Ukrainian Journal of Physical Optics
ISSUE 4
2024

Polarization of the Diffracted Optical Wave in the Crystals with Circular Optical Birefringence and Efficiency of Acousto-Optic Interaction Associated with Circularly Polarized Optical Waves in the Optically Active Crystals of Cubic System

O. Mys, M. Kostyrko, R. Vlokh

O.G. Vlokh Institute of Physical Optics of the Ivan Franko National University of Lviv,
23 Dragomanov Str., 79005, Lviv, Ukraine, e-mail: vlokh@ ifo.lviv.ua

Received: 22.08.2024

Abstract. The influence of the optical eigenwaves' polarization in the acousto-optic (AO) interaction in the optically active cubic crystals is analyzed in the present work on the example of cubic, optically active $NaClO_3$ crystals. It has been shown that when the incident optical wave is circularly polarized, the effective elasto-optic (EO) coefficient and AO figure of merit increase almost two times while the surfaces of these parameters undergo inflation compared with those manifested in the interaction of the linearly polarized optical waves. It has been found that six isotropic types of AO interaction are reduced to three types, i.e., the sign of the circular polarization does not affect the relation for the effective EO coefficient. Moreover, in this case, the relations for effective EO coefficients for the anisotropic AO interactions are the same as for isotropic types with respective AWs. We have shown that two approaches we recently developed, based on considering the ellipticity of optical eigenwaves and extraction of the circular polarization from the elliptical polarization state, agree when the diffracted optical wave's polarization is not considered. It has been found that in the presence of circular birefringence in the case when the polarization of the incident optical wave corresponds to the polarization of the eigenwave, the polarization of the diffracted optical wave is not necessarily the same as the polarization of the eigenwave, which corresponds to the phase-matching conditions. In the last case, the polarization of the diffracted wave is determined by the ratio between EO tensor components, which are included in the effective EO coefficient. In general, this polarization is elliptical and not the same as the polarization of the eigenwave.

Keywords: acousto-optic diffraction, acousto-optic figure of merit, effective elasto-optic coefficient, optical activity, circular polarization, polarization of the diffracted optical wave
UDC: 535.4, 534.2
DOI: 10.3116/16091833/Ukr.J.Phys.Opt.2024.04051

1. Introduction

The acousto-optic (AO) figure of merit is one of the most important parameters describing the efficiency of the AO Bragg diffraction. The efficiency of AO diffraction ($\eta = I_d / I_0$), i.e. the ratio of the intensity of diffracted wave (I_d) to the intensity of the incident wave (I_0) in the case of $\eta \ll 1$ is determined by the relation:

$$\eta = M_2 \frac{\pi^2 L}{2\lambda_0^2 H \cos^2\theta_B} P_{ac}\,, \tag{1}$$

where L – is the interaction length, H – is the height of the acoustic beam, θ_B – is the Bragg angle, λ_0 – is the wavelength of optical radiation in the vacuum and P_{ac} – is the acoustic wave (AW) power [1]. As it is seen from Eq. (1) the efficiency of AO diffraction is proportional to the AW power where the coefficient of proportionality is the AO figure of merit, which is determined as:

$$M_2=\frac{n_i^3 n_d^3 p_{eff}^2}{\rho v^3}, \quad (2)$$

where n_i and n_d are the refractive indices of the incident and diffracted optical waves, p_{eff} – is the effective elasto-optic (EO) coefficient, v is the velocity of the AW, and ρ – is the material density.

In our recent works [2-5], it has been shown that accounting of the ellipticity of optical eigenwaves caused by optical activity leads to a peak-like increase of effective EO coefficient and AO figure of merit whenever the ellipticity of eigenwaves approach unity. In the anisotropic crystals, such an increase is realized when the incident and diffracted optical waves propagate in the direction close to the optical axis. The incident optical wave should possess the same ellipticity as the eigenwave to realize such an AO interaction. In this case, new terms appear in the effective EO coefficient relation that are proportional to the ellipticity of incident optical wave. In the present work, we will analyze the AO interaction in the optically active crystals that belong to the cubic system, i.e., in crystals in which the linear birefringence is equal to zero and the ellipticity of eigenwaves is equal to ±1 in all directions of propagation.

The second task of the present work is as follows. In different works, we have used two approaches to analyze the effect of the ellipticity of eigenwaves on the efficiency of AO interaction. The first approach is based on accounting for the ellipticity of optical eigenwaves in the effective EO coefficient relations [2-5]. The second one considers the interaction of circular optical waves with acoustic waves (AW) at the wavelength of isotropic point [6], or at the propagation of optical waves close to the optical axis [7], i.e., at the conditions when the linear birefringence approaches zero. This approach is based on extracting the circularly polarized optical wave from the wave with elliptical polarization. Therefore, the second goal of the present work is to compare and match these two approaches. The last goal aims to answer the question: what is the polarization of the diffracted optical wave in the crystals with circular optical birefringence whenever the incident optical wave is of the same polarization as one of the optical eigenwaves?

2. Methods of analysis

Our analysis has been carried out on the example of $NaClO_3$ crystals, which belong to the space group of symmetry $P2_13$ (point group - 23) [8]. The crystals are optically active with a specific optical rotation equal to 2.3 deg/mm [9]. The refractive index for the wavelength 632.8 nm equals n = 1.514 [10]. The elastic modules at the normal conditions are C_{11}=4.85, C_{12}= 1.38, and C_{44}=1.17×10^{10}N/m^2, and the material density is ρ=2.49×10^{3}kg/m^3 [11]. The elasto-optic coefficients determined for the wavelength of Na yellow light are equal to p_{11}=0.163, p_{12}=0.24, p_{13}=0.2, p_{44}=−0.198 [12]. In the present work, we neglect the dispersion since the crystals are transparent in the visible spectral range (the absorption edge corresponds to 237.8 nm [13]).

