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Cubic-Quartic Optical Solitons with Kudryashov's Law of Self-Phase Modulation

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Abstract. This study aims to investigate cubic-quartic optical solitons with Kudryashov's law of self-phase modulation. Thus, the combination of third-order dispersion (3OD) and fourth-order dispersion (4OD) is assumed in the model to ensure the smooth existence of solitons. The study is implemented with the aid of two effective integration schemes known as the improved projective Riccati equations method and the soliton ansatz technique. The soliton solutions are derived based on two physical cases targeting the relation between 3OD and 4OD. In case 3OD is equivalent to fourfold frequency times 4OD, only dark and singular soliton profiles are extracted. However, if the former relation is not achieved, various structures of soliton pulses are generated, including kink-dark, singular, W-shaped, bright, dark, kink, and anti-kink solitons. The physical interpretations of retrieved optical solitons are represented by illustrating the wave behaviors with suitable values of model parameters. The results show that the combination of 3OD and 4OD has a significant effect on the dynamics of soliton propagation.

Keywords: optical solitons, cubic-quartic dispersion, Kudryashov's law, improved projective Riccati equations method, soliton ansatz
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1. Introduction

The study of data transmission through communication channels has received extensive attention as it possesses a wide range of applications in the industrial and engineering fields [1-4]. In the last three decades, soliton pulses have been used as potential carriers of an information signal in optical telecommunication systems [5-10]. However, soliton propagation along an optical fiber experiences some challenges, such as dispersion and attenuation [11-14]. Chromatic dispersion, for example, emerges due to the disparity in propagation velocity with wavelength, which brings about degraded signal quality and causes limitations in transmission distances and capacities [15-18]. Several effective mechanisms have been developed to escape the negative effect of chromatic dispersion, including the dispersion compensation method. Various dispersion compensation techniques are implemented, such as Bragg gratings dispersion, pure–cubic dispersion, pure–quartic dispersion, cubic–quartic (CQ) dispersion, etc [19-28]. The high intensity of propagating light leads to the emergence of nonlinear effects. One of these dominant effects in optical fiber communications systems is known as self-phase modulation (SPM), which occurs due to the change in the refractive index of the medium. In the literature, many authors worldwide deal with the effects of dispersion compensation technology on soliton pulses in the presence of distinct types of nonlinear laws of SPM. For more details, the reader is referred to references [29-37].

Kudryashov's law of SPM is a new novel structure arising from the nonlinear refractive index. The model including this nonlinear effect belongs to the nonlinear Schrödinger's equation (NLSE) family, and it can be known as NLSE with Kudryashov's proposed self-phase modulation. To diagnose the physical features of NLSE having this type of nonlinearity, numerous studies have been carried out in different fiber media such as Bragg gratings, birefringent fibers, and others [38-45]. For example, Zayed et al. [46] investigated chirped and chirp-free optical solitons in fiber BGs with dispersive reflectivity. Two types of soliton structures are detected, namely, dark and singular solitons. Recently, Al-Ghafri et al. [47] scrutinized the NLSE model in fiber BGs to examine the behaviors of chirped gap solitons, and they found distinct soliton profiles, including bright, dark, singular, W-shaped, kink, anti-kink, and Kink-dark solitons. Furthermore, the consequence of Kudryashov's law in birefringent fibers without four-wave mixing effects is discussed by Zayed et al. [48]. The results of their study revealed a variety of soliton structures, such as bright, dark, and singular solitons. Our current work sheds light on cubic-quartic nonlinear Schrodinger's equation (CQ–NLSE) with Kudryashov's proposed self-phase modulation. This means that both third-order dispersion (3OD) and fourth-order dispersion (4OD) are present in the model to compensate for a low count of chromatic dispersion.

The model of CQ–NLSE in the absence of group velocity dispersion is addressed as

$$iq_t + ia_1 q_{xxx} + a_2 q_{xxxx} + \left(\frac{b_1}{|q|^{2n}} + \frac{b_2}{|q|^n} + b_3 |q|^n + b_4 |q|^{2n} \right) q = 0, \tag{1}$$

where $q(x,t)$ is a complex-valued function indicating the wave profile, while the variables x and t denote the spatial and temporal coordinates, n - represents the power–law. In Eq. (1), the first term accounts for linear temporal evolution. The terms with real-valued coefficients a_1 and a_2 represent the 3OD and 4OD effects. The last four terms have the coefficients b_1, b_2, b_3, b_4 define Kudryashov's law that arises from nonlinear refractive index of an optical fiber and reflects essentially the influence of self-phase modulation in the medium. In the previous studies, for instance, Biswas et al. [49] investigated CQ solitons of the model (1) using the extended trial function method, and they obtained Jacobi's elliptic functions that degenerate to bright and singular optical solitons when the modulus of ellipticity reaches unity. By means of Lie symmetry analysis, the same model is discussed by two authors in [50]. Miscellaneous soliton profiles are secured, including dark, bright, singular and combo bright-singular solitons. Moreover, a group of scholars [51] utilized different forms of the F-expansion scheme which created solutions in terms of Weierstrass' elliptic functions and Jacobi's elliptic functions.

In addition to investigating CQ solitons, we examine the existence of pure bright soliton which is described by a single term of secant hyperbolic function. Two strategies, which are the improved projective Riccati equations method and the soliton ansatz technique, are employed to perform this study. The following sections of this paper are arranged as follows. In Section 2, the model of CQ–NLSE is analyzed and reduced to two integrable forms based on two assumptions discussing the relation between 3OD and 4OD. Section 3 describes the derivation of soliton solutions for the two discussed cases. In Section 4, the behaviors of optical solitons are displayed along with the physical interpretation. Finally, the conclusion of the work is given in Section 5.

2. Mathematical analysis of model

In order to derive the soliton solutions of CQ–NLSE defined by Eq. (1), we first attempt to analyze its complex form and convert it to a possibly integrable equation. Hence, we introduce the complex transformation of the form

$$q\left(x,t\right)=u\left(\xi\right)e^{i\psi\left(x,t\right)}, \tag{2}$$

where $u\left(\xi\right)$ accounts for the amplitude of the soliton wave while ξ is the wave variable given as $\xi=x-vt$. The function $\psi\left(x,t\right)$ stands for the phase component identified as $\psi\left(x,t\right)=-\kappa x+\omega t+\theta$. The parameters v, ω, κ and θ represent the soliton velocity, frequency, wavevector, and phase constant, respectively.

