Ukrainian Journal of Physical Optics
ISSUE 3
2024

Quiescent Bright Optical Solitons for Radhakrishnan–Kundu–Lakshmanan Equation with Nonlinear Chromatic Dispersion and Power—Law of Self–Phase Modulation by Lie Symmetry

Abdullahi Rashid Adem[1], Anjan Biswas[2,3,4,5], Yakup Yildirim[6,7], Anwar Jaafar Mohamad Jawad[8] & Ali Saleh Alshomrani[3]

[1]Department of Mathematical Sciences, University of South Africa, UNISA–0003, South Africa
[2]Department of Mathematics and Physics, Grambling State University, Grambling, LA 71245–2715, USA
[3]Mathematical Modeling and Applied Computation (MMAC) Research Group, Center of Modern Mathematical Sciences and their Applications (CMMSA), Department of Mathematics, King Abdulaziz University, Jeddah–21589, Saudi Arabia
[4]Department of Applied Sciences, Cross–Border Faculty of Humanities, Economics and Engineering, Dunarea de Jos University of Galati, 111 Domneasca Street, Galati–800201, Romania
[5]Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa–0204, South Africa
[6]Department of Computer Engineering, Biruni University, Istanbul–34010, Turkey
[7]Mathematics Research Center, Near East University, 99138 Nicosia, Cyprus
[8]Department of Computer Technical Engineering, Al-Rafidain University College, 10064 Baghdad, Iraq

Received: 08.11.2023

Abstract. The current paper recovers quiescent optical solitons for the Radhakrishnan–Kundu–Lakshmanan equation with a power law of self–phase modulation and nonlinear chromatic dispersion. The Lie symmetry analysis leads to stationary bright optical soliton solutions for linear and generalized temporal evolution. The parameter constraints for the existence of such solitons are enumerated.

Keywords: bright solitons, quiescent solitons, Lie symmetry
UDC: 535.32
DOI: 10.3116/16091833/Ukr.J.Phys.Opt.2024.03013

1. Introduction

The formation of quiescent optical solitons is a gigantic taboo in optoelectronics. The propagation of pulses through optical waveguides across trans-continental and trans–oceanic waveguides would be stalled, and catastrophic consequences would ensue. Therefore, it is imperative to study such solitons in optics. One of the leading causes for forming such kinds of solitons is when the chromatic dispersion (CD) is rendered nonlinear. The concept of quiescent solitons with nonlinear CD was first conceived in 2006, and later, a deluge of results started pouring in from this topic that received immense attention [1–15]. Various approaches have been implemented to address such quiescent solitons. The most popular approach is the one that implements Lie symmetry. Several models were also addressed to recover such solitons with nonlinear CD and a wide range of self–phase modulation (SPM) structures [1–13]. Some of these models that have been studied in this context are the nonlinear Schrödinger's equation, Lakshmanan–Porsezian–Daniel model, concatenation model, dispersive concatenation model, Sasa–Satsuma equation, and several others.

The current paper will focus on the Radhakrishnan–Kundu–Lakshmanan (RKL) equation with nonlinear CD and arbitrary intensity along with third–order dispersion (3OD) and fourth–order dispersion (4OD) terms. Incidentally, the RKL equation with nonlinear CD was already studied, and the integration methodologies were widely varied and completely different [13]. The wide variety of algorithms that were applied are sine–Gordon equation procedure, F–expansion approach, Riccati equation expansion, G'/G–expansion, Kudryashov's methodology, *exp*–expansion, and the extended Jacobi's elliptic function expansion. The model was, however, not considered with generalized temporal evolution. Only a special case of the nonlinear form of CD yielded results for the linear temporal evolution. This work strictly implements a far more robust approach to handle the model, namely Lie symmetry that reveals quiescent bright optical solitons. The paper is divided into two sections. First, the focus is on linear temporal evolution followed by generalized temporal evolution. In both cases, the parameter constraints for the existence of such solitons are enumerated. The details are exhibited in the rest of the paper.

2. Linear temporal evolution

The RKL equation with linear temporal evolution and nonlinear CD and general intensity is given as:

$$\begin{aligned} &iq_t + a\left(|q|^n q\right)_{xx} + b|q|^{2m} q \\ &= i\lambda\left(|q|^{2m} q\right)_x + i\mu\left(|q|^{2m}\right)_x q + i\delta|q|^{2m} q_x - i\gamma_1 q_{xxx} - \gamma_2 q_{xxxx} - i\gamma_3 q_{xxt} - \gamma_4 q_{xxxt}. \end{aligned} \tag{1}$$