2.1. Acoustic properties of $NaClO_3$ crystals

Since we will consider AO interaction in the *XZ* plane, let us consider the acoustic properties of $NaClO_3$ crystals in the same plane. The AW velocities can be determined by the equations derived based on Christoffel equation:

$$\upsilon_{QL} = \frac{C_{44}+C_{11}}{2} + \sqrt{\frac{\cos^2(2\theta)(C_{11}-C_{44})^2+\sin^2(2\theta)(C_{12}+C_{44})^2}{\rho}}, \quad (3)$$

for QL AW

$$\upsilon_{QT} = \frac{C_{44}+C_{11}}{2} - \sqrt{\frac{\cos^2(2\theta)(C_{11}-C_{44})^2+\sin^2(2\theta)(C_{12}+C_{44})^2}{\rho}}, \quad (4)$$

for QT AW polarized in *XZ* plane and

$$\upsilon_{PT} = \sqrt{\frac{C_{44}}{\rho}}. \quad (5)$$

for PT AW polarized parallel to the *Y* axis, where θ is the angle between the *X*-axis and acoustic wavevector. The angle of non-orthogonality for QT AW (or the angle of deviation from pure longitudinal state for QL AW) can be determined by the equation:

$$\alpha = 0.5\arctan\left\{\frac{\sin(2\theta)(C_{12}+C_{44})}{\cos(2\theta)(C_{11}-C_{44})}\right\}, \quad (6)$$

where $\alpha = \xi - \theta$ and ξ is the angle between *X*-axis and the displacement vector.

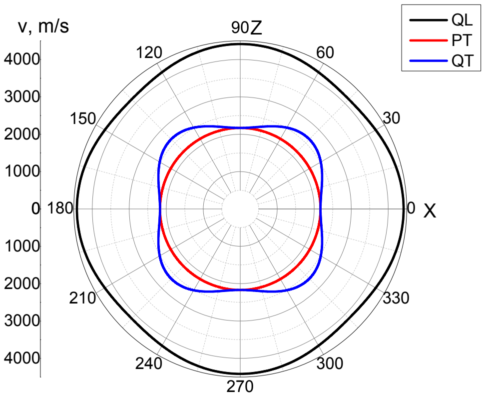


Fig. 1. Dependencies of AW velocities on the angle θ in the *XZ* plane.

It is seen (Fig. 1) that PT AW behaves isotropically with the change of angle θ and its velocity is equal to υ_{PT} =2167.67 m/s. The QT AW changes its value from the 2167.67 m/s in the case of its propagation along the principal axes to 2639.67 m/s in the case of its propagation along the bisector's directions between the principal axes. The anisotropy of velocity of the QL AW is about 265 m/s. The angle of non-orthogonality is equal to zero when the AWs are propagated along principal axes and the bisectors between them and reach its maximal value of $|5.225|$ deg at θ=65, 115 and 155 deg (Fig. 2).

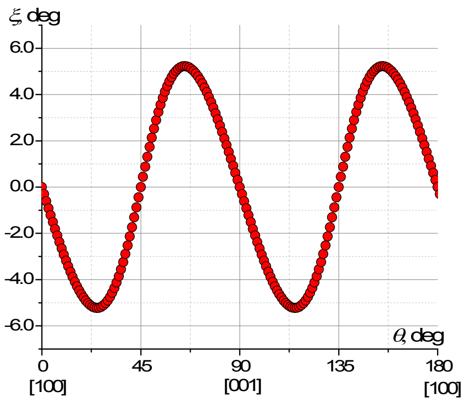


Fig. 2. Dependence of non-orthogonality angle on the angle θ in the *XZ* plane.

2.2. Determination of effective EO coefficients based on ellipticity of optical eigenwaves

Let us begin with the optically uniaxial state of crystals (Fig. 3a). In this case one can consider ordinary wave, polarized along *Y* axis and extraordinary wave polarized in the *XZ* plane. Now the ellipticity of incident optical eigenwaves, caused by the optical activity can be written as:

$$\chi_i = \frac{\sqrt{D_1^2 + D_3^2}}{D_2}, \tag{7}$$

(here D_i are the components of electrical induction of the incident optical wave) for ordinary polarized wave (the I, III and V types of AO interactions [14]) and:

$$\chi_i = \frac{D_2}{\sqrt{D_1^2 + D_3^2}}, \tag{8}$$

for extraordinary polarized optical wave (the II, IV and VI types of AO interactions [14]). Notice that at this stage of our analysis, we did not set any condition for the polarization of the diffracted optical wave. The Bragg angle has been accepted to be equal to one degree.

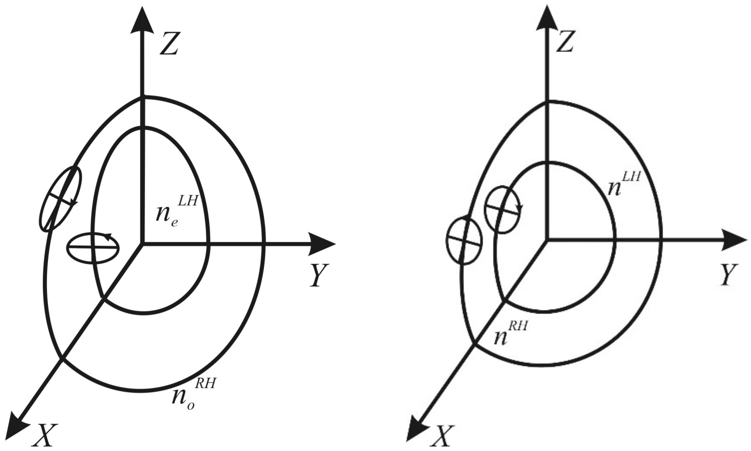


Fig. 3. The parts of the refractive indices surfaces for the optically active uniaxial crystals (a) and optically active crystals of the cubic system (b). The polarization is indicated by the ellipses and circles.