Substituting Eq. (2) into Eq. (1) results in two equations for real and imaginary parts given, respectively, as

$$\begin{aligned}&a_2u^{(4)}+3\omega\left(a_1-2a_2\omega\right)u''+\left(a_2\omega^4-a_1\omega^3-\kappa\right)u\\&+b_1u^{1-2n}+b_2u^{1-n}+b_3u^{n+1}+b_4u^{2n+1}=0,\end{aligned} \tag{3}$$

$$\left(v+3a_1\omega^2-4a_2\omega^3\right)u'-\left(a_1-4a_2\omega\right)u'''=0. \tag{4}$$

From Eq. (4), one can discuss two physical cases according to the second term that includes the relation between 3OD and 4OD in addition to frequency.

Case I. Considering that $a_1=4a_2\omega$, Eq. (4) yields the velocity of soliton as

$$v=-8a_2\omega^3. \tag{5}$$

To reach closed-form solutions, set

$$u\left(\xi\right)=P^{\frac{1}{n}}\left(\xi\right), \tag{6}$$

where $P=P\left(\xi\right)$ is the new dependent variable which reduces Eq. (3) to

$$\begin{aligned}&b_4n^4P^6+b_3n^4P^5-n^4\left(\omega+3a_2\omega^4\right)P^4+n^3\left(b_2n+6a_2\omega^2P''+a_2P^{(4)}\right)P^3\\&+\left\{b_1n^4-a_2n^2\left(n-1\right)\left(6\omega^2P'^2+4P'P'''+3P''^2\right)\right\}P^2\\&+a_2\left(n-1\right)\left(2n-1\right)\left(6nPP'^2P''-\left(3n-1\right)P'^4\right)=0.\end{aligned} \tag{7}$$

Case II. Considering that $a_1\neq 4a_2\omega$, Eq. (4) brings, after differentiating once

$$u^{(4)}=\frac{\left(v+3a_1\omega^2-4a_2\omega^3\right)u'}{\left(a_1-4a_2\omega\right)}, \tag{8}$$

from which Eq. (3) becomes

$$\begin{aligned}&b_1\left(a_1-4a_2\omega\right)u+b_2\left(a_1-4a_2\omega\right)u^{1+n}\\&+\left(a_1-4a_2\omega\right)\left(a_2\omega^4-a_1\omega^3-\kappa\right)u^{1+2n}\\&+b_3\left(a_1-4a_2\omega\right)u^{1+3n}+b_4\left(a_1-4a_2\omega\right)u^{1+4n}\\&+\left[3a_1^2\omega+20a_2^2\omega^3+a_2\left(v-15a_1\omega^2\right)\right]u^{2n}u''=0.\end{aligned} \tag{9}$$

As our aim is to create closed-form solutions, the transformation of Eq. (6) is applied to Eq. (9) to arrive at

$$\begin{aligned}&b_1n^2\left(a_1-4a_2\omega\right)+b_2n^2\left(a_1-4a_2\omega\right)P+n^2\left(a_1-4a_2\omega\right)\left(a_2\omega^4-a_1\omega^3-\kappa\right)P^2\\&+b_3n^2\left(a_1-4a_2\omega\right)P^3+b_4n^2\left(a_1-4a_2\omega\right)P^4\\&+\left[3a_1^2\omega+20a_2^2\omega^3+a_2\left(v-15a_1\omega^2\right)\right]\left(nPP''-\left(n-1\right)P'^2\right)=0.\end{aligned} \tag{10}$$

3. Retrieval of optical solitons

The target now is to derive the soliton solutions of CQ–NLSE model through obtaining the solutions of Eqs. (7) and (10) in the above two cases by using the improved projective Riccati equations method (IPRE). Assuming that these two equations have solutions in the form of a finite series as

$$U\left(\xi\right)=\alpha_0+\sum_{j=1}^{m}\left[\alpha_j f^j\left(\xi\right)+\beta_j g^j\left(\xi\right)\right], \tag{11}$$

where $\alpha_0,\alpha_j,\beta_j,\left(j=1,2,\ldots,m\right)$ are constants to be determined. The parameter m is a positive integer which can be identified by balancing the highest order derivative term with the nonlinear term in Eqs. (7) and (10).

The variables $f\left(\xi\right)$ and $g\left(\xi\right)$ satisfy the following improved projective Riccati equations

$$\begin{aligned}&f'\left(\xi\right)=\delta Ag^2\left(\xi\right),\qquad g'\left(\xi\right)=-Af\left(\xi\right)g\left(\xi\right)-\frac{B}{A}g\left(\xi\right)\left(R-Bf\left(\xi\right)\right),\\&g^2\left(\xi\right)=\delta\left[\frac{1}{A^2}\left(R-Bf\left(\xi\right)\right)^2-f^2\left(\xi\right)\right],\end{aligned} \tag{12}$$

where A,B and R are arbitrary constants and $\delta=\pm1$. The set of Eqs. (12) admits hyperbolic function solutions as well as trigonometric function solutions as mentioned in [52]. Herein, we only concentrate on hyperbolic function solutions from which soliton-type solutions are induced.

3.1. Solutions for Case I $(a_1=4a_2\omega)$

Balancing the terms of $P^{(4)}P^3$ and P^6 in Eq. (7) leads to $m=2$. Thus, the general solution (11) collapses to

$$P\left(\xi\right)=\alpha_0+\alpha_1 f\left(\xi\right)+\alpha_2 f^2\left(\xi\right)+\beta_1 g\left(\xi\right)+\beta_2 g^2\left(\xi\right). \tag{13}$$

Inserting Eq. (13) together with Eqs. (12) into Eq. (7) forms a polynomial having terms with $f^l g^s, (s=0,1;\ l=0,1,2,\ldots)$. Combining terms with the same power of $f^l g^s$ and equating them to zero, a system of algebraic equations is created. This system gives the following distinct solutions for the discussed model (1).

