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Sequel to "Stationary Optical Solitons with Nonlinear Chromatic Dispersion for Lakshmanan–Porsezian–Daniel Model Having Kerr Law of Nonlinear Refractive Index": Generalized Temporal Evolution

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Absract. The current paper studies the Lakshmanan–Porsezian–Daniel equation with nonlinear chromatic dispersion and Kerr law of self–phase modulation having generalized temporal evolution. The governing model is analyzed using Lie symmetry. The implicit solution is in terms of Appell hypergeometric function. The parameter constraints of the solutions are also enumerated.

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1. Introduction

One of the most important optoelectronics and quantum optics models is the Lakshmanan–Porsezian–Daniel (LPD) model [1, 2]. This equation has been extensively studied in various contexts. A modified version of this model was also studied when the chromatic dispersion (CD) was replaced by the third–order and fourth–order dispersion pair, and the solitons that emerged from this model were referred to as cubic–quartic solitons. In this context, the perturbed version of the cubic–quartic LPD model was analyzed using the semi-inverse variational principle [2]. The quiescent optical solitons, with nonlinear CD and Kerr law of self–phase modulation (SPM), were also addressed in 2021 [1]. However, the case with generalized temporal evolution was inadvertently omitted in the study of quiescent optical solitons. Only the linear temporal evolution was covered. The current paper, therefore, brings in the required closure. The Lie symmetry analysis is applied to the LPD model with generalized temporal evolution, and an explicit solution has been found. The parameter constraints are also enumerated for the solutions to exist. The details are jotted down in the subsequent section.

2. Generalized temporal evolution

The dimensionless form of the LPD equation with Kerr law nonlinearity having generalized temporal evolution and nonlinear chromatic dispersion is given as:

$$\begin{aligned} &i(q^l)_t + a\left(|q|^n q^l\right)_{xx} + b|q|^2 q^l = cq^l_{xxxx} + \alpha(q_x)^2(q^l)^* + \beta|q_x|^2 q^l \\ &+\gamma|q|^2(q^l)_{xx} + \delta q^2\left\{(q^l)^*\right\}_{xx} + \sigma|q|^4 q^l. \end{aligned} \tag{1}$$

In Eq. (1), the complex–valued function $q(x,t)$ represents the wave amplitude while the independent variables x and t account for the spatial and temporal co–ordinates, respectively. The generalized temporal evolution is characterized by the parameter l. If $l=1$, this model collapses to the regular version of LPD equation, with linear temporal evolution and Kerr law nonlinearity having nonlinear chromatic dispersion as it was studied during 2021. The first term is, therefore, the generalized temporal evolution, with its coefficient being $i=\sqrt{-1}$. The coefficient of a is the nonlinear chromatic dispersion, whereas n is the nonlinearity parameter. Next, the coefficient of b accounts for SPM that comes from Kerr's law of nonlinear refractive index change, c is real-valued constant, which yields the coefficient of higher-order dispersion, q_x is the inter-modal dispersion, q_{xx} is the chromatic dispersion, q_{xxxx} is the fourth–order dispersion. On the right-hand side, the terms are from the LPD model with the same physics [1].The coefficients of all terms for the model (1) are all real-valued constants.

In order to solve Eq. (1) for quiescent optical solitons, the following substitution is selected:

$$q(x,t) = \phi(x)\mathrm{e}^{i\lambda t}, \tag{2}$$

where $\phi(x)$ is the amplitude component of the waveform (2) and λ is the wave number of the soliton. Substituting Eq. (2) into Eq. (1) and decomposing into real and imaginary parts, the following pair of relations emerge:

$$\{\alpha + \delta l(l-1)\}\{\phi'(x)\}^2 + \delta l\phi(x)\phi''(x) = 0, \tag{3}$$

and

$$\begin{aligned} &a\{l^2 + l(2n-1) + n(n-1)\}\phi^{n+2}(x)\{\phi'(x)\}^2 + a(l+n)\phi^{n+3}(x)\phi''(x) + b\phi^6(x) \\ &-6cl(l-1)(l-2)\phi(x)\{\phi'(x)\}^2\phi''(x) - cl(l-1)(l-2)(l-3)\{\phi'(x)\}^4 \\ &-cl\phi^{(iv)}(x)\phi^3(x) - cl(l-1)\phi^2(x)\left[3\{\phi''(x)\}^2 + 4\phi'''(x)\phi'(x)\right] \\ &-\{\beta + \gamma l(l-1)\}\phi^4(x)\{\phi'(x)\}^2 - \gamma l\phi^5(x)\phi''(x) - \lambda l\phi^4(x) - \sigma\phi^8(x) = 0. \end{aligned} \tag{4}$$

