. (1) through the following expression:

$$q(x,t) = \left\{-\frac{3A_3}{8A_4} + \sqrt{\frac{27A_3^2 - 96A_2A_4}{32A_4^2}}\operatorname{sech}\left[\sqrt{\frac{9A_3^2 - 32A_2A_4}{32A_4}}(x - vt)\right]\right\}^{\frac{1}{2}} \\ \times e^{i\left(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \phi(\xi)\right)}, \tag{40}$$

and

$$q(x,t) = \left\{-\frac{3A_3}{8A_4} + \sqrt{-\frac{27A_3^2 - 96A_2A_4}{32A_4^2}}\operatorname{csch}\left[\sqrt{\frac{9A_3^2 - 32A_2A_4}{32A_4}}(x - vt)\right]\right\}^{\frac{1}{2}} \\ \times e^{i\left(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \phi(\xi)\right)}. \tag{41}$$

Case-2: If we set $\tau_1 = \tau_3 = 0$ and $\tau_0 = \frac{\tau_2^2}{4\tau_4}$, we get

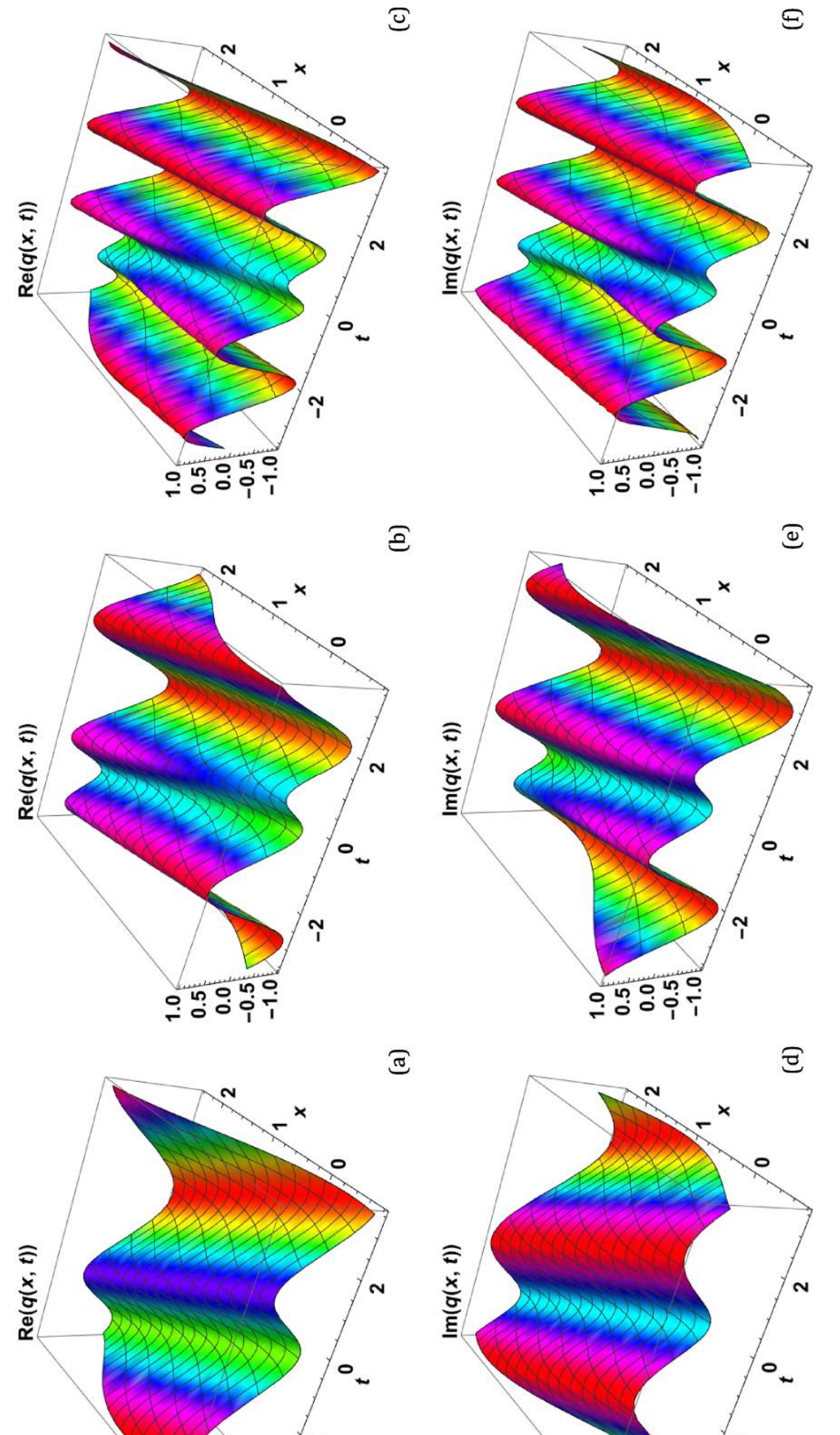

Fig. 4. 3D plot of a dark optical soliton solution in the presence of multiplicative white noise: (a) - $\sigma = 0$, (b) - $\sigma = 1.9$, (c) - $\sigma = 2.9$, (d) - $\sigma = 0$, (e) - $\sigma = 1.9$, (f) - $\sigma = 2.9$..

