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The Influence of Acoustic Activity and Elliptical Polarization of the Eigen Acoustic Waves on the Efficiency of Acousto-Optic Diffraction: The Example of Quartz Crystals

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Abstract. The present work is devoted to analyzing the influence of the ellipticity of acoustic waves caused by acoustic activity on the efficiency of acousto-optic interaction. It has been shown that the peak-like increase of the effective elastic-optic coefficients and acousto-optic figure of merit is manifested in the case when the acousto-optic diffraction appears on the quasi-transverse elliptical (or circular) acoustic waves that propagate in the directions that are close to the acoustic axes. In this paper, the analysis has been carried out on the example of quartz crystals, and it has been shown that the increase of acousto-optic figure of merit is peculiar for all types of acousto-optic interactions with the quasi-transverse acoustic waves. We have shown that accounting for the ellipticity of diffracted optical waves in the analysis of acousto-optic interaction in optically active crystals leads to more correct results than those obtained in our recent works.

Keywords: acoustic activity, acousto-optic diffraction, acousto-optic figure of merit, ellipticity of the eigen acoustic waves
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1. Introduction

The ellipticity of the eigen optical waves, caused by the optical activity, can essentially increase the effective elasto-optic (EO) coefficient and acousto-optic (AO) figure of merit [1-4]. It happened due to the inclusion into the relation for effective EO coefficients, the term, which is proportional to the square of the ellipticity of the eigenwaves and related additional components of the EO tensor. The effect is most manifested when incident and diffracted optical waves propagated close to the optical axis of optically active crystals where the ellipticity of eigenwaves is close to the unity. This fact makes the restriction on the direction of propagation of the acoustic wave (AW) that, due to the phase matching conditions at the tangential Bragg AO diffraction, have to propagate almost perpendicular to the optical axis. In some crystals, such geometry of interaction leads to an extremely high AO figure of merit. For example, in paratellurite crystal, the quasi-transverse (QT) AW, which propagates along <110> direction and is polarized in the plane perpendicular to the *Z* axis, possesses the velocity of 617 m/s [5] being slow enough that leads to the AO figure of merit equal to $1200\times10^{-15}s^3/kg$ [6] in the case of AO interaction of the circularly polarized optical waves with the mentioned shear AW. However, in other crystals, it cannot be so. For example, in the quartz crystal [2,4], the AO interaction between the circularly polarized optical waves does not lead to a significant gain in the AO figure of merit and does not exceed the maximal value

($2.38\times10^{-15}s^3/kg$) that is achieved without accounting of optical activity at the AO interaction in *XY* plane where ellipticity of the eigen optical waves approaches zero [7]. However, the optically active crystals as noncentrosymmetric media can possess acoustic activity, too. In this case, one can intuitively assume that accounting for the ellipticity of acoustic eigenwaves in the case of AO interaction with the AW with elliptical (circular) polarization would lead to an effect analogical to those caused by the optical activity. The ellipticity of the AWs approach unity in the case of their propagation along the acoustic axes where the velocities of QT waves are the same [8]. Still, the number of acoustic axes in crystals is much higher than the optical axes and can reach sixteen [9-12] instead of a maximum of two in crystal optics. It can increase the number of directions of AW propagation and the possibility of choosing the optimal direction, which corresponds to the highest AO figure of merit. Thus, the present work aims to analyze the influence of elliptical eigen polarization of AWs in acoustically active crystals on the AO figure of merit in the example of quartz crystals. Besides, in our recent works, including papers [1-3], the consideration of AO interaction has been done in the approximation of neglecting the exact polarization of the diffracted wave. It has been taken into account that the polarization of the diffracted wave has to correspond to a certain eigenwave. Still, the ratio of the electric field components in the direction of propagation of the diffracted wave that corresponds to the phase-matching conditions was not accounted for. In the present work, we will solve this problem.

2. Method of analysis

The effect of acoustic activity is well described in the literature (see, e.g. [8,13]) and has been experimentally studied in the number of crystals [14-17] to which belong SiO_2, $Bi_{12}GeO_{20}$,, $NaClO_3$, tellurium, and liquid crystals. Let us briefly remind features of this effect.