Set 1.

$$\begin{aligned}
&\alpha_1=\frac{2\alpha_0(A^2-B^2)}{BR},\ \alpha_1=\frac{\alpha_0(A^2-B^2)^2}{B^2R^2},\ \beta_1=\beta_2=0,\\
&b_1=\frac{4(n-1)(n-2)(3n-2)R^4A^4\alpha_0^2a_2}{B^4n^4},\\
&b_2=\frac{4(n-2)\left(3\omega^2n^2-4R^2(n^2-2n+2)\right)R^2A^2\alpha_0a_2}{B^2n^4},\\
&b_3=-\frac{4(n+2)\left(3\omega^2n^2-4R^2(n^2+2n+2)\right)R^2B^2a_2}{\alpha_0A^2n^4},\\
&b_4=-\frac{4(n+1)(n+2)(3n+2)R^4B^4a_2}{\alpha_0^2A^4n^4},\\
&\omega=\frac{\left(R^4(40n^2+96)-3\omega^2n^2(k^2n^2+16R^2)\right)a_2}{n^4}.
\end{aligned}\tag{14}$$

From Eqs. (14) along with (13), one can find soliton solutions to Eq. (1), given by

$$q(x,t)=\left\{\frac{\alpha_0A^2(B+A\tanh\left[R\left(x-vt\right)\right])^2}{B^2(A+B\tanh\left[R\left(x-vt\right)\right])^2}\right\}^{\frac{1}{n}}e^{i(-\kappa x+\omega t+\theta)},\tag{15}$$

and

$$q(x,t)=\left\{\frac{\alpha_0A^2(B+A\coth\left[R\left(x-vt\right)\right])^2}{B^2(A+B\coth\left[R\left(x-vt\right)\right])^2}\right\}^{\frac{1}{n}}e^{i(-\kappa x+\omega t+\theta)},\tag{16}$$

where $A\neq B$ and $A,B\neq 0$.

Set 2.

$$\begin{aligned}
&\beta_2=-\frac{\delta\alpha_0(A^2-B^2)}{R^2},\ \alpha_1=\alpha_2=\beta_1=0,\\
&b_1=\frac{4(n-1)(n-2)(3n-2)R^4A^4\alpha_0^2a_2}{n^4},\\
&b_2=\frac{4(n-2)\left(3\kappa^2n^2-4R^2(n^2-2n+2)\right)R^2\alpha_0a_2}{n^4},\\
&b_3=-\frac{4(n+2)\left(3\kappa^2n^2-4R^2(n^2+2n+2)\right)R^2a_2}{\alpha_0n^4},\\
&b_4=-\frac{4(n+1)(n+2)(3n+2)R^4a_2}{\alpha_0^2n^4},\\
&\omega=\frac{\left((40n^2+96)R^4-3k^2n^2(k^2n^2+16R^2)\right)a_2}{n^4}.
\end{aligned}\tag{17}$$

Making use of Eqs. (17) with (13), the soliton solutions to Eq. (1) are generated in the form

$$q(x,t)=\left\{\alpha_0-\frac{\alpha_0\left(A^2-B^2\right)\operatorname{sech}^2\left[R\left(x-vt\right)\right]}{(A+B\tanh\left[R\left(x-vt\right)\right])^2}\right\}^{\frac{1}{n}}e^{i\left(-\kappa x+\omega t+\theta\right)}, \tag{18}$$

and

$$q(x,t)=\left\{\alpha_0+\frac{\alpha_0\left(A^2-B^2\right)\operatorname{csch}^2\left[R\left(x-vt\right)\right]}{(A+B\coth\left[R\left(x-vt\right)\right])^2}\right\}^{\frac{1}{n}}e^{i\left(-\kappa x+\omega t+\theta\right)}, \tag{19}$$

where $A^2\neq B^2$.

Set 3.

$$\begin{aligned}
&\alpha_2=\frac{\alpha_1(A^2-B^2)}{2BR},\ \beta_2=\frac{\delta[\alpha_1BR-2\alpha_0(A^2-B^2)]}{2R^2},\ \beta_1=0,\\
&b_1=\frac{(n-1)(n-2)(3n-2)(2B\alpha_0+R\alpha_1)^2R^4a_2}{B^2n^4},\\
&b_2=\frac{2(n-2)\left(3\omega^2n^2-4R^2(n^2-2n+2)\right)(2B\alpha_0+R\alpha_1)R^2a_2}{Bn^4},\\
&b_3=-\frac{8(n+2)\left(3\omega^2n^2-4R^2(n^2+2n+2)\right)BR^2a_2}{n^4(2B\alpha_0+R\alpha_1)},\\
&b_4=-\frac{16(n+1)(n+2)(3n+2)B^2R^4a_2}{n^4(2B\alpha_0+R\alpha_1)^2},\\
&\omega=\frac{\left((40n^2+96)R^4-3\omega^2n^2(k^2n^2+16R^2)\right)a_2}{n^4}.
\end{aligned} \tag{20}$$

Applying Eqs. (20) to (13), this process is conducive to soliton solutions to Eq. (1)

$$q(x,t)=\left\{\frac{\left(2\alpha_0B+\alpha_1R\right)(B+A\tanh\left[R\left(x-vt\right)\right])^2}{2B(A+B\tanh\left[R\left(x-vt\right)\right])^2}\right\}^{\frac{1}{n}}e^{i\left(-\kappa x+\omega t+\theta\right)}, \tag{21}$$

and

$$q(x,t)=\left\{\frac{\left(2\alpha_0B+\alpha_1R\right)(B+A\coth\left[R\left(x-vt\right)\right])^2}{2B(A+B\coth\left[R\left(x-vt\right)\right])^2}\right\}^{\frac{1}{n}}e^{i\left(-\kappa x+\omega t+\theta\right)}, \tag{22}$$

where $B\neq 0$, $A\neq B$ and $2\alpha_0B\neq-\alpha_1R$.

3.2. Solutions for Case II ($a_1\neq 4a_2\omega$)

The soliton solutions of Eq. (1) when $a_1\neq 4a_2\omega$ are derived by two schemes, which are the IPRE method and the soliton ansatz technique.

3.2.1. Solution by IPRE method

The balance between the terms with PP'' and P^4 in Eq. (10) induces $m=1$. Accordingly, the general solution (11) becomes

$$P(\xi)=\alpha_0+\alpha_1f(\xi)+\beta_1g(\xi). \tag{23}$$

Similarly, the substitution of Eq. (23) in company with (12) into Eq. (10) produces a polynomial in $f^l g^s, (s=0,1;\quad l=0,1,2,\ldots)$. Collecting the coefficients of terms with the same order of $f^l g^s$ and equating them to zero yields a set of algebraic equations. By solving these equations simultaneously, one can produce the following results.