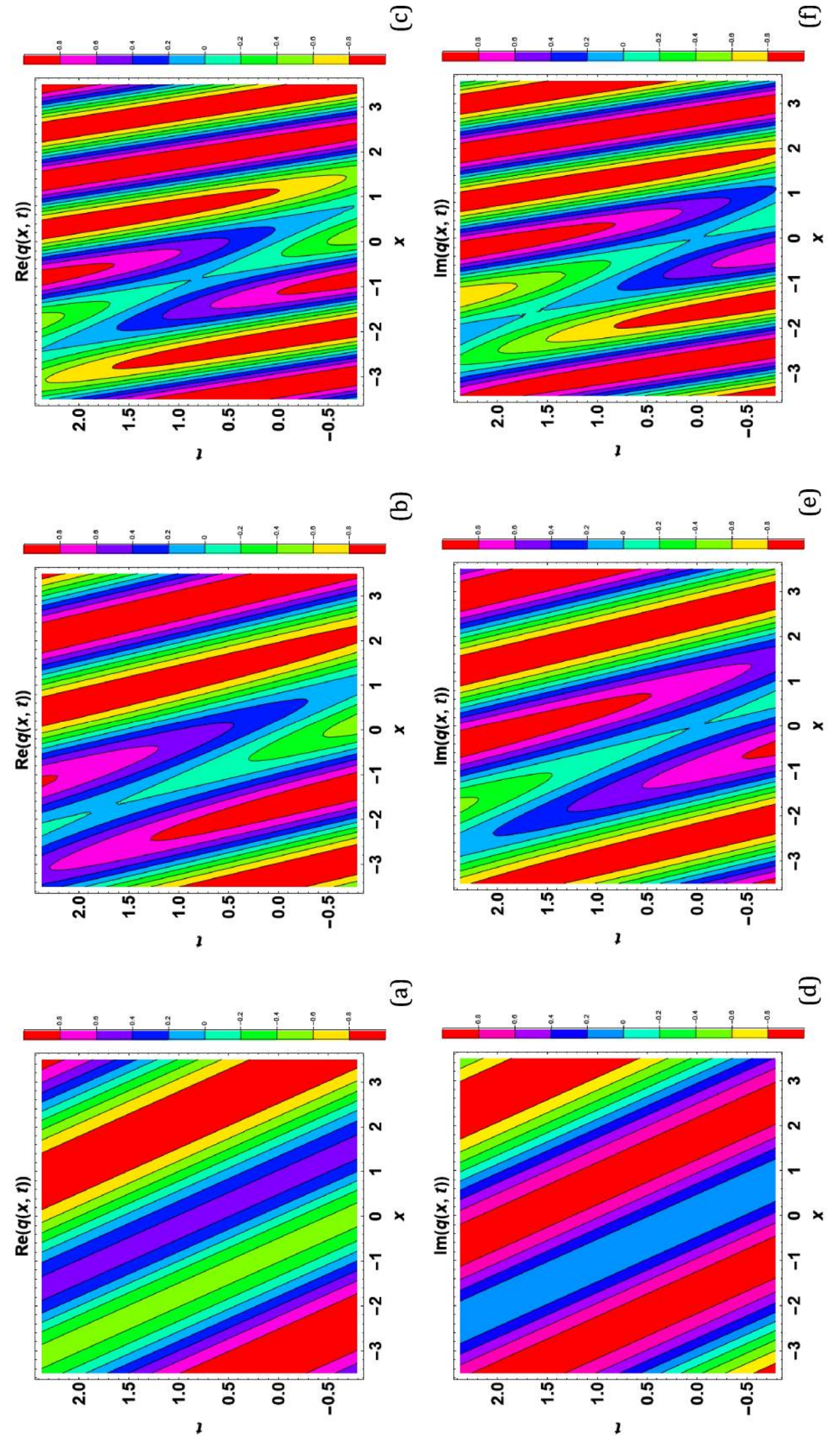

Fig. 5. Contour of a dark optical soliton solution in the presence of multiplicative white noise:
(a) - $\sigma = 0$, (b) - $\sigma = 1.9$, (c) - $\sigma = 2.9$, (d) - $\sigma = 0$, (e) - $\sigma = 1.9$, (f) - $\sigma = 2.9$.