Accounting for the first-order spatial dispersion, Hooke's law can be written in the view [8]

$$\begin{aligned}\sigma_{ij} &= C_{ijkl}e_{kl} + B_{ijklm}\frac{\partial e_{kl}}{\partial X_m} \\ &= C_{ijkl}\frac{\partial u_k}{\partial X_l} + B_{ijklm}\frac{\partial^2 u_k}{\partial X_l \partial X_m}\end{aligned}, \tag{1}$$

where σ_{ij} and e_{kl} are the stress and the strain caused by the acoustic wave, respectively, X_l and X_m are the coordinates, C_{ijkl} is the elastic stiffness tensor, u_k the displacement vector, and is B_{ijklm} the polar fifth-rank tensor describing the acoustical gyration. Then, the elastodynamic equation taking the spatial dispersion into account may be written as

$$C_{ijkl}\frac{\partial^2 u_k}{\partial X_i \partial X_l} + B_{ijklm}\frac{\partial^3 u_k}{\partial X_i \partial X_l \partial X_m} = \rho\frac{\partial^2 u_j}{\partial t^2}, \tag{2}$$

where ρ is the material density and t is the time. For the simplest case of plane waves with the unit polarization vector p_k, the amplitude *A*, the wave vector *m*, the velocity υ, and the wavelength Λ ($u_k = Ap_k e^{\frac{2\pi}{\Lambda}i(mr-\upsilon t)}$), Eq. (2) can be rewritten as:

$$\left(C_{ijkl}m_i m_l + \frac{2\pi}{\Lambda}iB_{ijklm}m_i m_l m_m\right)p_k = \rho\upsilon^2 p_j, \tag{3}$$

where $M_{ik}^a = C_{ijkl}m_i m_l + \frac{2\pi}{\Lambda}iB_{ijklm}m_i m_l m_m$ is the Christoffel tensor that accounts for the acoustical activity. The elastic stiffness tensor is Hermitian and includes an antisymmetric imaginary part, $i\frac{2\pi}{\Lambda}B_{ijklm}m_i m_l m_m$. Hence, the relation $B_{ijklm} = -B_{klijm}$ holds true, and the tensor reveals the internal symmetry {[V²]²}V. However, the tensor is symmetric with respect to permutations of the indices *i*, *l*, and *m* that correspond to the wave vectors. Then the internal symmetry of B_{ijklm} reduces to {V²}[V³]. As a consequence, it can be rewritten in terms of the axial fourth-rank acoustical gyration tensor g_{silm} with the internal symmetry εV[V³]:

$$\begin{aligned} g_{silm} &= \frac{\pi}{\Lambda}\delta_{sjk}B_{ijklm}, \\ \frac{2\pi}{\Lambda}B_{ijklm} &= \delta_{jks}g_{silm}, \end{aligned} \tag{4}$$

where δ_{sjk} is the unitary, fully antisymmetric axial Levi–Civita tensor. The axial vector of the acoustical activity is determined by the acoustical gyration tensor as

$$\phi_s = g_{silm}m_i m_l m_m. \tag{5}$$

Then, one can reduce Eq. (3) to the following form:

$$\left(M_{jk} + i\delta_{jks}\phi_s\right)p_k = \rho v^2 p_j, \tag{6}$$

where M_{jk} is the Christoffel tensor that does not take the acoustical activity into account. In the case when acoustic waves propagate along the acoustic axis, the velocities of the transverse waves are

$$\begin{aligned} v_{1,2}^2 &= \frac{1}{2}\left[(v_{01}^2 + v_{02}^2) \pm \sqrt{(v_{01}^2+v_{02}^2)^2 - 4v_{01}^2v_{02}^2 + 4\phi_3^2/\rho^2}\right]\Big|_{v_{01}=v_{02}=v_0} \\ &= v_0^2 \pm \frac{\phi_3}{\rho}. \end{aligned} \tag{7}$$

where $v_{01} = v_{02}$ are the velocities of eigen transverse AWs with orthogonal polarization. The ellipticities χ_{ac} of these waves are defined by the relation $-i\chi_{ac} = p_1/p_2$, being equal to ± 1. This describes the two circularly polarized waves with the opposite rotation directions of the displacement vector. The waves that pass through the sample with the thickness *d* acquire the phase difference $\Delta_c = \phi_3\omega d/\rho v_0^3$ (ω is the AW frequency, ρ is the density). Hence, the angle of rotation of the polarization plane is equal to $\beta = \Delta/2 = \frac{\phi_3\omega}{2\rho v_0^3}d$. When $v_{01} \neq v_{02}$ the ellipticity of eigenwaves is given by the relation $-i\chi_{ac} = p_1/p_2$ (or $i\chi_{ac} = p_1/p_2$), i.e.

$$\chi_{ac} = \begin{cases} \rho(v_{01}^2 - v_{02}^2)/\phi_3 \\ \phi_3/\rho(v_{01}^2 - v_{02}^2) \end{cases}. \tag{8}$$

Thus, the manifestation of the acoustic activity is similar to its optical analog.