The constitutive equations for the first and second types of AO interaction, i.e. the AO diffraction on QL AW, are as follows, respectively:

$$\begin{aligned} E_1 &= \chi_i \Delta B_{11} \sin\varphi_i D_1 + \Delta B_{12} D_2 + \chi_i \Delta B_{13} \cos\varphi_i D_3 \\ E_2 &= \chi_i \Delta B_{21} \sin\varphi_i D_1 + \Delta B_{22} D_2 + \chi_i \Delta B_{23} \cos\varphi_i D_3 \, , \\ E_3 &= \chi_i \Delta B_{31} \sin\varphi_i D_1 + \Delta B_{32} D_2 + \chi_i \Delta B_{33} \cos\varphi_i D_3 \end{aligned} \tag{9}$$

and

$$\begin{aligned} E_1 &= \Delta B_{11} \sin\varphi_i D_1 + \chi_i \Delta B_{12} D_2 + \Delta B_{13} \cos\varphi_i D_3 \\ E_2 &= \Delta B_{21} \sin\varphi_i D_1 + \chi_i \Delta B_{22} D_2 + \Delta B_{23} \cos\varphi_i D_3 \, , \\ E_3 &= \Delta B_{31} \sin\varphi_i D_1 + \chi_i \Delta B_{32} D_2 + \Delta B_{33} \cos\varphi_i D_3 \end{aligned} \tag{10}$$

where E_j represents the components of the electric field of the diffracted optical wave, φ_i is the angle between the *X* axis and direction of the wavevector of the incident optical wave,

ΔB_{ji} are the components of optical frequency impermeability tensor increment. The electric field of the diffracted wave is determined as:

$$E = \sqrt{E_1^2 + E_2^2 + E_3^2}\,. \tag{11}$$

As one can see at $\chi_i = 0$, i.e. at the linearly polarized eigen optical waves the Eqs. (9) and (10) are different. However, when linear anisotropy disappear and $\chi_i = \pm 1$ (Fig. 3b), the Eqs. (9) and (10) become the same. This means that in optically active crystals of cubic systems, the I and II, III and IV, and V and VI types of AO interaction are described using the same system of equations, respectively. Let us remind that at III and IV types of AO interactions, the AO diffraction is realized on the QT AW polarized in the *XZ* plane, while at the V and VI types of AO interaction, the diffraction occurs on the PT AW polarized along *Y* axis. This conclusion agree with the same making in our recent work [7] that six isotropic types of AO interaction of circularly polarized optical waves reduces to three types. From this conclusion, it follows that in the existence of linear optical anisotropy and optical activity, the $k_o \to k_o$ and $k_e \to k_e$ types of isotropic AO interactions are described by different effective EO coefficients (k_o and k_e are the wavevectors of ordinary and extraordinary elliptically polarized optical waves, respectively). In contrast, in the absence of linear anisotropy and the presence of optical activity, $k_r \to k_r$, and $k_l \to k_l$ types of AO, interactions are described by the same relations for effective AO coefficients (k_r and k_l are the wavevectors of right-handed (RH) and left-handed (LH) circularly polarized optical waves, respectively). Therefore for the I and II types of interactions (that are reduced to the I type), the relation for the effective EO coefficient can be written as:

$$\begin{aligned} p_{eff}^{2(I)} &= 0.5(p_{21}\cos\theta\cos\xi + p_{12}\sin\theta\sin\xi)^2 \\ &+0.5\chi_i^2 \left\{ \begin{aligned} &(p_{11}\cos\theta\cos\xi + p_{21}\sin\theta\sin\xi)^2\sin^2\varphi_i \\ &+(p_{12}\cos\theta\cos\xi + p_{11}\sin\theta\sin\xi)^2\cos^2\varphi_i \\ &+p_{55}^2\sin^2(\theta+\xi) + p_{44}\sin(\theta+\xi)\sin 2\varphi_i \\ &\times(p_{11}\cos\theta\cos\xi + p_{21}\sin\theta\sin\xi + p_{12}\cos\theta\cos\xi + p_{11}\sin\theta\sin\xi) \end{aligned} \right\}, \end{aligned} \tag{12}$$

$$\begin{aligned} p_{eff}^{2(II)} &= 0.5 \left\{ \begin{aligned} &(p_{11}\cos\theta\cos\xi + p_{21}\sin\theta\sin\xi)^2\sin^2\varphi_i \\ &+(p_{12}\cos\theta\cos\xi + p_{11}\sin\theta\sin\xi)^2\cos^2\varphi_i \\ &+p_{44}^2\sin^2(\theta+\xi) + p_{44}\sin(\theta+\xi)\sin 2\varphi_i \\ &\times(p_{11}\cos\theta\cos\xi + p_{21}\sin\theta\sin\xi + p_{12}\cos\theta\cos\xi + p_{11}\sin\theta\sin\xi) \end{aligned} \right\} \\ &+0.5\chi_i^2(p_{21}\cos\theta\cos\xi + p_{12}\sin\theta\sin\xi)^2, \end{aligned} \tag{13}$$

respectively. Only when $\chi_i = \pm 1$ these types of interaction are reduced to the one (I type of interaction) of the circularly polarized optical waves with QL AW and the Eqs. (12,13) become the same:

$$p_{eff}^{2(I)} = 0.5 \left\{ \begin{aligned} &(p_{11}\cos\theta\cos\xi + p_{21}\sin\theta\sin\xi)^2\sin^2\varphi_i \\ &+(p_{12}\cos\theta\cos\xi + p_{11}\sin\theta\sin\xi)^2\cos^2\varphi_i \\ &+p_{44}^2\sin^2(\theta+\xi) + (p_{21}\cos\theta\cos\xi + p_{12}\sin\theta\sin\xi)^2 \\ &+p_{44}\sin(\theta+\xi)\sin 2\varphi_i \\ &\times(p_{11}\cos\theta\cos\xi + p_{21}\sin\theta\sin\xi + p_{12}\cos\theta\cos\xi + p_{11}\sin\theta\sin\xi) \end{aligned} \right\}. \tag{14}$$

