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Quiescent Optical Solitons for the Complex Ginzburg–Landau Equation with Nonlinear Chromatic Dispersion and Kudryashov's Forms of Self–Phase Modulation

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Abstract. This paper recovers quiescent optical solitons for the complex Ginzburg–Landau equation, studied with nonlinear chromatic dispersion and three of Kudryashov's self–phase modulation structures. The enhanced Kudryashov's procedure has made retrieving a full spectrum of solitons possible.

Keywords: Kudryashov scheme, stationary solitons, bright solitons, dark solitons, singular solitons
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1. Introduction

The study of quiescent optical solitons for nonlinear chromatic dispersion (CD), together with a wide range of self–phase modulation (SPM) structures, has attracted a lot of attention during the past few years [1–10]. Several models in this context have touched base upon. These include the nonlinear Schrödinger's equation with Kudryashov's form of SPM, the Lakshmanan–Porsezian–Daniel model, the concatenation model, Radhakrishnan–Kundu–Lakshmanan (RKL) equation, complex Ginzburg–Landau equation (CGLE) and several many others. The implicit quiescent optical solitons for the CGLE with several forms of SPM were recovered and reported in 2022 [2]. However, it must be noted that all of the reported results were in terms of quadratures. The current paper implements Kudryashov's enhanced scheme to address the CGLE with nonlinear CD and three forms of SPM, which Kudryashov introduced. A full spectrum of quiescent solitons is retrieved for this model and presented. The results are exhibited after a quick introduction to the model.

2. The governing model and enhanced Kudryashov's procedure

The general dimensionless form of CGLE with nonlinear CD can be written as [2]:

$$iq_t + a\left(|q|^n q\right)_{xx} + F\left(|q|^2\right)q = \frac{1}{|q|^2 q^*}\left\{\alpha |q|^2 \left(|q|^2\right)_{xx} - \beta\left\{\left(|q|^2\right)_x\right\}^2\right\} + \gamma q. \tag{1}$$

The variable $q(x,t)$ is a dependent variable with a complex value. This variable is used to represent the wave profile in our research. Additionally, we consider the complex conjugate of $q(x,t)$ denoted as $q^*(x,t)$, where i represents the imaginary unit. Moreover, it is important to note that the spatial coordinate, denoted as x, and the temporal coordinate, denoted as t, are considered independent variables. The first term in the equation denotes the linear evolution. The coefficient of a represents the nonlinear chromatic dispersion, while the third term corresponds to the generalized nonlinear term. The term γ stems from detuning effect. The coefficients of α and β are from perturbative effects.

The real-valued algebraic functional F must possess smoothness of the complex-valued function $F(|q|^2)q$: $C \to C$. Treating the complex plane C as two-dimensional linear space R^2, the function $F(|q|^2)q$: is k times continuously differentiable provided:

$$F\left(|q|^2\right)q \in \bigcup_{m,n=1}^{\infty} C^k\left((-n,n)\times(-m,m);R^2\right).$$

Taking into consideration the following form of the nonlinear evolution equation:

$$G\left(u,u_x,u_t,u_{xt},u_{xx},...\right) = 0. \tag{2}$$

Here, G is a polynomial dependent on the unknown function u, as well as its time and space-independent variables. The function u is represented as $u(x,t)$.

This section presents the core procedure of the enhanced Kudryashov method. This method is a mathematical technique developed to obtain certain types of solutions. It has been widely developed and used in various fields of science and engineering. The core procedure involves several steps. The given equation is first transformed into a nonlinear ordinary differential equation (ODE) using appropriate transformations:

$$u(x,t) = U(\xi), \quad \xi = k(x - \upsilon t), \tag{3}$$

where k and υ are wave width and soliton speed variables to be determined later, the Eq. (1) is thus transformed to the ODE:

$$P\left(U,-\mu\upsilon U',\mu U',\mu^2 U'',...\right) = 0. \tag{4}$$

Step–1: Considering that the solution to (3) may be stated as:

$$U(\xi) = \lambda_0 + \sum_{l=1}^{N}\sum_{i+j=l}\lambda_{ij}Q^i(\xi)R^j(\xi), \tag{5}$$

where $\lambda_0, \lambda_{ij}\ (i,j = 0,1,...,N)$ are constants to be determined and the functions $R(\xi)$ and $Q(\xi)$ satisfy the following ODEs:

$$R'(\xi)^2 = R(\xi)^2(1 - \chi R(\xi)^2), \tag{6}$$

and

$$Q'(\xi)=Q(\xi)(\eta Q(\xi)-1). \tag{7}$$

The solutions to Eqs. (6) and (7) can be found by using these formulas:

$$R(\xi)=\frac{4c}{4c^2e^{\xi}+\chi e^{-\xi}}, \tag{8}$$

and

$$Q(\xi)=\frac{1}{\eta+de^{\xi}}, \tag{9}$$

where c,d,η and χ are arbitrary constants.

Step–2: Balance the highest-order derivatives and the nonlinear term in Eq. (4) to get the positive integer value N in Eq. (5).