Set 1.

$$\begin{aligned}
&\alpha_1=\frac{(A^2-B^2)\left(b_3(n+1)+2\alpha_0 b_4(n+2)\right)}{2BRb_4(n+2)},\ \beta_1=0,\\
&b_1=-\frac{(n-1)\left[\left(2\alpha_0 Ab_4(n+2)+Ab_3(n+1)\right)^2-B^2b_3^2(n+1)^2\right]^2}{16B^4b_4^3(n+2)^4(n+1)},\\
&b_2=\frac{b_3(n-2)\left[\left(2\alpha_0 Ab_4(n+2)+Ab_3(n+1)\right)^2-B^2b_3^2(n+1)^2\right]}{4B^2b_4^2(n+2)^3},\\
&\omega=\frac{2\alpha_0^2A^2b_4}{B^2(n+1)}+\frac{2\alpha_0A^2b_3-B^2\omega^3(n+2)(a_1-a_2\omega)}{B^2(n+2)}+\frac{b_3^2(n+1)(A^2-3B^2)}{2B^2b_4(n+2)^2},\\
&\nu=-\left[\frac{n^2A^2(a_1-4a_2\omega)(b_3(n+1)+2\alpha_0b_4(n+2))^2}{4B^2R^2a_2b_4(n+1)(n+2)^2}+\frac{\kappa(20a_2^2\omega^2-15a_1a_2\kappa+3a_1^2)}{a_2}\right].
\end{aligned}\tag{24}$$

Employing these findings to Eq. (23), one can secure soliton solutions to Eq. (1) presented as

$$q(x,t)=\left\{\frac{\alpha_0A\left(B+A\tanh\left[R(x-\nu t)\right]\right)}{B\left(A+B\tanh\left[R(x-\nu t)\right]\right)}+\frac{b_3\left(A^2-B^2\right)(n+1)\tanh\left[R(x-\nu t)\right]}{2Bb_4(n+2)\left(A+B\tanh\left[R(x-\nu t)\right]\right)}\right\}^{\frac{1}{n}}\times e^{i(-\kappa x+\omega t+\theta)},\tag{25}$$

and

$$q(x,t)=\left\{\frac{\alpha_0A\left(B+A\coth\left[R(x-\nu t)\right]\right)}{B\left(A+B\coth\left[R(x-\nu t)\right]\right)}+\frac{b_3\left(A^2-B^2\right)(n+1)\coth\left[R(x-\nu t)\right]}{2Bb_4(n+2)\left(A+B\coth\left[R(x-\nu t)\right]\right)}\right\}^{\frac{1}{n}}\times e^{i(-\kappa x+\omega t+\theta)},\tag{26}$$

where $A,B\neq 0$, $A\neq\pm B$ and $n\neq\{-1,-2\}$.

Set 2.

$$\begin{aligned}
&\alpha_0=-\frac{b_3(n+1)}{2b_4(n+2)},\ \alpha_1=0,\\
&b_1=\frac{b_3^2(n^2-1)\left[4\beta_1^2b_4^2R^2(n+2)^2-\delta b_3^2(A^2-B^2)(n+1)^2\right]}{16\delta b_4^3(A^2-B^2)(n+2)^4},\\
&b_2=\frac{b_3(n-2)\left[2\beta_1^2b_4^2R^2(n+2)^2-\delta b_3^2(A^2-B^2)(n+1)^2\right]}{4\delta b_4^2(A^2-B^2)(n+2)^3},\\
&\omega=\frac{\delta\beta_1^2R^2b_4}{(A^2-B^2)(n+1)}-\omega^3(a_1-a_2\omega)-\frac{3b_3^2(n+1)}{2b_4(n+2)^2},\\
&\nu=\frac{\delta n^2\beta_1^2b_4(a_1-4a_2\omega)}{a_2(A^2-B^2)(n+1)}-\frac{\kappa(20a_2^2\omega^2-15a_1a_2\omega+3a_1^2)}{a_2}.
\end{aligned}\tag{27}$$

Inserting Eqs. (27) into (23) yields soliton solutions to Eq. (1) in the form

$$q(x,t)=\left\{\frac{\beta_1 R\operatorname{sech}\left[R(x-vt)\right]}{A+B\tanh\left[R(x-vt)\right]}-\frac{b_3(n+1)}{2b_4(n+2)}\right\}^{\frac{1}{n}}e^{i(-\kappa x+\omega t+\theta)}, \tag{28}$$

and

$$q(x,t)=\left\{\frac{\beta_1 R\operatorname{csch}\left[R(x-vt)\right]}{A+B\coth\left[R(x-vt)\right]}-\frac{b_3(n+1)}{2b_4(n+2)}\right\}^{\frac{1}{n}}e^{i(-\kappa x+\omega t+\theta)}, \tag{29}$$

where $A\neq\pm B$ and $n\neq\{-1,-2\}$.

Set 3.

$$\begin{aligned}
&\alpha_0=-\frac{b_3(A^2-B^2)(n+1)-2\alpha_1 b_4 BR(n+2)}{2b_4(A^2-B^2)(n+2)},\ \beta_1=\alpha_1 A\sqrt{\frac{-\delta}{A^2-B^2}},\\
&b_1=-\frac{(n-1)\left[4\alpha_1^2 b_4^2 A^2R^2(n+2)^2-b_3^2(A^2-B^2)^2(n+1)^2\right]^2}{16b_4^3(A^2-B^2)^4(n+1)(n+2)^4},\\
&b_2=\frac{b_3(n-2)\left[4\alpha_1^2 b_4^2 A^2R^2(n+2)^2-b_3^2(A^2-B^2)^2(n+1)^2\right]}{4b_4^2(A^2-B^2)^2(n+2)^3},\\
&\omega=\frac{2\alpha_1^2A^2R^2b_4}{(A^2-B^2)^2(n+1)}-\kappa^3(a_1-a_2\omega)-\frac{3b_3^2(n+1)}{2b_4(n+2)^2},\\
&v=-\left[\frac{4n^2A^2\alpha_1^2 b_4(a_1-4a_2\omega)}{a_2(A^2-B^2)^2(n+1)}+\frac{\kappa(20a_2^2\omega^2-15a_1a_2\omega+3a_1^2)}{a_2}\right].
\end{aligned} \tag{30}$$

Using these outcomes with Eq. (23) provides soliton solutions to Eq. (1) as

$$q(x,t)=\left\{\frac{\alpha_1 AR\left(B+A\tanh\left[R(x-vt)\right]+\sqrt{B^2-A^2}\operatorname{sech}\left[R(x-vt)\right]\right)}{\left(A^2-B^2\right)\left(A+B\tanh\left[R(x-vt)\right]\right)}-\frac{b_3(n+1)}{2b_4(n+2)}\right\}^{\frac{1}{n}} \times e^{i(-\kappa x+\omega t+\theta)}, \tag{31}$$

and

$$q(x,t)=\left\{\frac{\alpha_1 AR\left(B+A\coth\left[R(x-vt)\right]+\sqrt{A^2-B^2}\operatorname{csch}\left[R(x-vt)\right]\right)}{\left(A^2-B^2\right)\left(A+B\coth\left[R(x-vt)\right]\right)}-\frac{b_3(n+1)}{2b_4(n+2)}\right\}^{\frac{1}{n}} \times e^{i(-\kappa x+\omega t+\theta)}, \tag{32}$$

where $A\neq 0$, $A\neq\pm B$ and $n\neq\{-1,-2\}$.