For III and IV types of AO interaction with QT AW the respective relation can be written as:

$$p_{eff}^{2(III)} = 0.5(p_{12}\sin\theta\cos\xi - p_{21}\cos\theta\sin^2\xi)^2 + 0.5\chi_i^2 \begin{Bmatrix} (p_{21}\sin\theta\cos\xi - p_{11}\cos\theta\sin\xi)^2\sin^2\varphi_i \\ +(p_{11}\sin\theta\cos\xi - p_{12}\cos\theta\sin\xi)^2\cos^2\varphi_i \\ +p_{44}^2\cos^2(\theta+\xi) + p_{44}\cos(\theta+\xi) \\ \times\sin 2\varphi_i \begin{pmatrix} p_{21}\sin\theta\cos\xi - p_{11}\cos\theta\sin\xi \\ +p_{11}\sin\theta\cos\xi - p_{12}\cos\theta\sin\xi \end{pmatrix} \end{Bmatrix}, \quad (15)$$

$$p_{eff}^{2(IV)} = 0.5 \begin{Bmatrix} (p_{21}\sin\theta\cos\xi - p_{11}\cos\theta\sin\xi)^2\sin^2\varphi_i \\ +(p_{11}\sin\theta\cos\xi - p_{12}\cos\theta\sin\xi)^2\cos^2\varphi_i \\ +p_{44}^2\cos^2(\theta+\xi) + p_{44}\cos(\theta+\xi) \\ \times\sin 2\varphi_i \begin{pmatrix} p_{21}\sin\theta\cos\xi - p_{11}\cos\theta\sin\xi \\ +p_{11}\sin\theta\cos\xi - p_{12}\cos\theta\sin\xi \end{pmatrix} \end{Bmatrix} + 0.5\chi_i^2(p_{12}\sin\theta\cos\xi - p_{21}\cos\theta\sin^2\xi)^2. \quad (16)$$

When $\chi_i = \pm 1$ these types of interaction are reduced to the II type of interaction of the circularly polarized optical waves with QT AW and the Eqs. (15,16) become the same:

$$p_{eff}^{2(II)} = 0.5 \begin{Bmatrix} (p_{21}\sin\theta\cos\xi - p_{11}\cos\theta\sin\xi)^2\sin^2\varphi_i \\ +(p_{11}\sin\theta\cos\xi - p_{12}\cos\theta\sin\xi)^2\cos^2\varphi_i + p_{44}^2\cos^2(\theta+\xi) \\ +(p_{12}\sin\theta\cos\xi - p_{21}\cos\theta\sin^2\xi)^2 + p_{44}\cos(\theta+\xi) \\ \times\sin 2\varphi_i \begin{pmatrix} p_{21}\sin\theta\cos\xi - p_{11}\cos\theta\sin\xi \\ +p_{11}\sin\theta\cos\xi - p_{12}\cos\theta\sin\xi \end{pmatrix} \end{Bmatrix}. \quad (17)$$

For V and VI types of interactions with the PT AW the equation for the effective EO coefficient has the view:

$$p_{eff}^{2(V)} = 0.5p_{44}^2 + 0.5\chi_i^2 p_{44}^2(\cos^2\theta\sin^2\varphi_i + \sin^2\theta\cos^2\varphi - 0.5\sin 2\theta\sin 2\varphi), \quad (18)$$

$$p_{eff}^{2(VI)} = 0.5p_{44}^2(\cos^2\theta\sin^2\varphi_i + \sin^2\theta\cos^2\varphi_i - 0.5\sin 2\theta\sin 2\varphi) + 0.5\chi_i^2 p_{44}^2. \quad (19)$$

At $\chi_i = \pm 1$ these equations are the same and describe the effective EO coefficient for the III type of interaction of AO interaction of the circularly polarized optical waves:

$$p_{eff}^{2(III)} = 0.5p_{44}^2 + 0.5p_{44}^2(\cos^2\theta\sin^2\varphi_i + \sin^2\theta\cos^2\varphi_i + 0.5\sin 2\theta\sin 2\varphi). \quad (20)$$

It has been found that the relations for the effective EO coefficients responsible for the anisotropic AO interaction between circularly polarized AWs with different signs of rotation of the electric field vector (i.e., $k_r \to k_l$, and $k_l \to k_r$) are same as Eqs. (14,17,20).

2.2. Determination of effective EO coefficients based on extraction of circularly polarized optical waves

Now, we will consider the derivation of effective EO coefficients based on the approach developed in our recent paper [7]. The incident LH wave can be presented by the system of equations for the components of its electric induction vectors D_1^{LH}, D_2^{LH}, and D_3^{LH}:

$$\begin{cases} D_1^{LH} = D_0\sin\varphi_i\cos\delta \\ D_3^{LH} = D_0\cos\varphi_i\cos\delta, \\ D_2^{LH} = D_0\sin\delta \end{cases} \quad (21)$$

where D_0 is the unit amplitude, and δ is the phase. The similar is true for the incident RH wave:

$$\begin{cases} D_1^{RH} = -D_0 \sin\varphi_i \cos\delta \\ D_2^{RH} = D_0 \sin\delta \\ D_3^{RH} = -D_0 \cos\varphi_i \cos\delta \end{cases} . \tag{22}$$

Then, the electric-field components E_1, E_2, and E_3 of the diffracted optical wave at the diffraction of the LH wave read as:

$$\begin{cases} E_1 = D_0(\Delta B_{12} \sin\delta + (\Delta B_{11} \sin\varphi_i + \Delta B_{13} \cos\varphi_i)\cos\delta) \\ E_2 = D_0(\Delta B_{22} \sin\delta + (\Delta B_{21} \sin\varphi_i + \Delta B_{23} \cos\varphi_i)\cos\delta) \\ E_3 = D_0(\Delta B_{32} \sin\delta + (\Delta B_{31} \sin\varphi_i + \Delta B_{33} \cos\varphi_i)\cos\delta) \end{cases} . \tag{23}$$