Step–3: Substitute (5) into (4) along with (6) and (7), we acquire the polynomials of $R(\xi),R'(\xi)$, and $Q(\xi)$ as a consequence of this substitution. We collect all terms that have the same power in this polynomial and equal them to zero to create an over-determined algebraic equation system that may be solved by Mathematica to determine the unknown parameters k, υ, c, d, η, χ, λ_0, λ_{ij}, $(i,j=0,1,...,N)$. Thus, we obtain the solutions to Eq. (2).

3. Application of the CGLE

To get stationary solitons, we set the wave transformation:

$$q(x,t)=U(kx)e^{i(\omega t+\theta_0)}, \tag{10}$$

where ω represents the frequency, and θ_0 stands for the phase constant, while k is a real nonzero constant that represents the wave width. Plugging (10) into (1) causes to:

$$\begin{aligned}&ak^2(n+1)U^{n+1}U''+ak^2n(n+1)U^nU'^2+F(U^2)U^2-2\alpha k^2UU''\\&-2k^2(\alpha-2\beta)U'^2+(\gamma-\omega)U^2=0.\end{aligned} \tag{11}$$

The following subsections will discuss Eq. (1) with different types of nonlinearity.

3.1. Kudryashov's first form of SPM

For Kudryashov's law of nonlinearity, Eq. (1) takes the form:

$$\begin{aligned}&iq_t+a(|q|^n q)_{xx}+\left(\frac{b_1}{|q|^{2m}}+\frac{b_2}{|q|^m}+b_3|q|^m+b_4|q|^{2m}\right)q\\&=\frac{1}{|q|^2q^*}\left\{\alpha|q|^2(|q|^2)_{xx}-\beta\left\{(|q|^2)_x\right\}^2\right\}+\gamma q,\end{aligned} \tag{12}$$

Where $b_1...b_4$ are all real-valued constants that give Kudryashov's first form of SPM, while m is referred to as the power law of nonlinearity parameter.

In this case, Eq. (11) becomes:

$$\begin{aligned}&ak^2(n+1)U^{2m+n+1}U''(\xi)+ak^2n(n+1)U^{2m+n}U'^2+b_1U^2+b_2U^{m+2}\\&+b_3U^{3m+2}+b_4U^{4m+2}-2\alpha k^2U^{2m+1}U''-2k^2(\alpha-2\beta)U^{2m}U'^2+(\gamma-\omega)U^{2m+2}=0.\end{aligned} \tag{13}$$

Choosing $n=m$, yields:

$$\begin{aligned}&(m+1)U^{3m+1}U''(\xi)+ak^2m(m+1)U^{3m}U'^2+b_1U^2+b_2U^{m+2}+b_3U^{3m+2}\\&+b_4U^{4m+2}-2\alpha k^2U^{2m+1}U''-2k^2(\alpha-2\beta)U^{2m}U'^2+(\gamma-\omega)U^{2m+2}=0.\end{aligned} \tag{14}$$

Using the transformation:

$$U = V^{\frac{1}{m}},$$

Eq. (14) becomes:

$$\begin{aligned} &V\left(ak^2(m+1)V'^2 + b_2 m^2\right) + ak^2 m(m+1)V^2 V'' + b_4 m^2 V^4 + b_3 m^2 V^3 + b_1 m^2 \\ &-2\alpha k^2 m V V'' + 2k^2\left(2\beta + \alpha(m-2)\right)V'^2 + m^2(\gamma - \omega)V^2 = 0. \end{aligned} \tag{15}$$

Balancing V^2V'' with V^4 in Eq. (15) gives $N = 2$. Consequently, we can write:

$$V(\xi) = \lambda_0 + \lambda_{01}R(\xi) + \lambda_{10}Q(\xi) + \lambda_{11}Q(\xi)R(\xi) + \lambda_{20}Q(\xi)^2 + \lambda_{02}R(\xi)^2. \tag{16}$$

Plugging (16) into (15) along with (6) and (7), we acquire the polynomials of $R(\xi), R'(\xi)$ and $Q(\xi)$ as a consequence of this substitution. We collect all terms with the same power in this polynomial and equal them to zero to create an over-determined algebraic equation system. Handling this system with appropriate software like Mathematica gives the following results:

Result–1:

$$\begin{aligned}
&\lambda_0 = -\frac{4a^2k^2(3m+2)(m+1)^3 + ab_3m^2\left(3m^2+5m+2\right) + 2b_4m^2\left(\alpha(m+4) - 4\beta\right)}{6ab_4m^2(m+1)^2}, \\
&\omega = -\frac{\begin{bmatrix} a^2b_3^2(m+2)\left(3m^2+5m+2\right)^2 m^4 \\ -4\left\{4a^4k^4(m+2)(3m+2)^2(m+1)^6 + b_4^2m^4\left[-8\alpha\beta\left(2m^2+21m+16\right)\right.\right. \\ \left.\left. +\alpha^2\left(-m^3+14m^2+88m+64\right) + 16\beta^2(5m+4)\right]\right\} \\ +4ab_3b_4\left(3m^2+5m+2\right)m^4\left(\alpha\left(m^2-6m-4\right) + 4\beta(2m+1)\right) \\ +3a^2b_4\gamma m^4(3m+2)(m+1)^4 \end{bmatrix}}{12a^2b_4m^4(m+1)^4(3m+2)}, \\
&b_1 = \frac{\begin{bmatrix} \left(2\beta + \alpha(m-2)\right)\begin{pmatrix} -8a^2k^2(3m+2)(m+1)^3 + ab_3m^2\left(3m^2+5m+2\right) \\ +2b_4m^2\left(\alpha(m+4) - 4\beta\right)\end{pmatrix} \\ \times\begin{pmatrix} 4a^2k^2(3m+2)(m+1)^3 + ab_3m^2\left(3m^2+5m+2\right) \\ +2b_4m^2\left(\alpha(m+4) - 4\beta\right)\end{pmatrix}^2 \end{bmatrix}}{54a^4b_4^2m^6(m+1)^7(3m+2)}, \\
&b_2 = \frac{\begin{bmatrix} -128a^6k^6(3m+2)^3(m+1)^9 \\ -96a^4b_4k^4m^2(3m+2)^2(m+1)^6\left(\alpha\left(3m^2-8m-8\right) + 4\beta(3m+2)\right) \\ -12ab_3m^2\left(3m^2+5m+2\right)(4a^4k^4(3m+2)^2(m+1)^6 \\ +b_4^2m^4(\alpha^2\left(-6m^3-7m^2+88m+80\right) - 8\alpha\beta(23m+20) + 16\beta^2(6m+5))) \\ +a^3b_3^3m^6\left(3m^2+5m+2\right)^3 + 6a^2b_3^2b_4m^6\left(3m^2+5m+2\right)^2 \\ \times\left(\alpha\left(3m^2-8m-8\right) + 4\beta(3m+2)\right) \\ +8b_4^3m^6\left(\alpha\left(9m^2-26m-32\right) + 4\beta(9m+8)\right)(\alpha(m+4) - 4\beta)^2 \end{bmatrix}}{108a^3b_4^2m^6(m+1)^6(3m+2)}, \\
&\lambda_{01} = \lambda_{11} = \lambda_{10} = \lambda_{20} = 0, \;\; \lambda_{02} = \frac{2ak^2\left(3m^2+5m+2\right)\chi}{b_4m^2}.
\end{aligned} \tag{17}$$

Inserting (17) together with (8) into (16) leads to a solution of the form:

$$q(x,t)=e^{i(\omega t+\theta_0)}\left\{\frac{32ac^2k^2(3m^2+5m+2)}{b_4m^2}\left(\frac{\chi e^{2kx}}{(4c^2e^{2kx}+\chi)^2}\right)\right.$$
$$\left.-\frac{4a^2k^2(3m+2)(m+1)^3+ab_3m^2(3m^2+5m+2)+2b_4m^2(\alpha(m+4)-4\beta)}{6ab_4m^2(m+1)^2}\right\}^{\frac{1}{m}}. \quad (18)$$

Setting $\chi=\pm 4c^2$ leads to the following bright and singular soliton solutions:

$$q(x,t)=e^{i(\omega t+\theta_0)}\left\{\frac{(2ak^2(3m^2+5m+2))}{b_4m^2}\operatorname{sech}^2(kx)\right.$$
$$\left.-\frac{4a^2k^2(3m+2)(m+1)^3+ab_3m^2(3m^2+5m+2)+2b_4m^2(\alpha(m+4)-4\beta)}{6ab_4m^2(m+1)^2}\right\}^{\frac{1}{m}}, \quad (19)$$

and

$$q(x,t)=e^{i(\omega t+\theta_0)}\left\{-\frac{(2ak^2(3m^2+5m+2))}{b_4m^2}\operatorname{csch}^2(kx)\right.$$
$$\left.-\frac{4a^2k^2(3m+2)(m+1)^3+ab_3m^2(3m^2+5m+2)+2b_4m^2(\alpha(m+4)-4\beta)}{6ab_4m^2(m+1)^2}\right\}^{\frac{1}{m}}. \quad (20)$$

Result–2:

$$\lambda_0=-\frac{a^2k^2(3m+2)(m+1)^3+ab_3m^2(3m^2+5m+2)+2b_4m^2(\alpha(m+4)-4\beta)}{6ab_4m^2(m+1)^2},$$

$$\lambda_{01}=\lambda_{11}=\lambda_{02}=0,\quad \lambda_{10}=\frac{2a\eta k^2(3m^2+5m+2)}{b_4m^2},\quad \lambda_{20}=-\frac{2a\eta^2k^2(3m^2+5m+2)}{b_4m^2}$$

$$\omega=\frac{\begin{bmatrix}a^4k^4(m+2)(3m+2)^2(m+1)^6+12a^2b_4\gamma m^4(3m+2)(m+1)^4\\ -a^2b_3^2m^4(m+2)(3m^2+5m+2)^2-4ab_3b_4m^4(3m^2+5m+2)\\ \times(\alpha(m^2-6m-4)+4\beta(2m+1))\\ -4b_4^2m^4(8\alpha\beta(2m^2+21m+16)+\alpha^2(m^3-14m^2-88m-64)-16\beta^2(5m+4))\end{bmatrix}}{12a^2b_4m^4(m+1)^4(3m+2)},$$