The SiO_2 crystals belong to the point group of symmetry 32. It is known that for crystals of pure axial point groups of symmetry, the polar and axial tensors are of the same structure. Thus, we can use the polar tensor V[V³] taken from [8] as the tensor g_{silm}. This tensor contains six independent components (see Table 1).

Table 1. View of the axial tensor of rank four with internal symmetry εV[V³] for the point group of symmetry 32.

	$m_1m_1m_1$	$m_2m_2m_2$	$m_3m_3m_3$	$m_1m_2m_2$	$m_2m_3m_3$	$m_3m_1m_1$	$m_1m_3m_3$	$m_2m_1m_1$	$m_3m_2m_2$	$m_1m_2m_3$
ϕ_1	$3\,g_{1122}$	0	0	g_{1122}	0	0	g_{2233}	0	0	g_{2311}
ϕ_2	0	$3\,g_{1122}$	0	0	g_{2233}	g_{2311}	0	g_{1122}	$-g_{2311}$	0
ϕ_3	0	$-g_{3211}$	g_{3333}	0	0	g_{3311}	0	g_{3211}	g_{3311}	

In the present work, we will consider AO interaction in the *YZ* plane since the relations for eigenvalues of Christoffel tensor in this plane can be obtained analytically. For this plane, the mentioned tensor can be reduced to the view presented in Table 2.

Table 2. The view of the axial tensor of rank four with internal symmetry εV[V³] for the point group of symmetry 32 for *YZ* plane.

	$m_2m_2m_2$	$m_3m_3m_3$	$m_2m_3m_3$	$m_3m_2m_2$
ϕ_2	$3\,g_{1122}$	0	g_{2233}	$-g_{2311}$
ϕ_3	$-g_{3211}$	g_{3333}	0	g_{3311}

The scalar gyration parameter in the *YZ* plane can be written as:

$$\begin{aligned} G_{ac} = g_{silm}l_sl_il_ll_m &= 3g_{2222}l_2^4 + g_{2233}l_2^2l_3^2 \\ &-g_{2322}l_2^3l_3 - g_{3222}l_3l_2^3 + g_{3333}l_3^4 + g_{3322}l_3^2l_2^2 \\ &= 3g_{1122}\sin^4\Theta + 0.25g_{2233}\sin^2 2\Theta - g_{2311}\sin^3\Theta\cos\Theta \\ &-g_{3211}\cos\Theta\sin^3\Theta + g_{3333}\cos^4\Theta + 0.25g_{3311}\sin^2 2\Theta, \end{aligned} \tag{9}$$

where $l_1 = \sin\Theta\cos\varphi$, $l_2 = \sin\Theta\sin\varphi$, $l_3 = \cos\Theta$ (for *YZ* plane $\varphi = 90\,\text{deg}$). As is seen from Eq. (9) at the propagation of AW along the *Z* axis $G_{ac} = g_{3333}$, while at it propagation along the *Y* axis $G_{ac} = 3g_{1122}$.

The system of Christoffel Eqs. (6) for *YZ* plane can be presented as:

$$\begin{cases} (M_{11} - \rho v^2)p_1 + (M_{12} + i\phi_3)p_2 + (M_{13} - i\phi_2)p_3 = 0 \\ (M_{21} - i\phi_3)p_1 + (M_{22} - \rho v^2)p_2 + M_{23}p_3 = 0 \\ (M_{31} - i\phi_2)p_1 + M_{32}p_2 + (M_{33} - \rho v^2)p_3 = 0 \end{cases}, \tag{10}$$

where

$$\begin{aligned} \phi_2 &= 3g_{1122}m_2^3 + g_{2233}m_2m_3^2 - g_{2311}m_3m_2^2, \\ \phi_3 &= -g_{3211}m_2^3 + g_{3333}m_3^3 + g_{3311}m_3m_2^2, \end{aligned} \tag{11}$$

From Eqs. (10,11) one can obtain the eigenvalues and eigenvectors of the Christoffel tensor and determine the AW velocities and their polarization. Unfortunately, these parameters can be derived only numerically, and their analytical presentation is impossible. Nonetheless, the acoustic activity makes a very small contribution to the velocities of AWs (about 2 m/s). Thus, we have used in our calculations the relations for AWs velocities obtained in our recent work [2]. Let us derive the deformation tensor components caused by the AWs. We will consider only AO interaction with QT AW since, in the quartz crystals, the quasi-longitudinal