The system of equations written for the case of diffraction of the RH wave is similar:

$$\begin{cases} E_1 = D_0(\Delta B_{12} \sin\delta - (\Delta B_{11} \sin\varphi_i + \Delta B_{13} \cos\varphi_i)\cos\delta) \\ E_2 = D_0(\Delta B_{22} \sin\delta - (\Delta B_{21} \sin\varphi_i + \Delta B_{23} \cos\varphi_i)\cos\delta) \\ E_3 = D_0(\Delta B_{32} \sin\delta - (\Delta B_{31} \sin\varphi_i + \Delta B_{33} \cos\varphi_i)\cos\delta) \end{cases} . \tag{24}$$

Then the field module is given by:

$$E = \left(E_1^2 + E_2^2 + E_3^2\right)^{1/2} . \tag{25}$$

As seen from Eqs. (23) and (24), the amplitudes of the field components E_1, E_2, and E_3 of the diffracted wave differ from each other, while their squares are the same since the mean values of $\langle \sin^2\delta \rangle$ and $\langle \cos^2\delta \rangle$ are equal to ½ and we have $\langle \sin 2\delta \rangle = 0$ for the averaged value $\sin 2\delta$. In general, these equations correspond to an elliptically polarized diffracted optical wave. On the other hand, it is well known [15] that any wave with elliptical polarization can be decomposed into two waves having circular polarizations, opposite rotation directions of their electric field vectors, and different amplitudes. The relation for the electric field components of these circular waves is as follows:

$$\begin{bmatrix} E^{LH} \\ E^{RH} \end{bmatrix} = \frac{Ee^{i\Delta}}{\sqrt{2}} \begin{bmatrix} (\cos\varepsilon - \sin\varepsilon)e^{i\beta} \\ (\cos\varepsilon + \sin\varepsilon)e^{-i\beta} \end{bmatrix}, \tag{26}$$

where E is the amplitude of elliptical oscillation given by Eq. (25), $\varepsilon = \mathrm{atan}\left(\frac{\sqrt{E_1^2 + E_3^2}}{E_2}\right)$ denotes the angle of ellipticity, β the orientation azimuth of the major axis of the polarization ellipse (i.e., the angle between the major axis of the polarization ellipse and the positive direction of the *X* axis), and Δ the absolute phase. The latter is determined by the angle between the initial direction of the electric field vector and the major axis of the polarization ellipse. In our case, one can accept the equalities $\beta = 0$ and $\Delta = 0$. Thus, Eq. (26) can be rewritten as

$$\begin{bmatrix} E^{LH} \\ E^{RH} \end{bmatrix} = \frac{E}{\sqrt{2}} \begin{bmatrix} \cos\varepsilon - \sin\varepsilon \\ \cos\varepsilon + \sin\varepsilon \end{bmatrix}. \tag{27}$$

However, if to set that the angle of ellipticity of the diffracted wave is equal to zero, that corresponds to the linearly polarized wave, which consists of equal portions of LH and RH waves Eq. (27) can be simplified:

$$\begin{bmatrix} E^{LH} \\ E^{RH} \end{bmatrix} = E \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{28}$$

On the other hand, Eq. (28) corresponds to the case of neglecting the operation of decomposition of an elliptically diffracted polarized optical wave on the circular components, i.e., to the non-accounting of the polarization of an eigenwave that appears in the process of diffraction. Therefore, the polarization of the diffracted optical wave is determined by the relation for ellipticity $\chi_d = \sqrt{E_1^2 + E_3^2}/E_2$ and, in the general case, can be elliptical and not the same as the polarization of the eigenwave. The ellipticity of the diffracted wave is determined by the relation between EO tensor components included in the relation for effective EO coefficient.

Finally, the effective EO coefficients are the same as determined based on the ellipticity of optical eigenwaves (see Eqs. (14,17,20):

$$p_{eff}^{2(I)} = 0.5 \left\{ \begin{array}{l} (p_{11}\cos\theta\cos\xi + p_{21}\sin\theta\sin\xi)^2 \sin^2\varphi_i \\ +(p_{12}\cos\theta\cos\xi + p_{11}\sin\theta\sin\xi)^2 \cos^2\varphi_i \\ +p_{44}^2 \sin^2(\theta+\xi) + (p_{21}\cos\theta\cos\xi + p_{12}\sin\theta\sin\xi)^2 \\ +p_{44}\sin(\theta+\xi)\sin 2\varphi_i \\ \times(p_{11}\cos\theta\cos\xi + p_{21}\sin\theta\sin\xi + p_{12}\cos\theta\cos\xi + p_{11}\sin\theta\sin\xi) \end{array} \right\}, \tag{29}$$

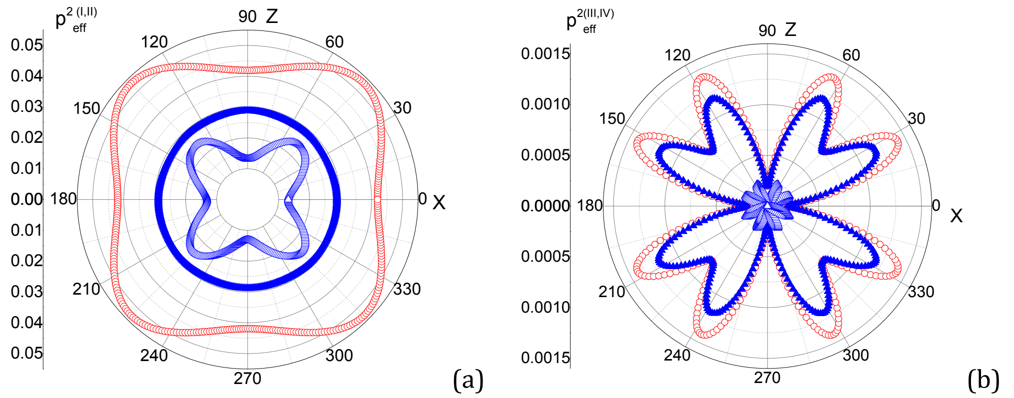
$$p_{eff}^{2(II)} = 0.5 \left\{ \begin{array}{l} (p_{21}\sin\theta\cos\xi - p_{11}\cos\theta\sin\xi)^2 \sin^2\varphi_i \\ +(p_{11}\sin\theta\cos\xi - p_{12}\cos\theta\sin\xi)^2 \cos^2\varphi_i + p_{44}^2\cos^2(\theta+\xi) \\ +(p_{12}\sin\theta\cos\xi - p_{21}\cos\theta\sin^2\xi)^2 \\ +p_{44}\cos(\theta+\xi) \\ \times\sin 2\varphi_i \begin{pmatrix} p_{21}\sin\theta\cos\xi - p_{11}\cos\theta\sin\xi \\ +p_{11}\sin\theta\cos\xi - p_{12}\cos\theta\sin\xi \end{pmatrix} \end{array} \right\}, \tag{30}$$