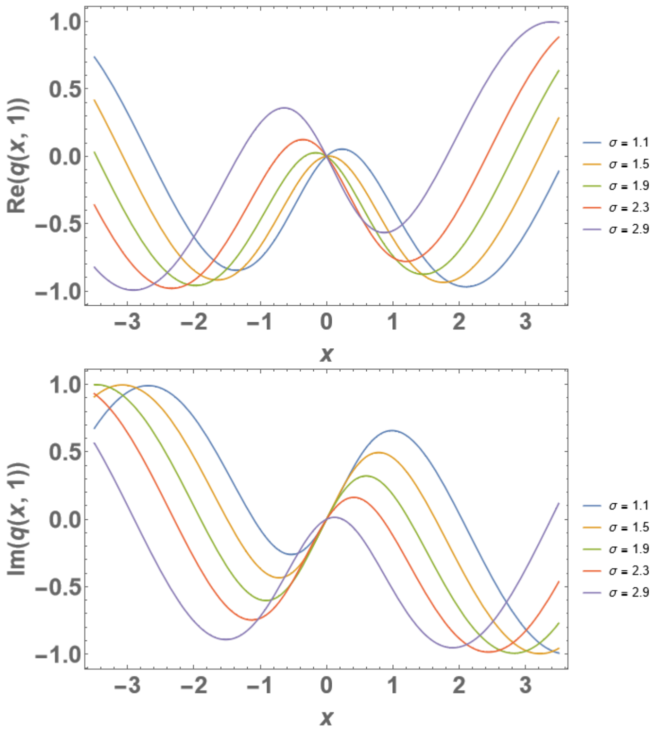

Fig. 6. 2D plot of a dark optical soliton solution in the presence of multiplicative white noise.

$$\alpha_0 = -\frac{3A_3}{8A_4}, \quad \beta_1 = 0, \quad A_1 = -\frac{3\left(3A_3^2 - 16A_2A_4\right)^2}{1024A_4^3},$$
$$\tau_2 = \frac{9A_3^2 - 32A_2A_4}{32A_4k^2}, \quad \tau_4 = -\frac{\alpha_1^2 A_4}{3k^2}. \tag{42}$$

The dark and singular soliton solutions to Eq. (1) can be articulated as follows, respectively:

$$q(x,t) = \left\{ -\frac{3A_3}{8A_4} + \sqrt{\frac{27A_3^2 - 96A_2A_4}{64A_4^2}} \tanh\left[\sqrt{\frac{32A_2A_4 - 9A_3^2}{64A_4}} (x - vt) \right] \right\}^{\frac{1}{2}} \times e^{i(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \phi(\xi))}, \tag{43}$$

and

$$q(x,t) = \left\{ -\frac{3A_3}{8A_4} + \sqrt{\frac{27A_3^2 - 96A_2A_4}{64A_4^2}} \coth\left[\sqrt{\frac{32A_2A_4 - 9A_3^2}{64A_4}} (x - vt) \right] \right\}^{\frac{1}{2}} \times e^{i(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \phi(\xi))}. \tag{44}$$

Case-3: If we set $\tau_2 = \tau_4 = 0$, we secure

$$\alpha_0 = \frac{-3A_3 \pm \sqrt{9A_3^2 - 32A_2A_4}}{8A_4}, \quad \alpha_1 = 0, \quad \beta_1 = \frac{\alpha_0^3(2\alpha_0A_4 + A_3) - 2A_1}{2\alpha_0k^2\tau_3},$$
$$\tau_0 = -\frac{A_4\left(\alpha_0^3(2\alpha_0A_4 + A_3) - 2A_1\right)^2}{12\alpha_0^2k^6\tau_3^2}, \quad \tau_1 = -\frac{(8\alpha_0A_4 + 3A_3)\left(\alpha_0^3(2\alpha_0A_4 + A_3) - 2A_1\right)}{12\alpha_0k^4\tau_3}. \tag{45}$$

Here is how the solution to Eq. (1) can be formulated:

$$q(x,t)=\left\{\frac{-3A_3\pm\sqrt{9A_3^2-32A_2A_4}}{8A_4}+\frac{\beta_1}{\wp\left(\frac{\sqrt{\tau_3}}{2}k(x-vt),g_2,g_3\right)}\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}, \tag{46}$$

where $g_2=\frac{4(8\alpha_0A_4+3A_3)\left(\alpha_0^3(2\alpha_0A_4+A_3)-2A_1\right)}{12\alpha_0k^4\tau_3^2}$ and $g_3=\frac{4A_4\left(\alpha_0^3(2\alpha_0A_4+A_3)-2A_1\right)^2}{12\alpha_0^2k^6\tau_3^3}$ are called invariants of the Weierstrass elliptic function.

Case-4: If we set $\tau_3=\tau_4=0$ and $\tau_0=\frac{\tau_1^2}{4\tau_2}$, we arrive at

$$\begin{gathered}\alpha_0=\frac{-3A_3A_4\mp\sqrt{27A_3^2A_4^2-96A_2A_4^3}}{8A_4^2},\quad \alpha_1=0,\quad A_1=-\frac{3\left(16A_2A_4-3A_3^2\right)^2}{1024A_4^3},\\ \tau_2=\frac{32A_2A_4-9A_3^2}{16A_4k^2},\tau_1=\pm\beta_1\sqrt{\frac{A_4^2\left(9A_3^2-32A_2A_4\right)}{12A_4^2k^4}}.\end{gathered} \tag{47}$$

To express the solution of Eq. (1), consider the following

$$q(x,t)=\left\{\frac{\frac{-3A_3A_4\mp\sqrt{27A_3^2A_4^2-96A_2A_4^3}}{8A_4^2}+\beta_1}{\mp\frac{2A_4\beta_1}{\sqrt{96A_2A_4-27A_3^2}}+\exp\left[\pm\sqrt{\frac{32A_2A_4-9A_3^2}{16A_4}}(x-vt)\right]}\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}. \tag{48}$$