AW is not affected by the acoustic activity. Let us consider the relations for effective EO coefficients for the case of AO interaction with QT_1 AW polarization, which belongs to the *YZ* plane in the case of neglecting the ellipticity of AW polarization. The ellipticity of this wave, caused by acoustic activity, is defined as:

$$\chi_{ac} = \frac{u_1}{\sqrt{u_2^2 + u_3^2}}, \tag{12}$$

where u_i is the unit displacement vector projections. The components of the deformation tensor that this AW induces are as follows:

$$\begin{aligned} e_2 &= -\cos\theta \sin\xi, \\ e_3 &= \sin\theta \cos\xi, \\ e_4 &= \cos(\theta + \xi), \\ e_5 &= \chi_{ac} \sin\theta, \\ e_6 &= \chi_{ac} \cos\theta, \end{aligned} \tag{13}$$

where θ is the angle between the *Y* axis and the wavevector of AW and ξ is the angle between the displacement vector and the *Y* axis. The angle $\theta = 90 - \Theta$, and the non-orthogonally angle is determined as $\zeta = 90 - \xi + \theta$. It should be noted that in our recent work [2], only the ellipticity of the incident optical wave has been taken into account, while the ellipticity of the diffracted one is not. In the present work, we account for the ellipticity of the diffracted wave. For the incident light wave, which is polarized parallel to the *X* axis (ordinary wave), the electric field of the diffracted wave can be written as:

$$\begin{aligned} E_1 &= \Delta B_{11} D_1 + \Delta B_{12} D_2 + \Delta B_{13} D_3 \\ E_2 &= E_1 \chi_d \sin\varphi_d \\ E_3 &= E_1 \chi_d \cos\varphi_d \end{aligned}, \tag{14}$$

where the ellipticity of the diffracted optical wave [18]:

$$\begin{aligned} \chi_d &= \frac{1}{2G_o(90-\varphi_d)}\left((n_o^2 - \mathrm{n'}_e^2) - \sqrt{(n_o^2 - \mathrm{n'}_e^2) + 4G_o(90-\varphi_d)^2}\right) \\ &\approx \frac{G_o(90-\varphi_d)}{n_o^2 - n'^2_e} \end{aligned}, \tag{15}$$

the scalar gyration parameter:

$$G_o(90-\varphi_d) = g_{33}\cos^2(90-\varphi_d) + g_{11}\sin^2(90-\varphi_d), \tag{16}$$

and the refractive index

$$\mathrm{n'}^2{}_e = \frac{n_o^2 n_e^2}{n_e^2 \cos^2(90-\varphi_d) + n_o^2 \sin^2(90-\varphi_d)}, \tag{17}$$

D_j is the electrical induction of the incident optical wave, ΔB_{ij} is the increment of the impermeability tensor and φ_d is the angle between *Y* axis and the wavevector of diffracted optical wave. In the further equations for effective EO coefficients, we will use the equalities: $\varphi_i = \theta - \theta_B - 90$, $\varphi_d = \theta + \theta_B - 90$, ($\theta_B = 0.1$ deg).It should be noted that these equations are strictly valid only for ordinary optical beams. Nonetheless, for the extraordinary beam, we have neglected the small deviation of the ellipse of the refractive index from the circle.

The effective EO coefficient peculiar for the III type of AO interaction can be written as:

$$\begin{aligned}
p_{eff}^{2(III)} &= 0.5\{(p_{13}\sin\theta\cos\xi - p_{12}\cos\theta\sin\xi - p_{14}\cos(\theta+\xi))^2\} \\
&+0.5\chi_i^2\chi_{ac}^2\begin{Bmatrix}(p_{14}\sin\theta + p_{66}\cos\theta)^2\sin^2\varphi_i \\ +(p_{44}\sin\theta + p_{41}\cos\theta)^2\cos^2\varphi \\ +(p_{14}\sin\theta + p_{66}\cos\theta)(p_{44}\sin\theta + p_{41}\cos\theta)\sin 2\varphi_i\end{Bmatrix} \\
&+0.5\chi_d^2\{(p_{13}\sin\theta\cos\xi - p_{12}\cos\theta\sin\xi - p_{14}\cos(\theta+\xi))^2\} \\
&+0.5\chi_i^2\chi_{ac}^2\chi_d^2\begin{Bmatrix}(p_{14}\sin\theta + p_{66}\cos\theta)^2\sin^2\varphi_i \\ +(p_{44}\sin\theta + p_{41}\cos\theta)^2\cos^2\varphi \\ +(p_{14}\sin\theta + p_{66}\cos\theta)(p_{44}\sin\theta + p_{41}\cos\theta)\sin 2\varphi_i\end{Bmatrix}
\end{aligned}, \quad (18)$$