$$b_1=\frac{\begin{bmatrix}(2\beta+\alpha(m-2))\begin{pmatrix}-2a^2k^2(3m+2)(m+1)^3+ab_3m^2(3m^2+5m+2)\\ +2b_4m^2(\alpha(m+4)-4\beta)\end{pmatrix}\\ \times\left(a^2k^2(3m+2)(m+1)^3+ab_3m^2(3m^2+5m+2)+2b_4m^2(\alpha(m+4)-4\beta)\right)^2\end{bmatrix}}{54a^4b_4^2m^6(m+1)^7(3m+2)},$$

$$b_2=\frac{\begin{bmatrix}-2a^6k^6(3m+2)^3(m+1)^9-6a^4b_4k^4m^2(3m+2)^2(m+1)^6\\ \times(\alpha(3m^2-8m-8)+4\beta(3m+2))\\ -3ab_3m^2(3m^2+5m+2)(a^4k^4(m+1)^6(3m+2)^2-4b_4^2m^4(\alpha^2(6m^3+7m^2-88m-80)\\ +8\alpha\beta(23m+20)-16\beta^2(6m+5)))+a^3b_3^3m^6(3m^2+5m+2)^3\\ +6a^2b_3^2b_4m^6(3m^2+5m+2)^2(\alpha(3m^2-8m-8)+4\beta(3m+2))\\ +8b_4^3m^6(\alpha(9m^2-26m-32)+4\beta(9m+8))(\alpha(m+4)-4\beta)^2\end{bmatrix}}{108a^3b_4^2m^6(m+1)^6(3m+2)}. \quad (21)$$

Inserting (21) together with (9) into (16) leads to a solution of the form:

$$q(x,t)=\left\{-\frac{a^2k^2(3m+2)(m+1)^3+ab_3m^2(3m^2+5m+2)+2b_4m^2(\alpha(m+4)-4\beta)}{6ab_4m^2(m+1)^2}\right.$$
$$\left.-\frac{2a\eta^2k^2(3m^2+5m+2)}{b_4m^2(de^{kx}+\eta)^2}+\frac{2a\eta k^2(3m^2+5m+2)}{b_4m^2(de^{kx}+\eta)}\right\}^{\frac{1}{m}}e^{i(\omega t+\theta_0)}. \tag{22}$$

Setting $\eta=\pm d$ leads to the following bright and singular soliton solutions:

$$q(x,t)=\left\{-\frac{a^2k^2(3m+2)(m+1)^3+ab_3m^2(3m^2+5m+2)+2b_4m^2(\alpha(m+4)-4\beta)}{6ab_4m^2(m+1)^2}\right.$$
$$\left.+\frac{(ak^2(3m^2+5m+2))}{2b_4m^2}\operatorname{sech}^2\left(\frac{kx}{2}\right)\right\}^{\frac{1}{m}}e^{i(\omega t+\theta_0)}, \tag{23}$$

and

$$q(x,t)=\left\{-\frac{a^2k^2(3m+2)(m+1)^3+ab_3m^2(3m^2+5m+2)+2b_4m^2(\alpha(m+4)-4\beta)}{6ab_4m^2(m+1)^2}\right.$$
$$\left.-\frac{(ak^2(3m^2+5m+2))}{2b_4m^2}\operatorname{csch}^2\left(\frac{kx}{2}\right)\right\}^{\frac{1}{m}}e^{i(\omega t+\theta_0)}. \tag{24}$$

3.2. Kudryashov's second form of SPM

For generalized quadratic–cubic with nonlocal nonlinearity, Eq. (1) takes the form:

$$iq_t+a(|q|^n q)_{xx}+\left(b_1|q|^m+b_2|q|^{2m}+b_3(|q|^m)_{xx}\right)q$$
$$=\frac{1}{|q|^2q^*}\left\{\alpha|q|^2(|q|^2)_{xx}-\beta\left\{(|q|^2)_x\right\}^2\right\}+\gamma q. \tag{25}$$

In this case, Eq. (11) becomes:

$$ak^2(n+1)U^{n+1}U''+ak^2n(n+1)U^nU'^2$$
$$+\left(b_1U^m+b_2U^{2m}+b_3k^2m\left(U^{m-1}U''+(m-1)U^{m-2}U'^2\right)\right)U^2$$
$$-2\alpha k^2UU''-2k^2(\alpha-2\beta)U'^2+(\gamma-\omega)U^2=0. \tag{26}$$

Choosing $n=m$ yields:

$$ak^2(m+1)U^{m+1}U''+ak^2m(m+1)U^mU'^2$$
$$+\left(b_1U^m+b_2U^{2m}+b_3k^2m\left(U^{m-1}U''+(m-1)U^{m-2}U'^2\right)\right)U^2$$
$$-2\alpha k^2UU''-2k^2(\alpha-2\beta)U'^2+(\gamma-\omega)U^2=0. \tag{27}$$

Using the transformation:

$$U=V^{\frac{1}{m}},$$

Eq. (27) becomes:

$$k^2mVV''\left(a(m+1)V-2\alpha+b_3mV\right)+k^2V'^2\left(a(m+1)V+4\beta+2\alpha(m-2)\right)$$
$$+m^2V^2\left(b_2V^2+b_1V+\gamma-\omega\right)=0. \tag{28}$$