Case-5: If we set $\tau_3=\tau_1=\tau_4=0$, we derive

$$\begin{gathered}\alpha_0=-\frac{3A_3}{8A_4},\quad \alpha_1=0,\quad A_1=\frac{9\left(15A_3^4-64A_2A_3^2A_4\right)}{4096A_4^3},\\ \tau_2=\frac{9A_3^2-32A_2A_4}{32A_4k^2},\quad \tau_0=-\frac{A_4\beta_1^2}{3k^2}.\end{gathered} \tag{49}$$

The singular soliton solution of Eq. (1) is expressed in the following manner:

$$q(x,t)=\left\{-\frac{3A_3}{8A_4}\pm\sqrt{\frac{96A_2A_4-27A_3^2}{32A_4^2}}\operatorname{csch}\left[\sqrt{\frac{32A_2A_4-9A_3^2}{32A_4}}(x-vt)\right]\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}. \tag{50}$$

Case-6: If we set $\tau_0=\tau_1=0$, we obtain

$$\begin{gathered}\alpha_0=-\frac{8A_2}{5A_3},\quad \alpha_1=-\frac{384A_2^3k^2\tau_3}{125A_1A_3^3},\quad \beta_1=0,\quad A_4=\frac{960A_2^3A_3^2-625A_1A_3^4}{4096A_2^4},\\ \tau_2=\frac{32A_2^3+125A_1A_3^2}{160A_2^2k^2},\quad \tau_4=\frac{12A_2^2\left(125A_1A_3^2-192A_2^3\right)k^2\tau_3^2}{3125A_1^2A_3^4}.\end{gathered} \tag{51}$$

The following expression encapsulates the straddled soliton solutions to Eq. (1):

$$q(x,t)=\left\{-\frac{8A_2}{5A_3}+\frac{12A_2\tau_3\left(32A_2^3+125A_1A_3^2\right)\operatorname{sech}^2\left(\sqrt{\dfrac{32A_2^3+125A_1A_3^2}{640A_2^2}}(x-vt)\right)}{625A_1A_3^3\left(\pm\sqrt{\tau_2\tau_4}\tanh\left(\sqrt{\dfrac{32A_2^3+125A_1A_3^2}{640A_2^2}}(x-vt)\right)+\tau_3\right)}\right\}^{\frac{1}{2}} \tag{52}$$
$$\times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}$$

and

$$q(x,t)=\left\{-\frac{8A_2}{5A_3}+\frac{12A_2\tau_3\left(32A_2^3+125A_1A_3^2\right)\operatorname{csch}^2\left(\sqrt{\dfrac{32A_2^3+125A_1A_3^2}{640A_2^2}}(x-vt)\right)}{625A_1A_3^3\left(\pm\sqrt{\tau_2\tau_4}\coth\left(\sqrt{\dfrac{32A_2^3+125A_1A_3^2}{640A_2^2}}(x-vt)\right)+\tau_3\right)}\right\}^{\frac{1}{2}} \tag{53}$$
$$\times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}.$$

4.2. Case-B: $n=2$

In this case Eq. (1) reduces to

$$\begin{aligned}&G_7U(\xi)^{12}+G_6U(\xi)^{10}+(G_3+G_5)U(\xi)^8+(G_2+G_4)U(\xi)^6\\&+G_1U(\xi)^4+G_0+k^2U(\xi)^3U''(\xi)=0.\end{aligned} \tag{54}$$

Using the transformation

$$U(\xi)=V(\xi)^{\frac{1}{4}},$$

we get

$$\begin{aligned}&16(G_2+G_4)V(\xi)^{\frac{5}{2}}+16G_6V(\xi)^{\frac{7}{2}}+16G_7V(\xi)^4\\&+16(G_3+G_5)V(\xi)^3+16G_1V(\xi)^2\\&+16G_0V(\xi)+4k^2V(\xi)V''(\xi)-3k^2V'(\xi)^2=0.\end{aligned} \tag{55}$$

For integrability, we set

$$G_2=-G_4,\ G_6=0.$$

Accordingly, Eq. (1) becomes

$$\begin{aligned}&B_4V(\xi)^4+B_3V(\xi)^3+B_2V(\xi)^2+B_1V(\xi)\\&+k^2\left(4V(\xi)V''(\xi)-3V'(\xi)^2\right)=0,\end{aligned} \tag{56}$$

where

$$B_4=16G_7,\ B_3=16(G_3+G_5),\ B_2=16G_1,\ B_1=16G_0. \tag{57}$$

Balancing $V(\xi)V''(\xi)$ with $V(\xi)^4$ in Eq. (56) gives $N=1$.