where χ_i is the ellipticity of the incident optical wave that is given by the relation

$$\begin{aligned}
\chi_i &= \frac{1}{2G_o(90-\varphi_i)}\left((n_o^2 - \mathrm{n}'^2_e) - \sqrt{(n_o^2 - \mathrm{n}'^2_e) + 4G_o(90-\varphi_i)^2}\right) \\
&\approx \frac{G_o(90-\varphi_i)}{n_o^2 - n'^2_e}
\end{aligned}, \quad (19)$$

G_o is the scalar gyration parameter and φ_i the angle between the Y axis and the propagation direction of the incident optical wave. Note that the parameter

$$G_o(90-\varphi_i) = g_{33}\cos^2(90-\varphi_i) + g_{11}\sin^2(90-\varphi_i), \quad (20)$$

and the refractive index

$$\mathrm{n}'^2{}_e = \frac{n_o^2 n_e^2}{n_e^2\cos^2(90-\varphi_i) + n_o^2\sin^2(90-\varphi_i)} \quad (21)$$

depend on the propagation direction of the incident optical wave. In our case, we accept the Bragg angle equal to $\theta_B = 0.1$ deg . Neglecting its small value leads to $\chi_i \approx \chi_d$.

For the isotropic AO interaction, the Eqs. (14) for the electric field of a diffracted optical wave are as follows:

$$E_1 = E_0\chi_d, \quad E_2 = E_0\sin\varphi_d, \quad E_3 = E_0\cos\varphi_d, \quad (22)$$

where $E_0 = \sqrt{E_2^2 + E_3^2}$ and $E_2 = \Delta B_{21}D_0\chi_i + \Delta B_{22}D_0\sin\varphi_i + \Delta B_{23}D_0\cos\varphi_i$, $E_3 = \Delta B_{31}D_0\chi_i + \Delta B_{32}D_0\sin\varphi_i + \Delta B_{33}D_0\cos\varphi_i$, D_0 is the amplitude of the electrical induction of the incident wave, $D_2 = D_0\sin\varphi_i$, $D_3 = D_0\cos\varphi_i$.

The fourth type of AO interaction of the extraordinary polarized optical wave with QT$_1$ AW is characterized by the effective EO coefficient:

$$\begin{aligned}
p_{eff}^{2(IV)} &= 0.5\begin{Bmatrix}(p_{13}\sin\theta\cos\xi - p_{11}\cos\theta\sin\xi - p_{14}\cos(\xi+\theta))^2\sin^2\varphi_i \\ +(p_{33}\sin\theta\cos\xi - p_{31}\cos\theta\sin\xi\cos\theta\sin\xi)^2\cos^2\varphi_i \\ +(p_{44}\cos(\xi+\theta) + p_{41}\cos\theta\sin\xi)^2 + (p_{44}\cos(\xi+\theta) + p_{41}\cos\theta\sin\xi) \\ \times\sin 2\varphi_i((p_{13}+p_{33})\sin\theta\cos\xi - (p_{31}+p_{11})\cos\theta\sin\xi - p_{14}\cos(\xi+\theta))\end{Bmatrix} \\
&+0.5\chi_i^2\chi_{ac}^2\{(p_{14}\sin\theta + p_{66}\cos\theta)^2 + (p_{44}\sin\theta + p_{41}\cos\theta)^2\} \\
&+0.5\chi_d^2\begin{Bmatrix}(p_{13}\sin\theta\cos\xi - p_{11}\cos\theta\sin\xi - p_{14}\cos(\xi+\theta))^2\sin^2\varphi_i \\ +(p_{33}\sin\theta\cos\xi - p_{31}\cos\theta\sin\xi\cos\theta\sin\xi)^2\cos^2\varphi_i + \left(\mathrm{p}_{44}\cos 2\theta\right)^2 \\ +(p_{44}\cos(\xi+\theta) + p_{41}\cos\theta\sin\xi)^2 + (p_{44}\cos(\xi+\theta) + p_{41}\cos\theta\sin\xi) \\ \times\sin 2\varphi_i((p_{13}+p_{33})\sin\theta\cos\xi - (p_{31}+p_{11})\cos\theta\sin\xi - p_{14}\cos(\xi+\theta))\end{Bmatrix} \\
&+0.5\chi_i^2\chi_{ac}^2\chi_d^2\{(p_{14}\sin\theta + p_{66}\cos\theta)^2 + (p_{44}\sin\theta + p_{41}\cos\theta)^2\}
\end{aligned} \quad (23)$$

