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Implicit Quiescent Optical Solitons for Complex Ginzburg–Landau Equation with Generalized Quadratic–Cubic Form of Self–Phase Modulation and Nonlinear Chromatic Dispersion by Lie Symmetry

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Abstract. This work is on the retrieval of quiescent optical solitons for the complex Ginzburg–Landau equation that is with nonlinear chromatic dispersion and generalized structure of quadratic–cubic form of self–phase modulation. The Lie symmetry is applied to make this retrieval possible. The model is studied with linear temporal evolutions as well as generalized temporal evolution.

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1. Introduction

The study of quiescent optical solitons with nonlinear chromatic dispersion (CD) for a variety of models has gained undivided attention for the past couple of decades. There are several approaches that have been implemented for the retrieval of such solitons apart from Lie symmetry and these are visible across a wide variety of reported results. Various models from optoelectronics have been studied in this context. These are the nonlinear Schrödinger's equation (NLSE), complex Ginzburg–Landau equation (CGLE), Lakshmanan–Porsezian–Daniel equation, NLSE with Kudryashov's form of self–phase modulation (SPM), concatenation model, the dispersive concatenation model, and many others. The results are visible all across the journals [1–13].

The current paper is a re-visitation of the quiescent optical solitons for the CGLE that has been addressed with several forms of non–Kerr laws of SPM. Incidentally, this work studies the form of a generalized quadratic–cubic structure that was inadvertently omitted in the past [8]. The current paper, therefore, bridges the gap. This work deals with the retrieval of quiescent optical solitons for the CGLE with nonlinear CD and having the generalized form of quadratic–cubic structure of SPM. The Lie symmetry approach will enlighten this derivation process. The model will be studied with linear temporal evolution as well as with generalized temporal evolution. In both cases, implicit forms of the quiescent optical solitons will emerge, and they are listed along with the relevant parameter constraints that are also enumerated. The details of the derivation follow through.

2. Linear temporal evolution

The dimensionless form of the complex Ginzburg–Landau equation (CGLE) with linear temporal evolution and nonlinear CD and having the generalized form of quadratic–cubic law of SPM is given as:

$$\begin{aligned} & iq_t + a\left(|q|^n q\right)_{xx} + \left(b_1|q|^m + b_2|q|^{2m}\right)q \\ & = \alpha\frac{|q_x|^2}{q^*} + \frac{\beta}{4|q|^2 q^*}\left[2|q|^2\left(|q|^2\right)_{xx} - \left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q. \end{aligned} \tag{1}$$

In Eq. (1), the dependent variable $q(x,t)$ is a complex–valued function and represents the wave envelope. The independent variables x and t are the spatial and temporal coordinates, respectively. The first term is the linear temporal evolution and its coefficient $i=\sqrt{-1}$. The coefficient of a is the nonlinear CD with n being the parameter of nonlinearity. Then, the coefficients of b_j for $j=1,2$, represent the generalized quadratic–cubic form of SPM with m representing the generalized parameter. If $m=1$, the special case collapses to CGLE with quadratic–cubic nonlinearity, and this was studied during 2022 [8]. The terms with α, β and γ emerge from nonlinear optoelectronic effects. The coefficients a, b_j, α, β and γ are all real–valued constants.

In order to address Eq. (1), the following pulse structure is chosen in the phase–amplitude format:

$$q(x,t) = \phi(x)e^{i\omega t}, \tag{2}$$

where $\phi(x)$ represents the pulse amplitude and ω its frequency. Upon substituting the wave structure given by Eq. (2) into Eq. (1) gives the ordinary differential equation (ODE) for pulse amplitude as:

$$\begin{aligned} & \alpha\{\phi'(x)\}^2 + \beta\phi(x)\phi''(x) + (\gamma+\omega)\phi^2(x) \\ & -a(n+1)\phi^{n+1}(x)\phi''(x) - an(n+1)\phi^n(x)\{\phi'(x)\}^2 \\ & -b_1\phi^{m+2}(x) - b_2\phi^{2m+2}(x) = 0. \end{aligned} \tag{3}$$