4.2.1. The enhanced Kudryashov procedure. The solution is formulated in the subsequent structure, in accordance with the enhanced Kudryashov technique:

$$V(\xi)=\sigma_0+\sigma_1R(\xi)+\rho_1\left(\frac{R'(\xi)}{R(\xi)}\right). \tag{58}$$

By substituting Eq. (28) and Eq. (15) into Eq. (10), a system of algebraic equations is obtained. By solving these equations by Mathematica, the following results are obtained:

$$a_0=-\frac{5B_3}{12B_4},\ a_1=\pm\frac{5B_3\sqrt{\chi}}{12B_4},\ b_1=0,$$
$$k=\frac{\sqrt{5}B_3}{12\sqrt{B_4}},\ B_1=-\frac{25B_3^3}{864B_4^2},\ B_2=\frac{25B_3^2}{144B_4}. \tag{59}$$

Presented below is the expression that signifies the solution to Eq. (1):

$$q(x,t)=\left\{-\frac{5B_3}{12B_4}\pm\frac{5B_3f\sqrt{\chi}}{3B_4\left(4f^2e^{\frac{(\sqrt{5}B_3)(x-vt)}{12\sqrt{B_4}}}+\chi e^{-\frac{(\sqrt{5}B_3)(x-vt)}{12\sqrt{B_4}}}\right)}\right\}^{\frac{1}{4}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}. \tag{60}$$

Choosing $\chi=4c^2$ facilitates the retrieval of bright soliton solution

$$q(x,t)=\left\{\frac{5B_3}{12B_4}\left(-1\pm\operatorname{sech}\left[\frac{(\sqrt{5}B_3)(x-vt)}{12\sqrt{B_4}}\right]\right)\right\}^{\frac{1}{4}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}. \tag{61}$$

4.2.2. The improved extended tanh function technique. The solution follows the structure outlined by the improved extended tanh function technique as expressed below:

$$V(\xi)=\alpha_0+\alpha_1\theta(\xi)+\frac{\beta_1}{\theta(\xi)}. \tag{62}$$

Inserting of the Eq. (62) together with Eq. (16) into Eq. (56), we get a system of algebraic equations. Solving these equations together yields the following results:

<u>Case-1</u>: If we set $\tau_0=\tau_1=\tau_3=0$, we obtain

$$\alpha_0=-\frac{12B_2}{5B_3},\quad \beta_1=0,\quad B_4=\frac{25B_3^2}{144B_2},$$
$$B_1=-\frac{24B_2^2}{25B_3},\quad \tau_2=\frac{B_2}{5k^2},\quad \tau_4=-\frac{5\alpha_1^2B_3^2}{144B_2k^2}. \tag{63}$$

The expression for the bright soliton solution to Eq. (1) is as follows:

$$q(x,t)=\left\{-\frac{12B_2}{5B_3}+\sqrt{\frac{144B_2^2}{25B_3^2}}\operatorname{sech}\left[\sqrt{\frac{B_2}{5}}(x-vt)\right]\right\}^{\frac{1}{4}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}. \tag{64}$$

<u>Case-2</u>: If we set $\tau_2=\tau_4=0$, we get

$$\alpha_0=\frac{-5B_2\pm\sqrt{25B_2^2+24B_1B_3}}{2B_3},\quad \alpha_1=0,$$
$$\beta_1=-\frac{\alpha_0(\alpha_0B_3+3B_2)}{6k^2\tau_3},\quad B_4=-\frac{5(\alpha_0B_3+B_2)}{6\alpha_0^2}, \tag{65}$$
$$\tau_0=\frac{(\alpha_0B_3+B_2)(\alpha_0B_3+3B_2)^2}{216k^6\tau_3^2},\quad \tau_1=-\frac{5\alpha_0B_3B_2+\alpha_0^2B_3^2+6B_2^2}{18k^4\tau_3}.$$

One can represent the solution of Eq. (1) through the following expression:

$$q(x,t)=\left\{\frac{-5B_2\pm\sqrt{25B_2^2+24B_1B_3}}{2B_3}+\frac{\beta_1}{\wp\left(\frac{\sqrt{\tau_3}}{2}k(x-vt),g_2,g_3\right)}\right\}^{\frac{1}{4}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}, \tag{66}$$

where $g_2=\frac{4\left(5\alpha_0B_3B_2+\alpha_0^2B_3^2+6B_2^2\right)}{18k^4\tau_3^2}$ and $g_3=-\frac{4(\alpha_0B_3+B_2)(\alpha_0B_3+3B_2)^2}{216k^6\tau_3^3}$ are called invariants of the Weierstrass elliptic function.

Case-3: If we set $\tau_0=\tau_1=0$, we secure

$$\begin{aligned}
&\alpha_0=\frac{3\left(-B_2\pm\sqrt{B_2^2+B_1B_3}\right)}{B_3},\quad \alpha_1=\frac{9\alpha_0k^2\tau_3}{5\alpha_0B_3+12B_2},\\
&\beta_1=0,\quad B_4=-\frac{5(2\alpha_0B_3+3B_2)}{9\alpha_0^2},\\
&\tau_2=\frac{\alpha_0B_3+3B_2}{3k^2},\quad \tau_4=\frac{9k^2\tau_3^2(2\alpha_0B_3+3B_2)}{(5\alpha_0B_3+12B_2)^2}.
\end{aligned} \tag{67}$$