Balancing V^2V'' with V^4 in Eq. (28) gives $N=2$. Consequently, we can write:

$$V(\xi)=\lambda_0+\lambda_{01}R(\xi)+\lambda_{10}Q(\xi)+\lambda_{11}Q(\xi)R(\xi)+\lambda_{20}Q(\xi)^2+\lambda_{02}R(\xi)^2. \tag{29}$$

Plugging (29) into (28) along with (6) and (7), we acquire the polynomials of $R(\xi),R'(\xi)$ and $Q(\xi)$ as a consequence of this substitution. We collect all terms with the same power in this polynomial and equal them to zero to create an over-determined algebraic equation system. Handling this system with appropriate software like Mathematica yields the following results:

Result–1:

$$\begin{aligned}
&\lambda_0=\lambda_{01}=\lambda_{11}=\lambda_{10}=\lambda_{20}=0,\\
&\lambda_{02}=-\frac{\chi\left(b_1\left(a(m+1)(3m+2)+3b_3m^2\right)+2b_2\left(\alpha(m+4)-4\beta\right)\right)}{2b_2\left(a(m+1)^2+b_3m^2\right)},\\
&\omega=\frac{4(\alpha-\beta)\left(b_1\left(a(m+1)(3m+2)+3b_3m^2\right)+2b_2\left(\alpha(m+4)-4\beta\right)\right)}{\left(a(m+1)^2+b_3m^2\right)\left(a(m+1)(3m+2)+3b_3m^2\right)},\\
&k=m\sqrt{-\frac{b_1\left(a(m+1)(3m+2)+3b_3m^2\right)+2b_2\left(\alpha(m+4)-4\beta\right)}{4\left(a(m+1)^2+b_3m^2\right)\left(a(m+1)(3m+2)+3b_3m^2\right)}}.
\end{aligned} \tag{30}$$

Inserting (30) together with (8) into (29) leads to a solution of the form:

$$q(x,t)=\left\{\begin{aligned}&-\frac{8c^2\left(b_1\left(a(m+1)(3m+2)+3b_3m^2\right)+2b_2\left(\alpha(m+4)-4\beta\right)\right)}{b_2\left(a(m+1)^2+b_3m^2\right)}\\ &\times\frac{\chi e^{2m\sqrt{-\frac{b_1\left(a(m+1)(3m+2)+3b_3m^2\right)+2b_2\left(\alpha(m+4)-4\beta\right)}{4\left(a(m+1)^2+b_3m^2\right)\left(a(m+1)(3m+2)+3b_3m^2\right)}}x}}{\left(4c^2e^{2m\sqrt{-\frac{b_1\left(a(m+1)(3m+2)+3b_3m^2\right)+2b_2\left(\alpha(m+4)-4\beta\right)}{4\left(a(m+1)^2+b_3m^2\right)\left(a(m+1)(3m+2)+3b_3m^2\right)}}x}+\chi\right)^2}\end{aligned}\right\}^{\frac{1}{m}} e^{i(\omega t+\theta_0)}. \tag{31}$$

Setting $\chi=\pm 4c^2$ leads to the following bright and singular soliton solutions:

$$q(x,t)=\left\{\begin{aligned}&-\frac{\left(b_1\left(a(3m^2+5m+2)+3b_3m^2\right)+2b_2\left(\alpha(m+4)-4\beta\right)\right)}{2b_2\left(a(m+1)^2+b_3m^2\right)}\\ &\times\operatorname{sech}^2\left[m\sqrt{-\frac{b_1\left(a(m+1)(3m+2)+3b_3m^2\right)+2b_2\left(\alpha(m+4)-4\beta\right)}{4\left(a(m+1)^2+b_3m^2\right)\left(a(m+1)(3m+2)+3b_3m^2\right)}}x\right]\end{aligned}\right\}^{\frac{1}{m}} \times e^{i(\omega t+\theta_0)}, \tag{32}$$

and

$$q(x,t)=\left\{\begin{aligned}&\frac{\left(b_1\left(a(3m^2+5m+2)+3b_3m^2\right)+2b_2\left(\alpha(m+4)-4\beta\right)\right)}{2b_2\left(a(m+1)^2+b_3m^2\right)}\\ &\times\operatorname{csch}^2\left[m\sqrt{-\frac{b_1\left(a(m+1)(3m+2)+3b_3m^2\right)+2b_2\left(\alpha(m+4)-4\beta\right)}{4\left(a(m+1)^2+b_3m^2\right)\left(a(m+1)(3m+2)+3b_3m^2\right)}}x\right]\end{aligned}\right\}^{\frac{1}{m}} \times e^{i(\omega t+\theta_0)}. \tag{33}$$

Result–2:

$$\begin{aligned}
&\lambda_0=\lambda_{01}=\lambda_{11}=\lambda_{10}=\lambda_{02}=0,\ \lambda_{20}=-\frac{2a\eta^2(m+1)(\gamma-\omega)}{5\alpha b_2m},\ b_3=-\frac{a\left(5m^2+9m+4\right)}{5m^2},\\
&\omega=\frac{4a^2\gamma(m+1)^2-10a\alpha b_1m(m+1)+25\alpha^2b_2m^2}{4a^2(m+1)^2},\ \beta=\frac{1}{8}\alpha(m+8),\ k=\sqrt{\frac{m(\omega-\gamma)}{2\alpha}}.
\end{aligned} \tag{34}$$