3.2.2. Solution by soliton ansatz

The soliton ansatz technique is implemented in Eq. (10) to extract optical solitons of Eq. (1). Suppose that Eq. (10) has a solution given by

$$P(\xi)=\eta_0+\frac{\eta_1\operatorname{sech}^2(\rho\xi)}{4-\left[1-\tanh(\rho\xi)\right]^2}+\frac{\eta_2\operatorname{sech}^4(\rho\xi)}{\left(4-\left[1-\tanh(\rho\xi)\right]^2\right)^2}, \tag{33}$$

where η_0, η_1, η_2 and ρ are constants to be determined. Upon substituting ansatz (33) into Eq. (10), an equation in $\operatorname{sech}(\rho\xi)\tanh(\rho\xi)$ of different orders is obtained. Equating all coefficients having the same order of $\operatorname{sech}(\rho\xi)\tanh(\rho\xi)$ to zero creates a system of algebraic equations that gives the following solutions.

Set 1.

$$\begin{gathered}\eta_2 = 0, \quad b_1 = \frac{4\mu\eta_0^2\rho^2(n-1)(2\eta_0+\eta_1)^2}{n^2\eta_1^2(a_1-4a_2\kappa)}, b_2 = -\frac{4\mu\eta_0\rho^2(n-2)(2\eta_0+\eta_1)(4\eta_0+\eta_1)}{n^2\eta_1^2(a_1-4a_2\kappa)}, \\ b_3 = \frac{8\mu\rho^2(n+2)(4\eta_0+\eta_1)}{n^2\eta_1^2(a_1-4a_2\kappa)}, \quad b_4 = -\frac{16\mu\rho^2(n+1)}{n^2\eta_1^2(a_1-4a_2\kappa)}, \\ \omega = \frac{4\mu\rho^2(24\eta_0^2+12\eta_0\eta_1+\eta_1^2)-n^2\eta_1^2\kappa^3(a_1-a_2\kappa)(a_1-4a_2\kappa)}{n^2\eta_1^2(a_1-4a_2\kappa)},\end{gathered} \tag{34}$$

where

$$\mu = 20a_2^2\omega^3 - 15a_1a_2\omega^2 + 3a_1^2\omega + a_2 v. \tag{35}$$

By virtue of Eqs. (34) together with (33), Eq. (1) possesses the optical soliton of the form

$$q(x,t) = \left\{\eta_0 + \frac{\eta_1 \operatorname{sech}^2[\rho(x-vt)]}{4-[1-\tanh[\rho(x-vt)]]^2}\right\}^{\frac{1}{n}} e^{i(-\kappa x+\omega t+\theta)}. \tag{36}$$

Set 2.

$$\begin{gathered}\eta_2 = -2\eta_1, \quad b_1 = \frac{4\mu\eta_0^2\rho^2(n-1)(8\eta_0+\eta_1)^2}{n^2\eta_1(a_1-4a_2\omega)}, b_2 = -\frac{4\mu\eta_0\rho^2(n-2)(12\eta_0+\eta_1)}{n^2\eta_1(a_1-4a_2\omega)}, \\ b_3 = \frac{16\mu\rho^2(n+2)}{n^2\eta_1(a_1-4a_2\omega)}, \quad b_4 = 0, \quad \omega = \frac{4\mu\rho^2(24\eta_0+\eta_1)-n^2\eta_1\omega^3(a_1-a_2\omega)(a_1-4a_2\omega)}{n^2\eta_1(a_1-4a_2\omega)},\end{gathered} \tag{37}$$

where μ is the same as in Eq. (35). Employing Eqs. (37) along with (33), one can reach an optical soliton to Eq. (1) as

$$q(x,t) = \left\{\eta_0 + \frac{\eta_1 \operatorname{sech}^2[\rho(x-vt)]}{4-[1-\tanh[\rho(x-vt)]]^2} - \frac{2\eta_1 \operatorname{sech}^4[\rho(x-vt)]}{\left(4-[1-\tanh[\rho(x-vt)]]^2\right)^2}\right\}^{\frac{1}{n}} \times e^{i(-\kappa x+\omega t+\theta)}. \tag{38}$$

4. Results and discussion

The section above discussed the optical solitons of CQ–NLSE defined by Eq. (1) by considering two assumptions, which are $a_1 = 4a_2\omega$ and $a_1 \neq 4a_2\omega$. Upon exploiting the IPRE method and soliton ansatz technique, various wave structures are constructed, including bright, dark, singular, W-shaped, kink, and kink-dark solitons. In comparison, the types of extracted solutions are distinct from the ones obtained in the previous studies [49-51]. To give a clear view of the behavior of optical solitons, the derived solutions are illustrated graphically by plotting the intensity profiles of solitons by choosing suitable values for the physical parameters. The undermined powers in Kudryashov's law of refractive index are examined with the values $n = \{-2, -1, 1, 2\}$ to give thorough insight into the dynamic evolutions of optical solitons.

Figs. 1-3 describes the intensity profiles of three types of dark and singular solitons for solutions of the Eqs. (15), (18) and (21) with the values of parameters given as $\omega = R = 0.5$, $a_2 = -0.5$ where $\alpha_0 = 0.5$; $\alpha_0 = 1$; $\alpha_0 = \alpha_1 = 1$, respectively. The changes in the values of n, A and B result in the two mentioned wave structures where $A = 2, B = 1$ leads to the dark soliton when $n = 1, 2$ and singular soliton when $n = -1, -2$ as shown in Figs. 1-3 (a) & (c) while