Represented below is the manner in which the straddled soliton solutions to Eq. (1) are expressed:

$$q(x,t)=\left\{\frac{3\left(-B_2\pm\sqrt{B_2^2+B_1B_3}\right)}{B_3}+\frac{9\alpha_0k^2\tau_3}{5\alpha_0B_3+12B_2}\left(\frac{-\tau_2\operatorname{sech}^2\left[\frac{\sqrt{\tau_2}}{2}k(x-vt)\right]}{\pm2\sqrt{\tau_2\tau_4}\tanh\left[\frac{\sqrt{\tau_2}}{2}k(x-vt)\right]+\tau_3}\right)\right\}^{\frac{1}{4}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))} \tag{68}$$

and

$$q(x,t)=\left\{\frac{3\left(-B_2\pm\sqrt{B_2^2+B_1B_3}\right)}{B_3}+\frac{9\alpha_0k^2\tau_3}{5\alpha_0B_3+12B_2}\left(\frac{\tau_2\operatorname{csch}^2\left[\frac{\sqrt{\tau_2}}{2}k(x-vt)\right]}{\pm2\sqrt{\tau_2\tau_4}\coth\left[\frac{\sqrt{\tau_2}}{2}k(x-vt)\right]+\tau_3}\right)\right\}^{\frac{1}{4}} \times e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\phi(\xi))}. \tag{69}$$

5. Conclusion

This paper focused on investigating the perturbed GI equation, specifically considering the incorporation of multiplicative white noise in the Ito sense. We have employed two highly effective methodologies to investigate the proposed model, with the aim of obtaining bright, dark, and singular solitons. The current study has demonstrated straddled solitons' availability and dynamic nature, showcasing their emergence and transformation. These findings contribute to the growing body of research on solitons and offer valuable insights into their behavior and potential applications. Overall, this investigation highlights the significance of employing a couple of techniques to understand complex models affected by white noise comprehensively and paves the way for further research in this area. Exploring the effects of noise on a range of wave phenomena, our study concentrated specifically on soliton solutions. Our analysis demonstrates that white noise predominantly impacts the phase component of solitons as recovered from the governing model.

Our study emphasizes the distinctiveness of our investigation into the impact of multiplicative white noise on soliton solutions within the context of the generalized GI equation. While prior research has explored the effects of noise on wave phenomena, our study delves into previously unexplored territory by applying multiplicative white noise to a novel model equation. Our paper aimed to examine the effects of noise on soliton solutions within the framework of the generalized GI equation and to uncover unique characteristics and behaviors of this equation under the influence of multiplicative white noise. By achieving this goal, our study offers a pioneering examination of the effects of noise on soliton solutions within this specific framework, contributing to a deeper understanding of wave phenomena in nonlinear optical systems. As a result, our study's novelty lies in applying multiplicative white noise to a previously unexplored model equation, the generalized GI equation, and the unveiling of optical soliton solutions within this new framework.

White noise is a type of noise that has a flat power spectrum and is characterized by random fluctuations over a wide range of frequencies. When solitons, which are self-sustaining waves that maintain their shape as they propagate, are subjected to white noise, their evolution can be affected in various ways. Some studies have shown that white noise can cause solitons to broaden or change their frequency, while others have shown that it can enhance their stability and prevent decay. The exact influence of white noise on soliton evolution depends on the specific characteristics of the soliton and the noise and is still an active area of research [20-25]. Further investigations are warranted to explore the full extent of solitons with white noise and their implications in various scientific and technological domains. These findings contribute to the existing body of knowledge and offer potential applications in various research areas and practical implementation in nonlinear optics [31–45]. Further exploration and experimentation are warranted to fully understand these stochastic solitons' implications and potential.

References

1. Biswas, A. (2009). Topological 1-soliton solution of the nonlinear Schrodinger's equation with Kerr law nonlinearity in 1+ 2 dimensions. *Communications in Nonlinear Science and Numerical Simulation*, *14*(7), 2845-2847.
2. Ricardo, W. (1997). Inverse scattering for the nonlinear schrodinger equation. *Communications in Partial Differential Equations*, *22*(11-12), 2089-2103.
3. Zayed, E. M. E., & Zedan, H. A. (2003). On the solution of the nonlinear Schrodinger equation. *Chaos, Solitons & Fractals*, *16*(1), 133-145.
4. Malomed, B. A., & Stenflo, L. (1991). Modulational instabilities and soliton solutions of a generalized nonlinear Schrodinger equation. *Journal of Physics A: Mathematical and General*, *24*(19), L1149.
5. Kudryashov, N. A. (2023). Conservation laws of the complex Ginzburg-Landau equation. *Physics Letters A*, *481*, 128994.