$$p^2{}_{eff}^{(III)} = 0.5p_{44}^2. + 5p_{44}^2(\cos^2\theta\sin^2\varphi_i + \sin^2\theta\cos^2\varphi_i + 0.5\sin 2\theta\sin 2\varphi). \tag{31}$$

Notice that the same view of Eqs. (14,17,20) and Eqs. (29,30,31), respectively, is reached for the case of neglecting the polarization of the diffracted optical wave. As a result, one can conclude that in the case when the optical eigenwaves are circularly polarized, Eqs. (14,17,20) and Eqs. (29,30,31) are the same only when one neglects the polarization of diffracted optical waves at their derivation. The same is not true when the linear birefringence appears.

3. Results and discussion

Figs. 4 and 5 present the dependencies of the square of effective EO coefficients and AO figure of merit, respectively. As seen, at the incidence of linearly polarized optical waves, the surfaces of effective EO coefficients and AO figures of merit differ. On the other hand, when an optical wave with circular eigenpolarization incident on the crystal, the surfaces become the same. Moreover, in the last case, the effective EO coefficients and AO figure of merit increase, leading to the inflation of the surfaces. The maximal value of the AO figure of merit (3.6×10^{-15}s^3/kg) is reached for the first type of AO interaction with the QL AW (Fig. 4a). The anisotropy of effective EO coefficients and AO figure of merit is caused only by the anisotropy of acoustic properties.



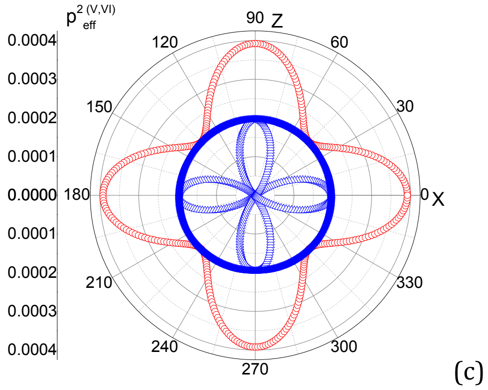
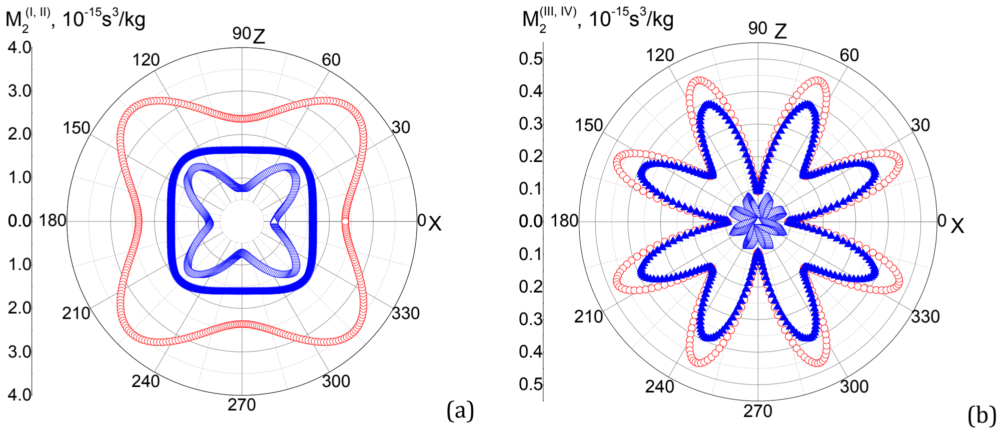


Fig. 4. The dependencies of the square of effective EO coefficient on angle θ in the *XZ* plane for $NaClO_3$ crystals. I and II types of AO interaction (a); III and IV types of AO interactions (b); V and VI types of AO interaction (c). Open (I, III, V types) and full (II, IV, VI types) triangles correspond to the interaction of the linearly polarized optical waves; open red circles correspond to the interaction of the circularly polarized eigen optical waves.



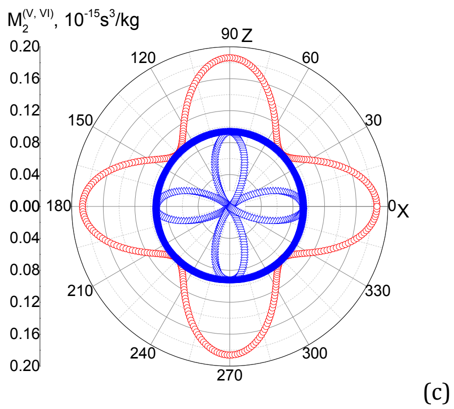


Fig. 5. The dependencies of the AO figure of merit on angle θ in the *XZ* plane for $NaClO_3$ crystals. I and II types of AO interaction (a); III and IV types of AO interactions (b); V and VI types of AO interaction (c). Open (I, III, V types) and full (II, IV, VI types) triangles correspond to the interaction of the linearly polarized optical waves; open red circles correspond to the interaction of the circularly polarized eigen optical waves.

In contrast to the optically uniaxial or biaxial crystals, where the influence of the polarization of the eigenwaves leads to the peak-like increase of the AO efficiency in the directions of optical axes, in the cubic crystals, this influence is manifested in the inflation of the surfaces of the effective EO coefficients and AO figure of merit. Obviously, it is caused by the fact that in the cubic optically active crystals, the ellipticity of eigenwaves is equal to unity in all directions.