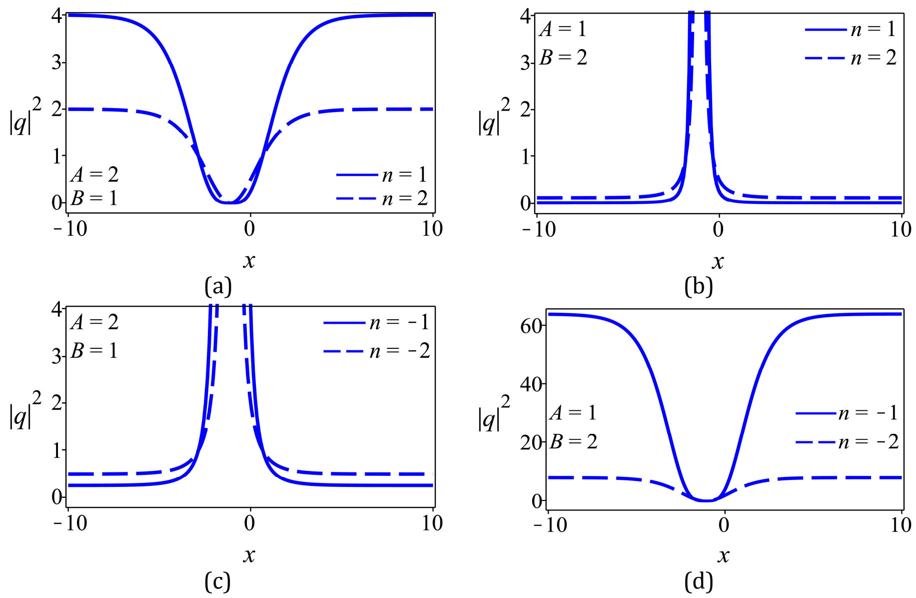

Fig. 1. Soliton intensity for solution of Eq. (15) with parameter values $\alpha_0 = \omega = R = 0.5$, $a_2 = -0.5$: (a), (d) - dark soliton; (b), (c) - singular soliton.

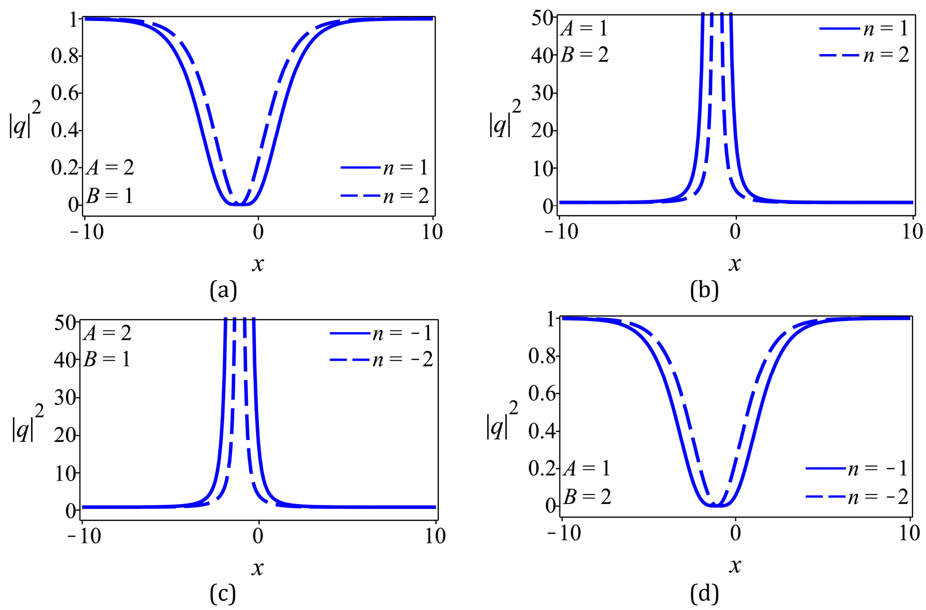

Fig. 2. Soliton intensity for the solution of Eq. (18) with parameter values $\alpha_0 = 1$, $\omega = R = 0.5$, $a_2 = -0.5$: (a), (d) - dark soliton; (b), (c) - singular soliton.

$A=1, B=2$ yields the singular soliton when $n=1,2$ and dark soliton when $n=-1,-2$ as exhibited in Figs. 1-3 (b,d). Fig. 4 displays the intensity profiles of kink-dark and singular solitons for the solution of Eq. (25) with the values of parameters given as $\alpha_0=\omega=0.5, a_1=a_2=-0.5, R=b_4=2, b_3=1, n=1,2$. The values $A=2$ and $B=1$ brings about the kink-dark wave, as demonstrated in Fig. 4 (a,b), whereas $A=1$ and $B=2$ induce

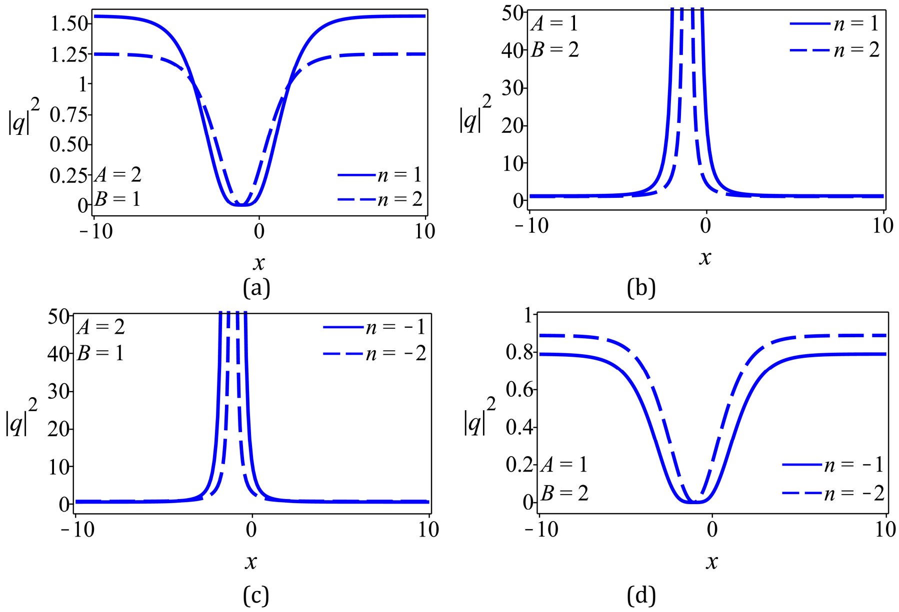

Fig. 3. Soliton intensity for the solution of Eq. (21) with parameter values $\alpha_0=\alpha_1=1,\ \omega=R=0.5,\ a_2=-0.5$: (a), (d) - dark soliton; (b), (c) - singular soliton.