For integrability of Eq. (3), set

$$n=-1. \tag{4}$$

The governing Eq. (1) therefore modifies to:

$$\begin{aligned} & iq_t + a\left(\frac{q}{|q|}\right)_{xx} + \left(b_1|q|^m + b_2|q|^{2m}\right)q \\ & = \alpha\frac{|q_x|^2}{q^*} + \frac{\beta}{4|q|^2 q^*}\left[2|q|^2\left(|q|^2\right)_{xx} - \left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q, \end{aligned} \tag{5}$$

and the corresponding ODE given by (3) reduces to:

$$\alpha\left\{\phi'(x)\right\}^2+\beta\phi(x)\phi''(x)+(\gamma+\omega)\phi^2(x)-b_1\phi^{m+2}(x)-b_2\phi^{2m+2}(x)=0. \tag{6}$$

Eq. (6) admits a single Lie point symmetry, namely $\partial/\partial x$. With the implementation of this translational Lie symmetry, Eq. (6) leads to the following form of implicit quiescent optical solitons:

$$x=\pm\frac{1}{m}\sqrt{-\frac{\alpha+\beta}{\gamma+\omega}}\ln\left(\frac{\phi^m}{A_1+A_2}\right), \tag{7}$$

where

$$A_1=(\gamma+\omega)(\alpha+\beta+\beta m)\left\{2\alpha+\beta(m+2)\right\}-b_1(\alpha+\beta)\phi^m(\alpha+\beta+\beta m), \tag{8}$$

$$A_2=\sqrt{B_1B_2},$$

$$B_1=(\gamma+\omega)(\alpha+\beta+\beta m)\left(2\alpha+\beta(m+2)\right), \tag{10}$$

$$\begin{aligned}B_2=&\left\{2\alpha+\beta(m+2)\right\}\left\{(\gamma+\omega)(\alpha+\beta+\beta m)-b_2(\alpha+\beta)\phi^{2m}\right\}\\&-2b_1(\alpha+\beta)\phi^m(\alpha+\beta+\beta m).\end{aligned} \tag{11}$$

Finally, Eq. (7) poses parameter constraints that must be satisfied, for these implicit quiescent optical solitons to exist, and these are:

$$(\alpha+\beta)(\gamma+\omega)<0, \tag{12}$$

and

$$A_1+A_2>0. \tag{13}$$

3. Generalalized temporal evolution

The CGLE given by Eq. (1) is now written with generalized temporal evolution as:

$$\begin{aligned}&i\left(q^l\right)_t+a\left(|q|^n q^l\right)_{xx}+\left(b_1|q|^m+b_2|q|^{2m}\right)q^l\\&=\alpha\frac{|q_x|^2}{\left(q^l\right)^*}+\frac{\beta}{4|q|^2\left(q^l\right)^*}\left[2|q|^2\left(|q|^2\right)_{xx}-\left\{\left(|q|^2\right)_x\right\}^2\right]+\gamma q^l.\end{aligned} \tag{14}$$

Here, in Eq. (14), the constant l is the parameter for the generalized temporal evolution. When $l=1$, Eq. (14) reduces to the case of linear temporal evolution studied in Eq. (1) . The same substitution given by Eq. (2) is applied to Eq. (14), and thus, the corresponding ODE for $\phi(x)$ reads:

$$\begin{aligned}&a\left\{l^2+l(2n-1)+(n-1)n\right\}\left\{\phi'(x)\right\}^2\phi^{2l+n}(x)\\&+a(l+n)\phi''(x)\phi^{2l+n+1}(x)+b_2\phi^{2(l+m+1)}(x)\\&+b_1\phi^{2l+m+2}(x)-(\gamma+\omega l)\phi^{2l+2}(x)-\alpha\phi^2(x)^2\left\{\phi'(x)\right\}^2-\beta\phi^3(x)=0.\end{aligned} \tag{15}$$