As it has been noticed above, the polarization of the diffracted optical wave is determined by the relation for ellipticity $\chi_d = \sqrt{E_1^2 + E_3^2}/E_2$ and, in the general case, can be elliptical and not the same as the polarization of the eigenwave. The ellipticity of the diffracted wave is determined by the relation between EO tensor components:

$$\chi_d = \frac{\sqrt{\begin{aligned}&(p_{11}\cos\theta\cos\xi + p_{21}\sin\theta\sin\xi)^2\sin^2\varphi_i + p_{44}^2\sin^2(\xi+\theta)\\&+(p_{12}\cos\theta\cos\xi + p_{11}\sin\theta\sin\xi)^2\cos^2\varphi_i\\&+p_{44}\sin(\xi+\theta)\sin 2\varphi_i(p_{11}\cos(\xi-\theta) + p_{21}\sin\theta\sin\xi + p_{12}\cos\theta\cos\xi)\end{aligned}}}{p_{11}\cos\theta\cos\xi + p_{12}\sin\theta\sin\xi}, \quad (32)$$

for the diffraction on the QL AW,

$$\chi_d = \frac{p_{12}\sin\theta\cos\xi - p_{11}\cos\theta\sin\xi}{\sqrt{\begin{aligned}&(p_{21}\sin\theta\cos\xi - p_{11}\cos\theta\sin\xi)^2\sin^2\varphi_i + p_{44}^2\cos^2(\xi+\theta)\\&+(p_{11}\sin\theta\cos\xi - p_{12}\cos\theta\sin\xi)^2\cos^2\varphi_i\\&+p_{44}\cos(\xi+\theta)\sin 2\varphi_i(p_{21}\sin\theta\sin\xi - p_{11}\sin(\xi-\theta) + p_{12}\cos\theta\cos\xi)\end{aligned}}}, \quad (33)$$

for the diffraction on the QT AW, and

$$\chi_d = \sin(\theta + \varphi_i), \quad (34)$$

for the diffraction on the PT AW.

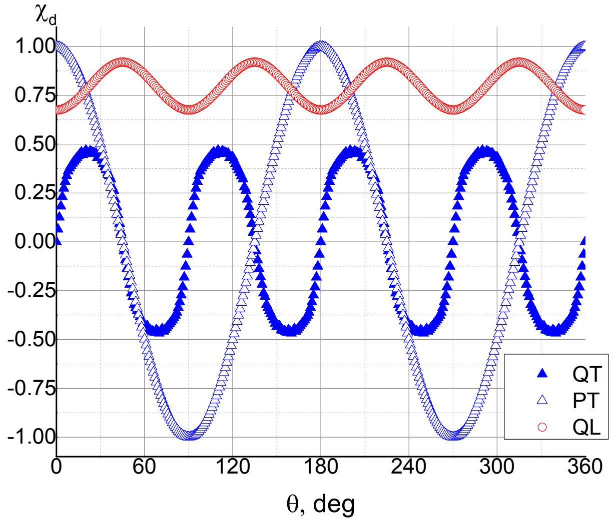


Fig. 6. Dependencies of ellipticity of the diffracted optical wave on angle θ at the AO diffraction in $NaClO_3$ crystals on the different acoustic eigenwaves.

As seen (Fig. 6), the ellipticity of the diffracted optical wave oscillates with the change of angle θ at the diffraction on all eigen AWs. The ellipticity is equal to ±1 only at the AO diffraction on the PT AW when the angle θ is almost equal to 0 and 90×n deg (n is the integer number). The diffracted wave is linearly polarized at θ≈45+90×n deg. The last is true for the diffraction on the QT AW. However, at other values of angle θ, the diffracted optical wave is elliptically polarized. At the diffraction on the QL AW, the ellipticity of the diffracted wave oscillates with a change of angle θ between values 0.675 and 0.92, with a period of 90 deg.

4. Conclusions

In the present work, we have analyzed the influence of the polarization of the optical eigenwaves participating in the AO interaction in the optically active cubic crystals. An analysis was carried out using the example of $NaClO_3$ crystals. It has been shown that when the incident optical wave is circularly polarized, corresponding to the polarization of one of the optical eigenwaves, the effective EO coefficient and AO figure of merit increase almost two times. The surfaces of these parameters undergo inflation in comparison with those manifested in the interaction of the linearly polarized optical waves. It has been shown that six isotropic types of AO interaction are reduced to three types, i.e., the sign of the circular polarization does not affect the relation for the effective EO coefficient. Moreover, the relations for effective EO coefficients for the anisotropic AO interactions in this case are the same as for isotropic types with respective AWs.

We have shown that two approaches we recently developed, based on considering the ellipticity of optical eigenwaves and extraction of the circular polarization from the elliptical polarization state, agree when the diffracted optical wave's polarization is not considered. The present work's results lead to important conclusions about the polarization of diffracted optical waves. When we deal with linear anisotropy, the polarization of the diffracted optical wave is determined by the phase-matching conditions and is always linear whenever the polarization of the incident optical wave is linear and corresponds to the polarization of one of the eigenwaves. In the presence of circular birefringence in the case when the polarization of the incident optical wave corresponds to the polarization of the eigenwave, the polarization of the diffracted optical wave is not necessarily the same as the polarization of the eigenwave, which corresponds to the phase-matching conditions. In the last case, the polarization of the diffracted wave is determined by the ratio between EO tensor components, which are included in the effective EO coefficient. In general, this polarization is elliptical and not the same as the polarization of the eigenwave.

Acknowledgement

The authors acknowledge the Ministry of Education and Science of Ukraine for financial support of this study (project #0123U101781).