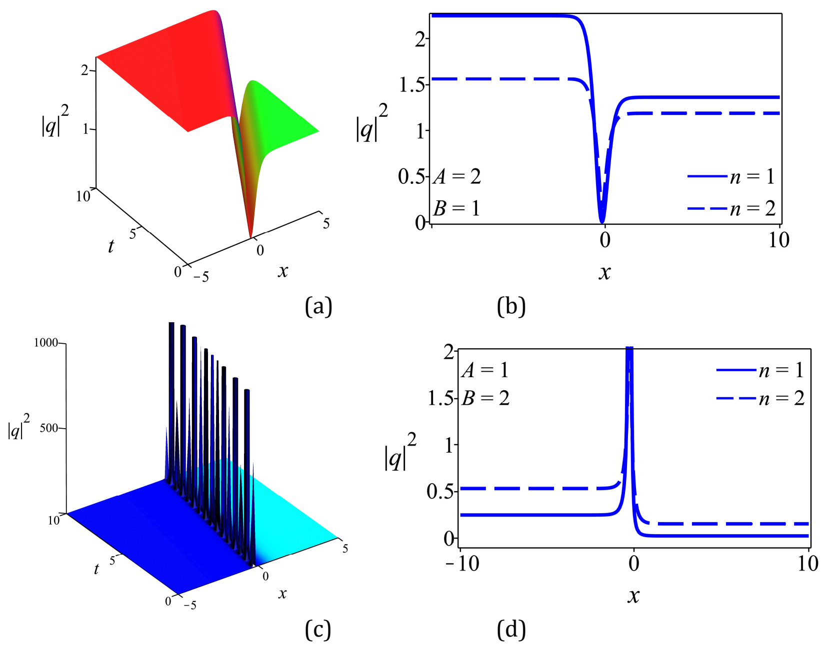

Fig. 4. Soliton intensity for the solution of Eq. (25) with parameter values $\alpha_0=\omega=0.5$, $a_1=a_2=-0.5,\ R=b_4=2,\ b_3=1$: (a), (b) - kink-dark soliton; (c), (d) - singular soliton.

the singular soliton as presented in Fig. 4 (c,d). Fig. 5 illustrates the intensity profiles of a W-shaped, and a bright solitons for the solution of Eq. (28) with the values of parameters given as $\omega = 0.5,\ a_1 = a_2 = -0.5,\ A = R = 2,\ B = b_3 = 1,\ b_4 = -1$. Herein, the variation of parameter β_1 affects the wave behavior, which generates a W-shaped soliton for $\beta_1 = -0.5$, as shown in Fig. 5 (a,b), and a bright soliton for $\beta_1 = 0.5$ as exhibited in Fig. 5 (c,d). Fig. 6 characterizes the intensity profiles of Kink and anti-Kink waves for the solution of Eq. (36) with the values

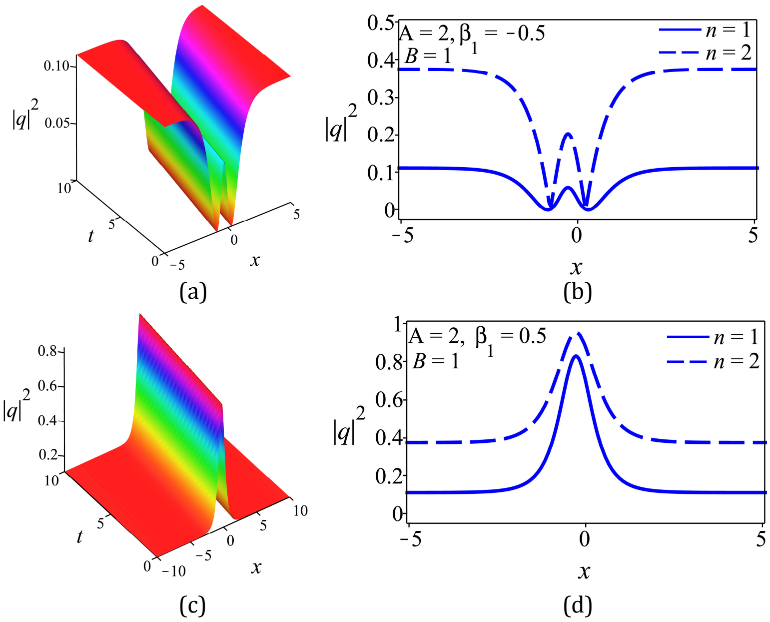

Fig. 5. Soliton intensity for the solution of Eq. (28) with parameter values $\omega = 0.5,\ a_1 = a_2 = -0.5,\ A = R = 2,\ B = b_3 = 1,\ b_4 = -1$: (a), (b) - W-shaped soliton; (c), (d) - bright soliton.

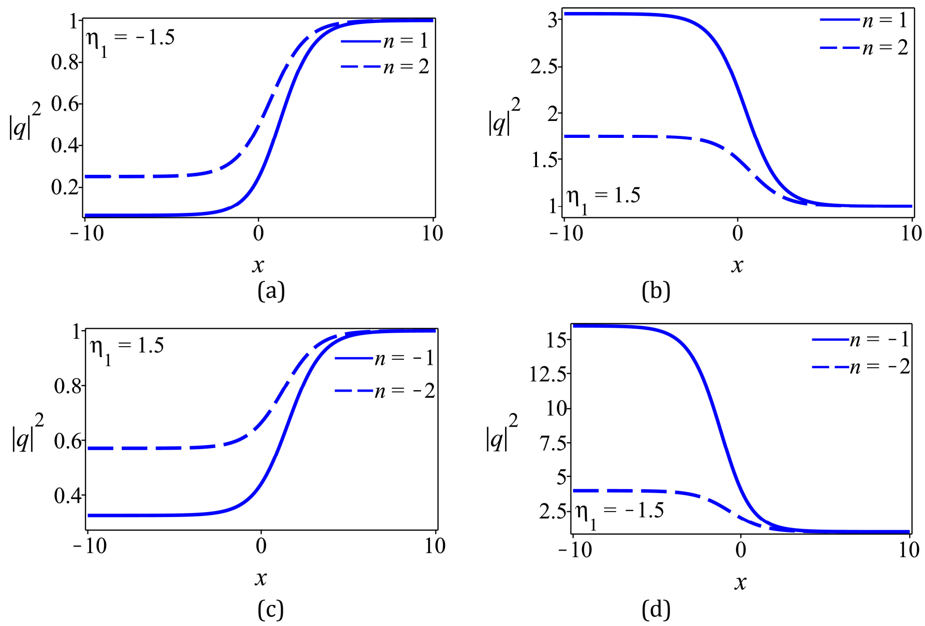

Fig. 6. Soliton intensity for the solution of Eq. (36) with parameter values $\eta_0 = b_3 = b_4 = 1,\ \omega = \nu = \rho = a_1 = 0.5,\ a_2 = -0.5$: (a), (c) - kink soliton; (b), (d) - anti-kink soliton.