6. Arnous, A. H., Seadawy, A. R., Alqahtani, R. T., & Biswas, A. (2017). Optical solitons with complex Ginzburg–Landau equation by modified simple equation method. *Optik*, *144*, 475-480.
7. Kudryashov, N. A. (2022). Exact solutions of the complex Ginzburg–Landau equation with law of four powers of nonlinearity. *Optik*, *265*, 169548.
8. Kudryashov, N. A. (2022). Exact solutions of the complex Ginzburg–Landau equation with law of four powers of nonlinearity. *Optik*, *265*, 169548.
9. Wazwaz, A. M., & Mehanna, M. (2021). Higher-order Sasa–Satsuma equation: Bright and dark optical solitons. *Optik*, *243*, 167421.
10. Kudryashov, N. A. (2021). Solitary waves of the generalized Sasa–Satsuma equation with arbitrary refractive index. *Optik*, *232*, 166540.
11. Alqahtani, R. T., Babatin, M. M., & Biswas, A. (2018). Bright optical solitons for Lakshmanan-Porsezian-Daniel model by semi-inverse variational principle. *Optik*, *154*, 109-114.
12. Biswas, A., Yıldırım, Y., Yaşar, E., & Alqahtani, R. T. (2018). Optical solitons for Lakshmanan–Porsezian–Daniel model with dual-dispersion by trial equation method. *Optik*, *168*, 432-439.
13. Biswas, A., Yildirim, Y., Yasar, E., Zhou, Q., Moshokoa, S. P., & Belic, M. (2018). Optical solitons for Lakshmanan–Porsezian–Daniel model by modified simple equation method. *Optik*, *160*, 24-32.
14. Biswas, A., Ekici, M., Sonmezoglu, A., Triki, H., Majid, F. B., Zhou, Q., Moshokoa, S.P., Mirzazadeh, M. & Belic, M. (2018). Optical solitons with Lakshmanan–Porsezian–Daniel model using a couple of integration schemes. *Optik*, *158*, 705-711.
15. El-Sheikh, M. M. A., Ahmed, H. M., Arnous, A. H., Rabie, W. B., Biswas, A., Alshomrani, A. S., Ekici, M., Zhou, Q. & Belic, M. R. (2019). Optical solitons in birefringent fibers with Lakshmanan–Porsezian–Daniel model by modified simple equation. *Optik*, *192*, 162899.
16. Arshed, S. (2018). Two reliable techniques for the soliton solutions of perturbed Gerdjikov–Ivanov equation. *Optik*, *164*, 93-99.
17. Kaur, L., & Wazwaz, A. M. (2018). Optical solitons for perturbed Gerdjikov–Ivanov equation. *Optik*, *174*, 447-451.
18. Biswas, A., & Alqahtani, R. T. (2017). Chirp-free bright optical solitons for perturbed Gerdjikov–Ivanov equation by semi-inverse variational principle. *Optik*, *147*, 72-76.
19. Onder, I., Secer, A., Ozisik, M., & Bayram, M. (2023). Investigation of optical soliton solutions for the perturbed Gerdjikov-Ivanov equation with full-nonlinearity. *Heliyon*, *9*(2).
20. Zayed, E. M., Arnous, A. H., Biswas, A., Yıldırım, Y., & Asiri, A. (2023). Optical solitons for the concatenation model with multiplicative white noise. *Journal of Optics*, 1-10.
21. Secer, A. (2022). Stochastic optical solitons with multiplicative white noise via Itô calculus. *Optik*, *268*, 169831.
22. Li, Z., & Peng, C. (2023). Dynamics and embedded solitons of stochastic quadratic and cubic nonlinear susceptibilities with multiplicative white noise in the Itô sense. *Mathematics*, *11*(14), 3185.
23. Cakicioglu, H., Ozisik, M., Secer, A., & Bayram, M. (2023). Stochastic dispersive Schrödinger–Hirota equation having parabolic law nonlinearity with multiplicative white noise via Ito calculus. *Optik*, *279*, 170776.
24. Zayed, E. M., Shohib, R. M., & Alngar, M. E. (2022). Dispersive optical solitons in magneto-optic waveguides with stochastic generalized Schrôdinger–Hirota equation having multiplicative white noise. *Optik*, *271*, 170069.
25. Zayed, E. M., Gepreel, K. A., & El-Horbaty, M. (2023). Highly dispersive optical solitons in fiber Bragg gratings with stochastic perturbed Fokas–Lenells model having spatio-temporal dispersion and multiplicative white noise. *Optik*, *286*, 170975.
26. Itô, K. (1944). 109. stochastic integral. *Proceedings of the Imperial Academy*, *20*(8), 519-524.
27. Arnous, A. H. (2021). Optical solitons with Biswas–Milovic equation in magneto-optic waveguide having Kudryashov's law of refractive index. *Optik*, *247*, 167987.
28. Yang, Z., & Hon, B. Y. (2006). An improved modified extended tanh-function method. *Zeitschrift für Naturforschung A*, *61*(3-4), 103-115.
29. Kudryashov, N. A. (2020). Traveling wave solutions of the generalized Gerdjikov–Ivanov equation. *Optik*, *219*, 165193.
30. Farahat, S. E., Shazly, E. E., El-Kalla, I. L., & Kader, A. A. (2023). Bright, dark and kink exact soliton solutions for perturbed Gerdjikov–Ivanov equation with full nonlinearity. *Optik*, *277*, 170688.
31. Wang, S. (2023). Novel soliton solutions of CNLSEs with Hirota bilinear method. *Journal of Optics*, 1-6.
32. Kopçasız, B., & Yaşar, E. (2023). The investigation of unique optical soliton solutions for dual-mode nonlinear Schrödinger's equation with new mechanisms. *Journal of Optics*, *52*(3), 1513-1527.
33. Tang, L. (2023). Bifurcations and optical solitons for the coupled nonlinear Schrödinger equation in optical fiber Bragg gratings. *Journal of Optics*, *52*(3), 1388-1398.