For the AW polarized along the *X*-axis (PT_2 AW), the deformation tensor components are as follows:

$$\begin{aligned} &e_1 = -\chi_{ac}\sin 2\theta, \quad e_2 = -\chi_{ac}\sin 2\theta, \quad e_3 = \chi_{ac}\sin 2\theta, \\ &e_4 = \chi_{ac}\cos 2\theta, \quad e_5 = \sin\theta, \quad e_6 = \cos\theta. \end{aligned} \tag{24}$$

It should be noted that this AW is the pure transverse wave (i.e. PT_2 AW). The ellipticity of this wave is determined by the relation:

$$\chi_{ac} = \frac{\sqrt{u_2^2 + u_3^2}}{u_1}. \tag{25}$$

For the V type of isotropic AO interaction where the ordinary polarized optical wave interact with QT_1 AW the effective EO coefficient can be written as:

$$\begin{aligned} p_{eff}^{2(V)} &= 0.5\chi_{ac}^2\left\{((p_{13} - p_{12} - p_{11})\sin 2\theta - p_{14}\cos 2\theta)^2 + p_{44}^2\cos^2 2\theta\right\} \\ &+0.5\chi_i^2\left\{\begin{aligned}&(p_{14}\sin\theta + p_{66}\cos\theta)^2\sin^2\varphi_i \\ &+(p_{44}\sin\theta + p_{41}\cos\theta)^2\cos^2\varphi_i \\ &+(p_{14}\sin\theta + p_{66}\cos\theta)(p_{44}\sin\theta + p_{41}\cos\theta)\sin 2\varphi_i\end{aligned}\right\} \\ &+0.5\chi_{ac}^2\chi_d^2\left\{((p_{13} - p_{12} - p_{11})\sin 2\theta - p_{14}\cos 2\theta)^2 + p_{44}^2\cos^2 2\theta\right\} \\ &+0.5\chi_i^2\chi_d^2\left\{\begin{aligned}&(p_{14}\sin\theta + p_{66}\cos\theta)^2\sin^2\varphi_i + \\ &(p_{44}\sin\theta + p_{41}\cos\theta)^2\cos^2\varphi_i + \\ &(p_{14}\sin\theta + p_{66}\cos\theta)(p_{44}\sin\theta + p_{41}\cos\theta)\sin 2\varphi_i\end{aligned}\right\}. \end{aligned} \tag{26}$$

For the VI type of isotropic AO interaction where the extraordinary polarized optical wave interact with PT_2 AW the effective EO coefficient can be written as:

$$\begin{aligned} p_{eff}^{2(VI)} &= 0.5\chi_{ac}^2\left\{\begin{aligned}&((p_{13} - p_{12} - p_{11})\sin 2\theta - p_{14}\cos 2\theta)^2\sin^2\varphi_i \\ &+(p_{33} - 2p_{31})^2\sin^2 2\theta\cos^2\varphi_i + (p_{44}\cos 2\theta)^2 \\ &+p_{44}\cos 2\theta\sin 2\varphi_i((p_{13} - p_{12} - p_{11} + p_{33} - 2p_{31})\sin 2\theta - p_{14}\cos 2\theta)\end{aligned}\right\} \\ &+0.5\chi_i^2\left\{(p_{14}\sin\theta + p_{66}\cos\theta)^2 + (p_{44}\sin\theta + p_{41}\cos\theta)^2\right\} \\ &+0.5\chi_{ac}^2\chi_d^2\left\{\begin{aligned}&((p_{13} - p_{12} - p_{11})\sin 2\theta - p_{14}\cos 2\theta)^2\sin^2\varphi_i \\ &+(p_{33} - 2p_{31})^2\sin^2 2\theta\cos^2\varphi_i + (p_{44}\cos 2\theta)^2 \\ &+p_{44}\cos 2\theta\sin 2\varphi_i((p_{13} - p_{12} - p_{11} + p_{33} - 2p_{31})\sin 2\theta - p_{14}\cos 2\theta)\end{aligned}\right\} \\ &+0.5\chi_i^2\chi_d^2\left\{(p_{14}\sin\theta + p_{66}\cos\theta)^2 + (p_{44}\sin\theta + p_{41}\cos\theta)^2\right\}. \end{aligned} \tag{27}$$