Inserting (34) together with (9) into (29) leads to a solution of the form:

$$q(x,t)=\left\{\left(\frac{4a^2\gamma(m+1)^2-10a\alpha b_1 m(m+1)+25\alpha^2 b_2 m^2}{4a^2(m+1)^2}-\gamma\right)\frac{2a\eta^2(m+1)}{5\alpha b_2 m\left(de^{\sqrt{\frac{m(\omega-\gamma)}{2\alpha}}x}+\eta\right)^2}\right\}^{\frac{1}{m}} \tag{35}$$
$$\times e^{i(\omega t+\theta_0)}.$$

Setting $\eta=\pm d$ leads to the following bright and singular soliton solutions:

$$q(x,t)=\left\{\frac{1}{8}\left(\frac{5\alpha m}{a(m+1)}-\frac{2b_1}{b_2}\right)\left(1-\tanh\left[\frac{1}{2}\sqrt{\frac{m(\omega-\gamma)}{2\alpha}}x\right]\right)^2\right\}^{\frac{1}{m}} e^{i(\omega t+\theta_0)}, \tag{36}$$

and

$$q(x,t)=\left\{\frac{1}{8}\left(\frac{5\alpha m}{a(m+1)}-\frac{2b_1}{b_2}\right)\left(1-\coth\left[\frac{1}{2}\sqrt{\frac{m(\omega-\gamma)}{2\alpha}}x\right]\right)^2\right\}^{\frac{1}{m}} e^{i(\omega t+\theta_0)}. \tag{37}$$

Result–3:

$$\begin{gathered}\lambda_{01}=\lambda_{11}=\lambda_{10}=\lambda_{02}=0,\ \lambda_0=-\frac{5\alpha m}{2a(m+1)},\ \lambda_{20}=\frac{5\alpha\eta^2 m}{2a(m+1)},\\ b_3=-\frac{a(5m^2+9m+4)}{5m^2},\quad b_2=\frac{ab_1(m+1)}{5\alpha m},\\ \omega=\frac{4a\gamma(m+1)-5\alpha b_1 m}{4a(m+1)},\ \beta=\alpha\left(1-\frac{m}{2}\right),\ k=\pm\frac{m}{2}\sqrt{\frac{5b_1}{2a(m+1)}}.\end{gathered} \tag{38}$$

Inserting (38) together with (9) into (29) leads to a solution of the form:

$$q(x,t)=\left(\frac{5\alpha\eta^2 m}{2a(m+1)\left(de^{\pm\frac{m}{2}\sqrt{\frac{5b_1}{2a(m+1)}}x}+\eta\right)^2}-\frac{5\alpha m}{2a(m+1)}\right)^{\frac{1}{m}} e^{i(\omega t+\theta_0)}. \tag{39}$$

Setting $\eta=\pm d$ leads to the following bright and singular soliton solutions:

$$q(x,t)=\left\{\frac{5\alpha m}{8a(m+1)}\left(\left(1\pm\tanh\left[\frac{m}{4}\sqrt{\frac{5b_1}{2a(m+1)}}x\right]\right)^2-4\right)\right\}^{\frac{1}{m}} e^{i(\omega t+\theta_0)}, \tag{40}$$

and

$$q(x,t)=\left\{\frac{5\alpha m}{8a(m+1)}\left(\left(1\pm\coth\left[\frac{m}{4}\sqrt{\frac{5b_1}{2a(m+1)}}x\right]\right)^2-4\right)\right\}^{\frac{1}{m}} e^{i(\omega t+\theta_0)}. \tag{41}$$

3.3. Kudryashov's third form of SPM

For this nonlinearity, Eq. (1) takes the form

$$\begin{aligned}&iq_t+a(|q|^n q)_{xx}+\left(b_1|q|^m+b_2|q|^{2m}+b_3|q|^{3m}+b_4|q|^{4m}+b_5|q|^{5m}+b_6|q|^{6m}\right)q\\&=\frac{1}{|q|^2 q^*}\left\{\alpha|q|^2(|q|^2)_{xx}-\beta\left\{(|q|^2)_x\right\}^2\right\}+\gamma q.\end{aligned} \tag{42}$$

In this case, Eq. (11) becomes

$$\begin{aligned}&ak^2(n+1)U^{n+1}U''+ak^2n(n+1)U^nU'^2\\&+\left(b_1U^m+b_2U^{2m}+b_3U^{3m}+b_4U^{4m}+b_5U^{5m}+b_6U^{6m}\right)U^2\\&-2\alpha k^2UU''-2k^2(\alpha-2\beta)U'^2+(\gamma-\omega)U^2=0,\end{aligned}\tag{43}$$

$$n=4m,$$

$$\begin{aligned}&ak^2(4m+1)U^{4m+1}U''+4ak^2m(4m+1)U^{4m}U'^2\\&+b_1U^{m+2}+b_2U^{2m+2}+b_3U^{3m+2}+b_4U^{4m+2}+b_5U^{5m+2}\\&+b_6U^{6m+2}-2\alpha k^2UU''-2k^2(\alpha-2\beta)U'^2+(\gamma-\omega)U^2=0,\end{aligned}\tag{44}$$

$$U=V^{\frac{1}{m}},$$

$$\begin{aligned}&k^2V'^2\left(a\left(12m^2+7m+1\right)V^4+4\beta+2\alpha(m-2)\right)+k^2mVV''\left(a(4m+1)V^4-2\alpha\right)\\&+m^2V^2\left(b_6V^6+b_5V^5+b_4V^4+b_3V^3+b_2V^2+b_1V+\gamma-\omega\right)=0.\end{aligned}\tag{45}$$