For integrability, the following constraints must hold:

$$\alpha + \beta = 0, \tag{5}$$

$$\delta + \gamma = 0, \tag{6}$$

and

$$c = 0. \tag{7}$$

With these parameter constraints in place, the governing model (1) shrinks to:

$$\begin{aligned} &i(q^l)_t + a\left(|q|^n q^l\right)_{xx} + b|q|^2 q^l \\ &= \beta\left\{|q_x|^2 q^l - (q_x)^2(q^l)^*\right\} + \gamma\left[|q|^2(q^l)_{xx} - q^2\left\{(q^l)^*\right\}_{xx}\right] + \sigma|q|^4 q^l, \end{aligned} \tag{8}$$

and thus Eq. (4) collapses to:

$$a\left\{l^2+l(2n-1)+n(n-1)\right\}\phi^n(x)\left\{\phi'(x)\right\}^2+a(l+n)\phi^{n+1}(x)\phi''(x)$$
$$+b\phi^4(x)-\lambda l\phi^2(x)-\sigma\phi^6(x)=0. \tag{9}$$

The above Eq. (9) admits a single Lie point symmetry, namely $\partial/\partial x$.

This symmetry will be used in the integration process and it leads to the following implicit solution in terms of the Appell hypergeometric function of two variables:

$$x=\pm\frac{1}{n}\sqrt{\frac{2a(l+n)(2l+n)\phi^n}{l\lambda}}F_1\left(\frac{n}{4};\frac{1}{2},\frac{1}{2};\frac{4+n}{4};\frac{A_1}{bA_2-\sqrt{A_2A_3}},\frac{A_1}{bA_2+\sqrt{A_2A_3}}\right), \tag{10}$$

where

$$A_1=2\sigma\phi^2\left\{4l^2+4l(n+1)+n(n+2)\right\}, \tag{11}$$

$$A_2=4l^2+4l(n+2)+n(n+4), \tag{12}$$

and

$$A_3=A_2b^2-4\lambda l\sigma(2l+n+2)^2. \tag{13}$$

The Appell hypergeometric function of two variables:

$$F_1(a;b_1,b_2;c;x,y), \tag{14}$$

has a primary definition through the hypergeometric series:

$$x^m y^n\left(\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(a)_{m+n}(b_1)_m(b_2)_n}{(c)_{m+n}m!n!}\right), \tag{15}$$

where $(a)_m$ is the Pochhammer symbol and the series is convergent inside the region:

$$\max(|x|,|y|)<1. \tag{16}$$

The parametric restriction (16) translates to:

$$\max\left(\frac{A_1}{bA_2-\sqrt{A_2A_3}},\frac{A_1}{bA_2+\sqrt{A_2A_3}}\right)<1, \tag{17}$$

together with:

$$A_2A_3>0, \tag{18}$$

and

$$bA_2\neq\left|\sqrt{A_2A_3}\right|. \tag{19}$$

The Appell hypergeometric function (10) stands valid with the condition:

$$a\lambda>0. \tag{20}$$

3. Conclusions

The current paper addressed the LPD equation with nonlinear CD and the Kerr law of SPM with generalized temporal evolution. This yielded the implicit solution in terms of the Appell hypergeometric function. The results are consistent with those reported earlier with linear temporal evolution. Upon setting the generalized temporal evolution parameter to unity, it is observed that the results scale back to the previously reported results with linear temporal evolution. The results of the paper and the model serve as monumental prospects down the road. The model will be studied numerically later, and the corresponding bifurcation analysis will be carried out. Later, this model will be further analyzed for quiescent optical solitons and the power law of SPM. These results will be aligned with the pre-existing ones and reported [3–5].

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Анотація. *У цій статті досліджується рівняння Лакшманана–Порсезіана–Деніела з нелінійною хроматичною дисперсією та законом Керра самофазової модуляції, що має узагальнену часову еволюцію. Модель аналізується за допомогою симетрії Лі. Неявний розв'язок отримано в термінах гіпергеометричної функції Аппеля. Перераховані також обмеження параметрів рішень.*

Ключові слова: *стаціонарні солітони, симетрія Лі*