34. Thi, T. N., & Van, L. C. (2023). Supercontinuum generation based on suspended core fiber infiltrated with butanol. *Journal of Optics*, *52*(4), 2296-2305.
35. Li, Z., & Zhu, E. (2023). Optical soliton solutions of stochastic Schrödinger–Hirota equation in birefringent fibers with spatiotemporal dispersion and parabolic law nonlinearity. *Journal of Optics*, 1-7.
36. Han, T., Li, Z., Li, C., & Zhao, L. (2023). Bifurcations, stationary optical solitons and exact solutions for complex Ginzburg–Landau equation with nonlinear chromatic dispersion in non-Kerr law media. *Journal of Optics*, *52*(2), 831-844.
37. Tang, L. (2023). Phase portraits and multiple optical solitons perturbation in optical fibers with the nonlinear Fokas–Lenells equation. *Journal of Optics*, 1-10.
38. Nandy, S., & Lakshminarayanan, V. (2015). Adomian decomposition of scalar and coupled nonlinear Schrödinger equations and dark and bright solitary wave solutions. *Journal of Optics*, *44*, 397-404.
39. Chen, W., Shen, M., Kong, Q., & Wang, Q. (2015). The interaction of dark solitons with competing nonlocal cubic nonlinearities. *Journal of Optics*, *44*, 271-280.
40. Xu, S. L., Petrović, N., & Belić, M. R. (2015). Two-dimensional dark solitons in diffusive nonlocal nonlinear media. *Journal of Optics*, *44*, 172-177.
41. Dowluru, R. K., & Bhima, P. R. (2011). Influences of third-order dispersion on linear birefringent optical soliton transmission systems. *Journal of Optics*, *40*, 132-142.
42. Singh, M., Sharma, A. K., & Kaler, R. S. (2011). Investigations on optical timing jitter in dispersion managed higher order soliton system. *Journal of Optics*, *40*, 1-7.
43. Janyani, V. (2008). Formation and Propagation-Dynamics of Primary and Secondary Soliton-Like Pulses in Bulk Nonlinear Media. *Journal of Optics*, *37*, 1-8.
44. Hasegawa, A. (2004). Application of Optical Solitons for Information Transfer in Fibers—A Tutorial Review. *Journal of Optics*, *33*(3), 145-156.
45. Mahalingam, A., Uthayakumar, A., & Anandhi, P. (2013). Dispersion and nonlinearity managed multisoliton propagation in an erbium doped inhomogeneous fiber with gain/loss. *Journal of Optics*, *42*, 182-188.

Manar S. Ahmed, Ahmed H. Arnous & Yakup Yıldırım. (2024). Optical Solitons of the Generalized Stochastic Gerdjikov-Ivanov Equation in the Presence of Multiplicative White Noise. *Ukrainian Journal of Physical Optics*, *25*(5), S1111 – S1130.
doi: 10.3116/16091833/Ukr.J.Phys.Opt.2024.S1111

Анотація. *Це дослідження спрямоване на вивчення збуреного рівняння Герджикова-Іванова, яке включає мультиплікативний білий шум в сенсі Іто. Ми використовуємо два ефективних методи для дослідження запропонованої моделі з метою генерації світлих, темних та сингулярних солітонів. Крім того, виявлено виникнення та трансформація розмежованих солітонів Розмежовані солітони характеризуються своєю унікальною здатністю перетворюватися у солітони. Крім того, ми отримали розв'язки подвійно періодичного типу Вейерштрасса та експоненціальні розв'язки. Для цього дослідження застосовуються дві методики: покращений метод Кудряшова та покращений модифікований метод розширеної функції гіперболічного тангенсу. Ці методи використовуються для досягнення конкретних цілей дослідження, зокрема тих, що стосуються солітонів. Ці солітонні розв'язки використовуються як цінні інструменти для вивчення різних явищ за наявності білого шуму. Наше дослідження стосується впливу шуму на різні хвильові явища, особливо зосереджуючись на солітонних рішеннях. Результати демонструють, що білий шум, в першу чергу, впливає на фазову складову солітонів, отриманих з моделі. У нашій роботі вперше представлені оптичні солітонні розв'язки, отримані зі збуреного рівняння Герджикова-Іванова під впливом мультиплікативного білого шуму. Новизна нашого дослідження полягає в застосуванні цього ефекту до раніше не дослідженого рівняння моделі та розкритті рішень оптичних солітонів у цій новій структурі. Ці результати є значним прогресом у розумінні хвильових явищ у нелінійних оптичних системах.*

Ключові слова: *процес Вінера, метод Кудряшова, стохастичне рівняння Герджикова–Іванова, інтенсивність шуму*