Balancing V^5V'' with V^8 in Eq. (45) gives $N=1$. Consequently, we can write:

$$V(\xi)=\lambda_0+\lambda_{01}R(\xi)+\lambda_{10}Q(\xi).\tag{46}$$

Plugging (46) into (45) along with (6) and (7), we acquire the polynomials of $R(\xi), R'(\xi)$ and $Q(\xi)$ as a consequence of this substitution. To create an over-determined algebraic equation system, we collect all terms with the same power in this polynomial and equal them to zero. Handling this system with appropriate software like Mathematica leads to the following results:

Result–1:

$$\begin{aligned}&\lambda_{10}=0,\ \lambda_0=-\frac{b_5(5m+1)}{2b_6(9m+2)},\ \lambda_{01}=\pm\sqrt{\frac{b_5^2(5m+1)^2\chi}{2b_6^2(9m+2)^2}},\\&k=b_5m\sqrt{\frac{5m+1}{2ab_6(4m+1)(9m+2)^2}},\\&b_1=-\frac{6\alpha b_5m}{a\left(36m^2+17m+2\right)},\ b_2=-\frac{4\alpha b_6m}{a\left(20m^2+9m+1\right)}-\frac{b_5^4(3m+1)(5m+1)^3}{16b_6^3(9m+2)^4},\\&b_3=0,\quad b_4=\frac{b_5^2\left(20m^2+9m+1\right)}{b_6(9m+2)^2},\\&\omega=\frac{2\alpha b_5^2m(5m+1)}{ab_6(4m+1)(9m+2)^2}+\gamma,\ \beta=\alpha\left(1-\frac{m}{2}\right).\end{aligned}\tag{47}$$

Inserting (47) together with (8) into (46) leads to a solution of the form:

$$q(x,t)=\left(\frac{\pm4ce^{b_5m\sqrt{\frac{5m+1}{2ab_6(4m+1)(9m+2)^2}}x}\sqrt{\frac{b_5^2(5m+1)^2\chi}{b_6^2(9m+2)^2}}}{\sqrt{2}\left(4c^2e^{2b_5m\sqrt{\frac{5m+1}{2ab_6(4m+1)(9m+2)^2}}x}+\chi\right)}-\frac{b_5(5m+1)}{2b_6(9m+2)}\right)^{\frac{1}{m}}e^{i(\omega t+\theta_0)}.\tag{48}$$

Setting $\chi=\pm4c^2$ leads to the following bright soliton solution:

$$q(x,t)=\left\{-\frac{b_5(5m+1)}{2b_6(9m+2)}\left(1\mp\sqrt{2}\operatorname{sech}\left[b_5m\sqrt{\frac{5m+1}{2ab_6(4m+1)(9m+2)^2}}\,x\right]\right)\right\}^{\frac{1}{m}}e^{i(\omega t+\theta_0)}.\tag{49}$$

Result–2:

$$\lambda_0 = 0,\ \lambda_{01} = 0,\ \lambda_{10} = -\frac{b_5\eta(5m+1)}{b_6(9m+2)},\ k = \pm b_5 m\sqrt{-\frac{5m+1}{ab_6(4m+1)(9m+2)^2}},$$
$$b_1 = \frac{2\alpha b_5 m}{a(36m^2+17m+2)},\ b_4 = \frac{b_5^2(4m+1)(5m+1)}{b_6(9m+2)^2},\ \beta = \frac{1}{2}\alpha(m+2),\ b_3 = 0, \quad (50)$$
$$\gamma = \frac{2\alpha b_5^2 m(5m+1)}{ab_6(4m+1)(9m+2)^2} + \omega.$$

Inserting (50) together with (9) into (46) leads to a solution of the form:

$$q(x,t) = \left(-\frac{b_5\eta(5m+1)}{b_6(9m+2)\left(de^{\pm b_5 m\sqrt{-\frac{5m+1}{ab_6(4m+1)(9m+2)^2}}x} + \eta\right)}\right)^{\frac{1}{m}} e^{i(\omega t+\theta_0)}. \quad (51)$$

Setting $\eta = \pm d$ leads to the following dark and singular soliton solutions:

$$q(x,t) = \left\{-\frac{b_5(5m+1)}{2b_6(9m+2)}\left(1 \pm \tanh\left[\frac{b_5 m}{2}\sqrt{-\frac{5m+1}{ab_6(4m+1)(9m+2)^2}}\,x\right]\right)\right\}^{\frac{1}{m}} \times e^{i(\omega t+\theta_0)}, \quad (52)$$

and

$$q(x,t) = \left\{-\frac{b_5(5m+1)}{2b_6(9m+2)}\left(1 \pm \coth\left[\frac{b_5 m}{2}\sqrt{-\frac{5m+1}{ab_6(4m+1)(9m+2)^2}}\,x\right]\right)\right\}^{\frac{1}{m}} \times e^{i(\omega t+\theta_0)}. \quad (53)$$