For the integrability of ODE (15), one must choose

$$l=-n \tag{16}$$

and this reduces the governing model Eq. (14) to

$$\begin{aligned}&\frac{i}{\left(q^n\right)_t}+a\left(\frac{|q|^n}{q^n}\right)_{xx}+\frac{\left(b_1|q|^m+b_2|q|^{2m}\right)}{q^n}\\&=\alpha|q_x|^2\left(q^n\right)^*+\frac{\beta\left(q^n\right)^*}{4|q|^2}\left[2|q|^2\left(|q|^2\right)_{xx}-\left\{\left(|q|^2\right)_x\right\}^2\right]+\frac{\gamma}{q^n},\end{aligned} \tag{17}$$

while the corresponding ODE, given by Eq. (15), condenses to:

$$\begin{aligned} & a\left\{n^2+(n-1)n-(2n-1)n\right\}\frac{\left\{\phi'(x)\right\}^2}{\phi^n(x)} \\ & +b_1\phi^{m-2n+2}(x)+b_2\phi^{2(m-n+1)}(x) \\ & -(\gamma-\omega n)\phi^{2-2n}(x)-\alpha\phi^2(x)\left\{\phi'(x)\right\}^2-\beta\phi^3(x)\phi''(x)=0. \end{aligned} \tag{18}$$

The above equation admits a single Lie point symmetry, namely $\partial/\partial x$. This symmetry will be used in the integration process and it leads to the following implicit solution in terms of the Appell hypergeometric function of two variables

$$\begin{aligned} x = & \pm\frac{\phi^{n+1}}{n+1}\sqrt{\frac{\alpha-n\beta}{n\omega-\gamma}} \\ & \times F_1\left(\frac{1+n}{m};\frac{1}{2},\frac{1}{2};\frac{1+m+n}{m};-\frac{A_1}{A_2-A_3},-\frac{A_1}{A_2+A_3}\right) \end{aligned} \tag{19}$$

where

$A_1=b_2\phi^m(\alpha-\beta n)\left\{2\alpha+\beta(m-2n)\right\}$, $A_2=\alpha^2 b_1+\alpha\beta b_1 m-\beta^2 b_1 mn+\beta^2 b_1 n^2-2\alpha\beta b_1 n$,

$A_3=\sqrt{(\alpha-n\beta)\left\{\alpha+\beta(m-n)\right\}\left[b_2(\gamma-n\omega)\left\{2\alpha+\beta(m-2n)\right\}^2+b_1^2(\alpha-n\beta)\left\{\alpha+\beta(m-n)\right\}\right]}$.

The implicit solution given by Eq. (19) poses a parameter constraint given by

$$(\alpha-n\beta)(n\omega-\gamma)>0. \tag{20}$$

Finally, the Appell hypergeometric function of two variables is defined as:

$$F_1(a;b_1,b_2;c;x,y) \tag{21}$$

is formulated through the hypergeometric series:

$$x^m y^n\left(\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(a)_{m+n}(b_1)_m(b_2)_n}{(c)_{m+n}m!n!}\right), \tag{22}$$

which is convergent inside the region

$$\max(|x|,|y|)<1 \tag{23}$$

and the Pochhammer symbol is:

$$(p)_n=\begin{cases}1 & n=0, \\ p(p+1)\cdots(p+n-1) & n>0.\end{cases} \tag{24}$$

The convergence criteria given by Eq. (23), for Eq. (20), transforms to:

$$\max\left(\frac{A_1}{A_2-A_3},\frac{A_1}{A_2+A_3}\right)<1. \tag{25}$$

4. Conclusions

The paper retrieved quiescent optical solitons for the CGLE with nonlinear CD and the generalized form of the quadratic–cubic law of SPM. The Lie symmetry has made this retrieval possible for linear temporal evolution as well as generalized temporal evolution. The parameter constraints that naturally emerged from the solution structures are also enlisted. These solutions lead to the conclusion that the CD must never be rendered to be nonlinear, deliberately or inadvertently. This would only cause the solitons to be stationary, and the information transfer across intercontinental distances would consequently be stalled. Thus, catastrophic consequences would ensue. These results would be further extended with additional forms of SPM for the CGLE and would be later disclosed after aligning them with the pre–existing works [14–24].

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Анотація. *Ця робота присвячена отриманню стаціонарних оптичних солітонів для комплексного рівняння Гінзбурга–Ландау з нелінійною хроматичною дисперсією та узагальненою структурою квадратично-кубічної форми самомодуляції фази. Для досягнення цього використовується симетрія Лі. Модель досліджується з лінійною часовою еволюцією, а також з узагальненою часовою еволюцією.*

Ключові слова*: спокійні солітони, симетрія Лі*