Here, in Eq. (1), $q = q(x,t)$ is a complex-valued function representing the soliton profile, λ represents the coefficient of self-steepening for short pulses, μ is the higher-order dispersion coefficient, δ is the inter-modal dispersion, the first term is the linear evolution term with its coefficient being $i = \sqrt{-1}$. The coefficient of a is the nonlinear CD with n being the parameter of nonlinearity. For $n = 0$, the CD is linear. The coefficient of b is the SPM with the parameter m representing the general intensity. The dispersive parameters are given by the coefficients of γ_j for $1 \le j \le 4$, which account for 3OD and 4OD for both, spatial and spatiotemporal. Eq. (1) does not support mobile solitons because the CD is nonlinear. Therefore, to locate the stationary solitons, the following substitution is selected:

$$q(x,t) = \phi(x)e^{i\omega t}. \tag{2}$$

where $\phi(x)$ and ω represent the amplitude component and frequency of the soliton, respectively. Upon substituting Eq. (2) into Eq. (1) and decomposing the resulting equation into real and imaginary components reveals the following pair of relations:

$$\begin{aligned} &a(n+1)\phi^n(x)\left[n\{\phi'(x)\}^2 + \phi(x)\phi''(x)\right] + b\phi(x)^{2m+2} \\ &+\gamma_2\phi^{(iv)}(x)\phi(x) - \gamma_3\omega\phi(x)\phi''(x) - \omega\phi^2(x) = 0, \end{aligned} \tag{3}$$

and

$$(\gamma_1 + \gamma_4\omega)\phi'''(x) - \{\delta + (2m+1)\lambda + 2m\mu\}\phi'(x)\phi^{2m}(x) = 0. \tag{4}$$

For integrability of Eq. (1) the following constraint conditions on the parameters must hold:

$$\gamma_1 + \gamma_4\omega = 0, \tag{5}$$

$$\gamma_2 = 0, \tag{6}$$

$$n = -1, \tag{7}$$

and

$$\delta + (2m+1)\lambda + 2m\mu = 0. \tag{8}$$

Implementing these constraints, the governing model given by Eq. (1) transforms to:

$$\begin{aligned} & iq_t + a\left(\frac{q}{|q|}\right)_{xx} + b|q|^{2m} q \\ & = i\lambda\left(|q|^{2m} q\right)_x + i\mu\left(|q|^{2m}\right)_x q - i\{\lambda(2m+1) + 2m\mu\}|q|^{2m} q_x \\ & + i\gamma_4 \omega q_{xxx} - i\gamma_3 q_{xxt} - \gamma_4 q_{xxxt}, \end{aligned} \tag{9}$$

while Eq. (3) simplifies to:

$$\gamma_3 \omega \phi''(x) + \omega \phi(x) - b\phi^{2m+1}(x) = 0. \tag{10}$$

The above equation admits a single Lie point symmetry, namely $\partial / \partial x$. With the implementation of this symmetry, the following solution is yielded:

$$\phi(x) = \left[\frac{(m+1)\omega}{b}\right]^{\frac{1}{2m}} \operatorname{sech}^{\frac{1}{m}}\left(\frac{mx}{\sqrt{-\gamma_3}}\right). \tag{11}$$

Thus, the quiescent bright 1–soliton solution is given by:

$$q(x,t) = \left[\frac{(m+1)\omega}{b}\right]^{\frac{1}{2m}} \operatorname{sech}^{\frac{1}{m}}\left(\frac{mx}{\sqrt{-\gamma_3}}\right) e^{i\omega t}, \tag{12}$$

which remains valid for

$$\omega b > 0, \tag{13}$$

and

$$\gamma_3 < 0. \tag{14}$$

3. Generalized temporal evolution

The RKL equation with generalized temporal evolution reads:

$$\begin{aligned} & i\left(q^l\right)_t + a\left(|q|^n q^l\right)_{xx} + b|q|^{2m} q^l \\ & = i\lambda\left(|q|^{2m} q^l\right)_x + i\mu\left(|q|^{2m}\right)_x q^l + i\delta|q|^{2m}\left(q^l\right)_x - i\gamma_1\left(q^l\right)_{xxx} \\ & -\gamma_2\left(q^l\right)_{xxxx} - i\gamma_3\left(q^l\right)_{xxt} - \gamma_4\left(q^l\right)_{xxxt}. \end{aligned} \tag{15}$$