4. Conclusions

The current paper addressed the evolution of quiescent optical solitons with a nonlinear form of CD and Kudryashov's three proposed forms of SPM. The enhanced Kudryashov's approach retrieved such solitons to the model with linear temporal evolution. Unlike in the previous works, where CGLE revealed results in terms of quadratures, the current work revealed a wide spectrum of explicit quiescent optical solitons. The results are thus nice and interesting. Therefore, the paper's results paved the way for the future. One of the foreseeable pathways is to address the model with generalized temporal evolution. The results will be recovered with generalized temporal evolution, which will be reported next. Later, the model will be studied using differential group delay and dense wavelength division multiplexing topology. The model will also be considered in other optoelectronic devices such as magneto-optic waveguides, optical couplers, optical metamaterials, Bragg gratings, and others. The results of those research activities will be sequentially disseminated.

References

1. Adem, A. R., Ntsime, B. P., Biswas, A., Khan, S., Alzahrani, A. K., & Belic, M. R. (2021). Stationary optical solitons with nonlinear chromatic dispersion for Lakshmanan–Porsezian–Daniel model having Kerr law of nonlinear refractive index. *Ukr. J. Phys. Opt.*, *22*(2), 83-86.
2. Adem, A. R., Ntsime, B. P., Biswas, A. N. J. A. N., Ekici, M., Yildirim, Y. A. K. U. P., & Alshehri, H. M. (2022). Implicit quiescent optical solitons with complex Ginzburg–Landau equation having nonlinear chromatic dispersion. *Journal of Optoelectronics and Advanced Materials*, *24*(September-October 2022), 450-462.
3. Adem, A. R., Biswas, A., Yildirim, Y., & Asiri, A. (2023). Implicit quiescent optical solitons for the dispersive concatenation model with nonlinear chromatic dispersion by lie symmetry. *Contemporary Mathematics*, 666-674.

4. Adem, A. R., Biswas, A., Yildirim, Y., Jawad, A. J. M. & Alshomrani, A. S.. (2024). Sequel to "Stationary optical solitons with nonlinear chromatic dispersion for Lakshmanan–Porsezian–Daniel model having Kerr law of nonlinear refractive index": generalized temporal evolution. *Ukr.J.Phys.Opt.* (submitted).
5. Arnous, A. H., Nofal, T. A., Biswas, A., Khan, S., & Moraru, L. (2022). Quiescent optical solitons with Kudryashov's generalized quintuple-power and nonlocal nonlinearity having nonlinear chromatic dispersion. *Universe*, *8*(10), 501.
6. Arnous, A. H., Biswas, A., Yildirim, Y., Moraru, L., Aphane, M., Moshokoa, S. P. & Alshehri, H. M. (2023). Quiescent optical solitons with Kudryashov's generalized quintuple-power and nonlocal nonlinearity having nonlinear chromatic dispersion: generalized temporal evolution. *Ukr.J.Phys.Opt. 24*(2), 105–113.
7. Ekici, M. (2022). Stationary optical solitons with complex Ginzburg–Landau equation having nonlinear chromatic dispersion and Kudryashov's refractive index structures. *Physics Letters A*, *440*, 128146.
8. Ekici, M. (2023). Stationary optical solitons with Kudryashov's quintuple power law nonlinearity by extended Jacobi's elliptic function expansion. *Journal of Nonlinear Optical Physics & Materials*, *32*(01), 2350008.
9. Kudryashov, N. A. (2022). Stationary solitons of the model with nonlinear chromatic dispersion and arbitrary refractive index. *Optik*, *259*, 168888.
10. Yalçı, A. M., & Ekici, M. (2022). Stationary optical solitons with complex Ginzburg–Landau equation having nonlinear chromatic dispersion. *Optical and Quantum Electronics*, *54*(3), 167.

Ahmed M. Elsherbeny, Ahmed H. Arnous, Anjan Biswas, Yakup Yildirim, Anwar Jaafar Mohamad Jawad & Ali Saleh Alshomrani. (2024). Quiescent Optical Solitons for the Complex Ginzburg–Landau Equation with Nonlinear Chromatic Dispersion and Kudryashov's Forms of Self–Phase Modulation. *Ukrainian Journal of Physical Optics*, *25*(2), 02109 – 02119. doi: 10.3116/16091833/Ukr.J.Phys.Opt.2024.02109

***Анотація**. У цій статті відновлюються стаціонарні оптичні солітони для комплексного рівняння Гінзбурга–Ландау, яке вивчається з нелінійною хроматичною дисперсією та трьома структурами самомодуляції фази Кудряшова. Покращена процедура Кудряшова зробила можливим отримання повного спектру солітонів.*

***Ключові слова**: схема Кудряшова, нерухомі солітони, яскраві солітони, темні солітони, сингулярні солітони*