References

1. Magdich, L. N. (1989). Acoustooptic devices and their applications. CRC Press.
2. Mys, O., Kostyrko, M., Adamenko, D., Martynyuk-Lototska, I., Skab, I., & Vlokh, R. (2022). Effect of ellipticity of optical eigenwaves on the enhancement of efficiency of acousto-optic Bragg diffraction. A case of optically active $Pb_5Ge_3O_{11}$ crystals. AIP Advances, 12(5), 055130.
3. Mys, O., Adamenko, D., & Vlokh, R. (2023). Enhancement of acousto-optic diffraction efficiency in SiO_2 crystals due to ellipticity of optical eigenwaves: isotropic acousto-optic interaction. Ukrainian Journal of Physical Optics, 24(2), 124-134.
4. Mys, O., Adamenko, D., & Vlokh, R. (2023). Increase in the acousto-optic figure of merit in SiO_2 crystals due to optical activity. Anisotropic acousto-optic interactions. Ukrainian Journal of Physical Optics, 24(3), 262-275.
5. Mys, O., Adamenko, D., & Vlokh, R. (2023). Influence of Faraday elliptical birefringence on the acousto-optic diffraction efficiency: a case of isotropic interaction with quasi-longitudinal acoustic waves in KH_2PO_4 crystals. Errata. Ukrainian Journal of Physical Optics, 24(4), 04087-04087.
6. Mys, O., Martynyuk-Lototska, I., Pogodin, A., Dudok, T., Adamenko, D., Krupych, O., Skab, I., and Vlokh, R. (2019). Acousto-optic interaction between circularly polarized optical eigenwaves: example of $AgGaS_2$ crystals. Applied Optics, 58(22), 6012-6018 .

7. Mys, O., Krupych, O., Martynyuk-Lototska, I., Orykhivskyi, I., Kostyrko, M. and Vlokh, R. (2021). Types of acousto-optic interactions between acoustic and circularly polarized optical waves: case of $Pb_5Ge_3O_{11}$ crystals. Applied Optics, 60(10), 2846-2853
8. Karppinen, M. (2024). Origin of Optical Activity in Cubic $NaClO_3$ and $NaBrO_3$ Crystals at 296K. European Journal of Applied Sciences, 12(1), 174-184.
9. Chandrasekhar, S., & Madhava, M. S. (1967). Optical rotatory dispersion of crystals of sodium chlorate and sodium bromate. Acta Crystallographica, 23(6), 911-913.
10. Narasimhamurty, T. S. (1954, October). Photoelastic studies of optically active crystals: $NaClO_3$ and $NaBrO_3$. In Proceedings of the Indian Academy of Sciences-Section A (Vol. 40, pp. 167-175). Springer India.
11. Belomestnykh, V. N., & Ul'Yanov, V. L. (1992). Temperature dependence of the moduli of elasticity of $NaClO_3$ and $NaBrO_3$ single crystals. Russian Physics Journal, 35(12), 1134-1139.
12. Michael Bass, Carolyn MacDonald, Guifang Li, Casimer M. DeCusatis, Virendra N. Mahajan, (2010). Handbook of Optics, Volume V: Atmospheric Optics, Modulators, Fiber Optics, X-Ray and Neutron Optics. McGraw-Hill, Inc.
13. Chandrasekhar, S., & Madhava, M. S. (1969). Optical rotatory dispersion of a mixed crystal of sodium chlorate-sodium bromate. Materials Research Bulletin, 4(8), 489-493.
14. Mys, O., Adamenko, D., Krupych, O. and Vlokh, R. (2018). Effect of deviation from purely transverse and longitudinal polarization states of acoustic waves on the anisotropy of acousto-optic figure of merit: the case of Tl_3AsS_4 crystals. Applied Optics, 57(28), 8320-8330.
15. Azzam, R. M. A. and Bashara, N. M. (1977). Ellipsometry and polarized light. North Holland Publishing Company.

O. Mys, M. Kostyrko, R. Vlokh. (2024). Polarization of the Diffracted Optical Wave in the Crystals with Circular Optical Birefringence and Efficiency of Acousto-Optic Interaction Associated with Circularly Polarized Optical Waves in the Optically Active Crystals of Cubic System**.** *Ukrainian Journal of Physical Optics*, *25*(4), 04051 – 04062.
doi: 10.3116/16091833/Ukr.J.Phys.Opt.2024.04051

Анотація. *У роботі проаналізовано вплив поляризації власних оптичних хвиль на акустооптичну (АО) взаємодію в оптично активних кубічних кристалах на прикладі кубічних оптично активних кристалів $NaClO_3$. Показано, що при циркулярній поляризації падаючої оптичної хвилі ефективний пружнооптичний (ПО) коефіцієнт і коефіцієнт АО якості збільшуються майже вдвічі, а поверхні цих параметрів зазнають здуття порівняно з тими, що проявляються при взаємодії лінійно поляризованих оптичних хвиль. Встановлено, що шість ізотропних типів АО взаємодії зводяться до трьох типів, тобто знак кругової поляризації не впливає на співвідношення для ефективного ЕО коефіцієнта. Крім того, у цьому випадку співвідношення для ефективних ЕО коефіцієнтів для анізотропних АО взаємодій є такими ж, як і для ізотропних типів з відповідними акустичними хвилями. Показано, що два підходи, які ми нещодавно розробили, засновані на розгляді еліптичності оптичних власних хвиль і на виділенні кругової поляризації зі стану еліптичної поляризації, узгоджуються, коли не враховується поляризація дифрагованої оптичної хвилі. Встановлено, що за наявності циркулярного двозаломлення у випадку, коли поляризація падаючої оптичної хвилі відповідає поляризації власної хвилі, поляризація дифрагованої оптичної хвилі не обов'язково збігається з поляризацією власної хвилі, що відповідає умовам синхронізму. В останньому випадку поляризація дифрагованої хвилі визначається співвідношенням між компонентами ЕО тензора, які входять в ефективний ЕО коефіцієнт. Загалом, ця поляризація є еліптичною і не збігається з поляризацією власної хвилі.*

Ключові слова*: акустооптична дифракція, коефіцієнт акустооптичної якості, ефективний пружнооптичний коефіцієнт, оптична активність, кругова поляризація, поляризація дифрагованої оптичної хвилі*