In Eq. (15), the parameter l represents the generalized temporal evolution. For $l = 1$, one recovers linear temporal evolution as given by Eq. (1). Applying the same transformation as given by Eq. (2), the real and imaginary components that come out of Eq. (15) are:

$$\begin{aligned} & a(l+n)(l+n-1)\{\phi'(x)\}\phi^{n+2}(x) + b\phi^{2m+4}(x) + l\omega\phi^4(x) \\ & + 4\gamma_2 l(l-1)\phi'\phi'''(x)\phi^2(x) + 6\gamma_2 l(l-1)(l-2)\phi(x)\{\phi'(x)\}^2\phi''(x) \\ & + \gamma_2 l(l-1)(l-2)(l-3)\{\phi'(x)\}^4 + \gamma_2 l\phi^{(iv)}(x)\phi^3(x) \\ & - \gamma_3\omega l^2(l-1)\{\phi'(x)\}^2\phi^2(x) + 3\gamma_2 l(l-1)\{\phi''(x)\}^2\phi^2(x) \\ & + a(l+n)\phi''(x)\phi^{n+3}(x) - \gamma_3\omega l^2\phi''(x)\phi^3(x) = 0, \end{aligned} \tag{16}$$

and

$$
\begin{aligned}
& l(\gamma_1+\gamma_4 l\omega) \\
& \times\left[\phi'''(x)\phi^2(x)+3(l-1)\phi''(x)\phi'(x)\phi(x)+(l-1)(l-2)\{\phi'(x)\}^3\right] \\
& -\{\delta l+(2m+l)\lambda+2m\mu\}\phi'(x)\phi^{2m+2}(x)=0.
\end{aligned} \tag{17}
$$

From Eqs. (16) and (17), the following constraints naturally emerge for integrability:

$$\gamma_1+\gamma_4 l\omega=0, \tag{18}$$

$$n+l=0, \tag{19}$$

$$\delta l+(2m+l)\lambda+2m\mu=0, \tag{20}$$

and Eq. (6). Based on these relations, the governing model (15) reformulates as:

$$
\begin{aligned}
& \frac{i}{(q^n)_t}+a\left(\frac{|q|^n}{q^n}\right)_{xx}+b\frac{|q|^{2m}}{q^n} \\
& =i\lambda\left(\frac{|q|^{2m}}{q^n}\right)_x+i\mu\frac{\left(|q|^{2m}\right)_x}{q^n} \\
& +i\left\{\frac{\lambda(2m-n)+2m\mu}{n}\right\}\frac{|q|^{2m}}{(q^n)_x}-\frac{i\gamma_4 n\omega}{(q^n)_{xxx}}-\frac{i\gamma_3}{(q^n)_{xxt}}-\frac{\gamma_4}{(q^n)_{xxxt}},
\end{aligned} \tag{21}
$$

while Eq. (16) simplifies to:

$$l^2\gamma_3\omega\left[\phi''(x)\phi(x)+(l-1)\{\phi'(x)\}^2\right]+l\omega\phi^2(x)-b\phi^{2m+2}(x)=0. \tag{22}$$

The above equation again admits a single Lie point symmetry, namely $\partial/\partial x$. Using this symmetry and performing the integration leads to its solution:

$$\phi(x)=\left[\frac{(m-n)\omega}{b}\right]^{\frac{1}{2m}}\operatorname{sech}^{\frac{1}{m}}\left(\frac{mx}{l\sqrt{-\gamma_3}}\right), \tag{23}$$

where the negative sign is discarded since the soliton would be pointing downward. Thus, the bright quiescent 1–soliton solution is given as:

$$q(x,t)=\left[\frac{(m-n)\omega}{b}\right]^{\frac{1}{2m}}\operatorname{sech}^{\frac{1}{m}}\left(\frac{mx}{l\sqrt{-\gamma_3}}\right)e^{i\omega t}. \tag{24}$$

This quiescent soliton solution remains valid for the same constraint conditions as Eqs. (13) and (14), and in addition to Eq. (13), one must have

$$m>n. \tag{25}$$

4. Conclusions

The paper identified bright quiescent optical solitons for the RKL equation with arbitrary intensity and 3OD and 4OD terms. The Lie symmetry analysis was the integration algorithm that made this retrieval possible. Unlike several other models, where only implicit quiescent optical solitons were recovered, the RKL model gave way to bright quiescent optical solitons [1–12]. Linear temporal evolution and generalized temporal evolution were considered. The parameter constraints for the existence of such solitons are also presented. The results of this paper are indeed very encouraging. Later, the model will be addressed with different forms of SPM for the RKL equation. Additionally, this study will combine several other models for different optoelectronic devices. The research results of such findings will be disseminated after aligning them with the pre–existing works [16–28].

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Анотація. *У цій статті продемонстрована можливість існування стаціонарних оптичні солітонів в моделі рівняння Радхакрішнана–Кунду–Лакшманана зі степеневим законом самофазової модуляції та нелінійною хроматичною дисперсією. Аналіз симетрії Лі приводить до появи стаціонарних світлих оптичних солітонних розв'язків при лінійній та узагальненій часовій еволюції. Встановлені параметричні обмеження для існування таких солітонів.*

Ключові слова: *світлі солітони, стаціонарні солітони, симетрія Лі*