of parameters given as $\eta_0 = b_3 = b_4 = 1,\ \omega = \nu = \rho = a_1 = 0.5, a_2 = -0.5$. The kink soliton is shown in Fig. 6 (a) & (c) when $\eta_1 = -1.5, n = 1,2$ and $\eta_1 = 1.5,\ n = -1,-2$, respectively, while the anti-kink soliton is plotted in Fig. 6 (b) & (d) when $\eta_1 = 1.5,\ n = 1,2$ and $\eta_1 = -1.5,\ n = -1,-2$, respectively. Finally, Fig. 7 represents the intensity profiles of bright and dark solitons for solution of the Eq. (38) with the values of parameters given as $\eta_0 = 1,\ \omega = \nu = \rho = a_1 = 0.5,\ a_2 = -0.5$. The bright soliton is delineated in Fig. 7 (a) & (d) when $\eta_1 = 1.5,\ n = 1, 2$ and $\eta_1 = -1.5,\ n = -1, -2$ respectively, while the dark soliton is plotted in Fig. 7 (b) & (c) when $\eta_1 = -1.5,\ n = 1, 2$ and $\eta_1 = 1.5,\ n = -1, -2$ respectively.

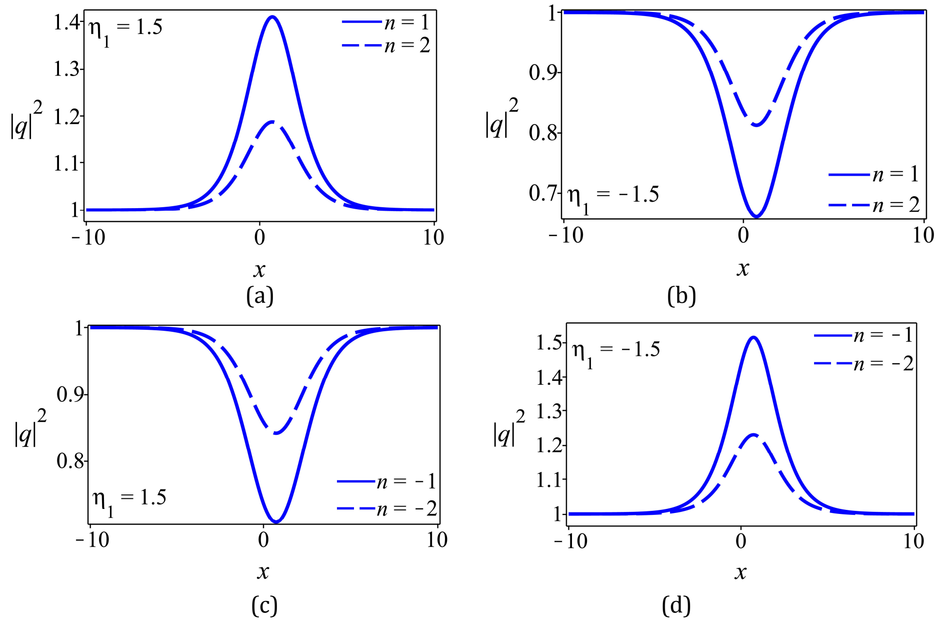

Fig. 7. Soliton intensity for the solution of Eq. (38) with parameter values $\eta_0 = 1$, $\omega = \nu = \rho = a_1 = 0.5, a_2 = -0.5$: (a), (d) - bright soliton; (b), (c) - dark soliton.

According to the derived solutions and their physical interpretations, assumption $a_1 = 4a_2\omega$ has provided two wave solutions only, namely, dark and singular solitons to the model of CQ–NLSE using IPRE method while soliton ansatz technique could not generate any solution. Contrarily, in the case of $a_1 \neq 4a_2\omega$ the IPRE scheme has given rise to four types of soliton structures, including kink-dark, singular, W-shaped, and bright solitons. Additionally, the implementation of the soliton ansatz technique, in the latter case, has created attractive soliton profiles containing kink, anti-kink, bright and dark solitons.

It is worth mentioning that although the mathematical tools applied in the current and previous studies have yielded distinct types of bright solitons, they couldn't extract the pure bright soliton, which is presented by a single term in $\operatorname{sech}(\xi)$.

5. Conclusion

In this study, we have dealt with the dynamics of solitons in polarization-preserving fibers dominated by Kudryashov's refractive index law. The cubic-quartic optical solitons are discussed based on a relation combining the effects of 3OD, 4OD, and frequency. Using the

IPRE scheme, only two types of solitons are secured, including dark and singular solitons, when considering 3OD is proportional to fourfold 4OD frequency. If the former relation is unrealized, miscellaneous soliton waves are created, such as kink-dark, singular, W-shaped, bright, dark, kink, and anti-kink solitons. The behaviors of optical solitons undergo remarkable evolutions due to the variations of model parameters. The current results are significant and can contribute to improving the field of optoelectronics.

In future work, the discussed model can be extended to include perturbation terms of Hamiltonian type. Various physical features can be revealed due to the presence of self–steepening terms, higher–order dispersion, and nonlinear dispersion. Soliton propagation will be investigated by means of effective integration schemes. Further to this, the modulation instability (MI) of the model can be examined in addition to deriving the expression of the MI gain spectrum.

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Анотація. *Метою цього дослідження є вивчення кубічно-квартичних оптичних солітонів з використанням закону Кудряшова щодо самомодуляції фази. Для забезпечення неперервного існування солітонів в моделі передбачено комбінацію дисперсії третього (3OD) і четвертого (4OD) порядків. Дослідження проводиться за допомогою двох ефективних методів інтегрування, відомих як метод покращених проективних рівнянь Ріккаті та техніки анзацу солітона. Рішення солітонів, отримані на основі двох фізичних випадків, спрямованих на встановлення співідношення між 3OD і 4OD. У випадку, коли 3OD дорівнює чотирикратному значенню хвильового вектора 4OD, отримуються лише темні та сингулярні профілі солітонів. Однак, якщо це співвідношення не виконується,тоді генеруються різні структури солітонних імпульсів, включаючи кінк-темні, сингулярні, W-подібні, яскраві, темні, кінк та антикінк солітони. Фізичні інтерпретації отриманих оптичних солітонів представлені шляхом ілюстрації хвильової поведінки при певних значеннях параметрів моделі. Результати показують, що поєднання 3OD і 4OD має значний вплив на динаміку поширення солітонів.*

Ключові слова: *оптичні солітони, кубічно-квартова дисперсія, закон Кудряшова, вдосконалений метод проективних рівнянь Ріккаті, солітонний анзац.*