For the anisotropic diffraction in the case of incident ordinary wave and diffracted extraordinary optical wave the Eqs. (14) are the same as Eqs. (22), while relations for electric field components of diffracted wave are as follows $E_2 = \Delta B_{21}D_1 + \Delta B_{22}\chi_i D_1\sin\varphi_i + \Delta B_{23}\chi_i D_1\cos\varphi_i$, $E_3 = \Delta B_{31}D_1 + \Delta B_{32}\chi_i D_1\sin\varphi_i + \Delta B_{33}\chi_i D_1\cos\varphi_i$, $D_2 = \chi_i D_1\sin\varphi_i$, $D_3 = \chi_i D_1\cos\varphi_i$.

It is seen (Eqs. (18,23)) that for the III and IV types of interaction at the neglecting ellipticity of eigen AWs caused by the acoustic activity, the enhancement of effective EO coefficient is stimulated only by the ellipticity of the diffracted optical wave. On the other side (Eqs. (26,27)) for the V and VI types of interaction, at the neglecting ellipticity of eigen AWs caused by the acoustic activity, the enhancement of effective EO coefficient is stimulated only by the ellipticity of the incident optical wave.

For the VIII type of anisotropic diffraction on the QT$_1$ AW, the effective EO coefficient can be written as:

$$
\begin{aligned}
p_{eff}^{2(VIII)} &= 0.5\chi_{ac}^2\left\{(p_{66}\cos\theta + p_{14}\sin\theta)^2 + (p_{41}\cos\theta + p_{44}\sin\theta)^2\right\}\\
&+0.5\chi_i^2\left\{\begin{aligned}&(p_{41}\cos\theta\sin\xi + p_{44}\cos(\theta+\xi))^2\\ &+(p_{23}\sin\theta\cos\xi - p_{22}\cos\theta\sin\xi - p_{14}\cos(\theta+\xi))^2\sin^2\varphi_i\\ &+(p_{33}\sin\theta\cos\xi - p_{31}\cos\theta\sin\xi)^2\cos^2\varphi_i\\ &+\sin 2\varphi_i((p_{23}+p_{33})\sin\theta\cos\xi - (p_{22}+p_{31})\cos\theta\sin\xi - p_{14}\cos(\theta+\xi))\\ &\times(p_{41}\cos\theta\sin\xi + p_{44}\cos(\theta+\xi))\end{aligned}\right\}\\
&+0.5\chi_{ac}^2\chi_d^2\left\{(p_{66}\cos\theta + p_{14}\sin\theta)^2 + (p_{41}\cos\theta + p_{44}\sin\theta)^2\right\}\\
&+0.5\chi_i^2\chi_d^2\left\{\begin{aligned}&(p_{41}\cos\theta\sin\xi + p_{44}\cos(\theta+\xi))^2\\ &+(p_{23}\sin\theta\cos\xi - p_{22}\cos\theta\sin\xi - p_{14}\cos(\theta+\xi))^2\sin^2\varphi_i\\ &+(p_{33}\sin\theta\cos\xi - p_{31}\cos\theta\sin\xi)^2\cos^2\varphi_i\\ &+\sin 2\varphi_i((p_{23}+p_{33})\sin\theta\cos\xi - (p_{22}+p_{31})\cos\theta\sin\xi - p_{14}\cos(\theta+\xi))\\ &\times(p_{41}\cos\theta\sin\xi + p_{44}\cos(\theta+\xi))\end{aligned}\right\},
\end{aligned}
\tag{28}
$$

where φ_i is the angle between the Y axis and the wavevector of the incident optical wave.

For the eight type of anisotropic AO interaction with the PT$_2$ AW, the effective EO coefficient is written as follows:

$$
\begin{aligned}
p_{eff}^{2(IX)} &= 0.5\left\{(p_{66}\cos\theta + p_{14}\sin\theta)^2 + (p_{41}\cos\theta + p_{44}\sin\theta)^2\right\}\\
&+0.5\chi_d^2\left\{(p_{66}\cos\theta + p_{14}\sin\theta)^2 + (p_{41}\cos\theta + p_{44}\sin\theta)^2\right\}\\
&+0.5\chi_i\chi_{ac}^2\left\{\begin{aligned}&(p_{44}\cos 2\theta)^2\\ &+((p_{13}-p_{11}-p_{12})\sin 2\theta - p_{14}\cos 2\theta)^2\sin^2\varphi_i\\ &+(p_{33}-2p_{31})^2\sin^2 2\theta\cos^2\varphi_i\\ &\sin 2\varphi_i(p_{13}-p_{11}-p_{12}+p_{33}-2p_{31})\sin 2\theta - p_{14}\cos 2\theta)\\ &\times(p_{44}\cos 2\theta)\end{aligned}\right\}\\
&+0.5\chi_i^2\chi_d^2\chi_{ac}^2\left\{\begin{aligned}&(p_{44}\cos 2\Theta)^2\\ &+((p_{13}-p_{11}-p_{12})\sin 2\theta - p_{14}\cos 2\theta)^2\sin^2\varphi_i\\ &+(p_{33}-2p_{31})^2\sin^2 2\theta\cos^2\varphi_i\\ &+\sin 2\varphi_i(p_{13}-p_{11}-p_{12}+p_{33}-2p_{31})\sin 2\theta - p_{14}\cos 2\theta)\\ &\times(p_{44}\cos 2\theta)\end{aligned}\right\}.
\end{aligned}
\tag{29}
$$

It is seen (Eqs. (28,29)) that at the VIII type of AO interaction, the effective EO coefficient foremost is proportional to χ_i^2, while at the IX type – to χ_d^2.

It should be noted that in our recent work [2], the relations for effective EO coefficients contain some logic errors; thus, the results of the present work with those, obtained in [2] cannot be comparable. The AO figure of merit depends on p_{eff} according to the relation:

$$M_2 = \frac{n_i^3 n_d^3 p_{eff}^2}{\rho v^3}, \tag{30}$$

where n_i and n_d are the refractive indices associated respectively with the incident and diffracted optical waves.

The quartz crystals are optically active, with the specific rotation equal to $\rho = 18.8\,\mathrm{deg/mm}$ at the optical wavelength $\lambda = 632.8\,\mathrm{nm}$ [19,20]. The gyration tensor components at this wavelength are equal to $g_{33} = 10.06 \pm 0.07 \times 10^{-5}$ and $g_{11} = -4.8 \pm 0.5 \times 10^{-5}$ for a dextrorotary modification of the crystals [19]. The refractive indices amount to $n_o = 1.543$ and $n_e = 1.552$ [7]. The components p_{ijkl} of the EO tensor at $\lambda = 538\,\mathrm{nm}$ are as follows: $p_{11} = 0.16$, $p_{13} = 0.27$, $p_{12} = 0.27$, $p_{14} = -0.030$, $p_{31} = 0.29$, $p_{33} = 0.10$, $p_{41} = -0.047$ and $p_{44} = -0.079$ (it is supposed that the dispersion of EO coefficients in the region 538–632.8 nm is week enough). The elastic stiffnesses C_{mnkl} are given by $C_{11} = 87.486$, $C_{33} = 107.2$, $C_{12} = 6.244$, $C_{13} = 11.91$, $C_{44} = 57.98$, $C_{66} = 40.626$ and $C_{14} = -18.09$ GPa [21,22]. All of these coefficients have been measured under the condition of constant electric induction. Finally, the density of α-SiO_2 amounts to $\rho = 2649\,\mathrm{kg/m^3}$. The acoustic activity of quartz crystal has been studied in [14], where the component of the acoustic gyration tensor g_{3333} was determined to be equal $5.58\times10^{8}\,\mathrm{N/m^2}$. Other components, which are needed for calculation, are not available in the literature. Therefore, we have taken these components by the random principle to be equal: $g_{1122}=0.66\times10^{7}$, $g_{2233}=4.0\times10^{7}$, $g_{2311}=6.0\times10^{7}$, $g_{3211}=4.0\times10^{7}$, $g_{3311}=9.0\times10^{7}\,\mathrm{N/m^2}$.

3. Results and discussion

3.1. Isotropic diffraction

In the case of isotropic diffraction, one deals with the four types of AO interactions with transverse AWs, which are (III)-(VI) types of AO interactions. For the III type of AO interaction (Fig. 1), the effective AO coefficient and AO figure of merit acquire pick like maximum at $\theta = 0\,\mathrm{deg}$ due to the action of ellipticity of diffracted optical wave. The AO figure of merit (Fig. 1b) increases from the value $0.65\times10^{-15}\,\mathrm{s^3/kg}$ up to the value $1.29\times10^{-15}\,\mathrm{s^3/kg}$. However, the effect of the ellipticity of the QT_2 acoustic wave is negligibly small, since, in Eq. 18, the square of the ellipticity of the acoustic wave appears at the

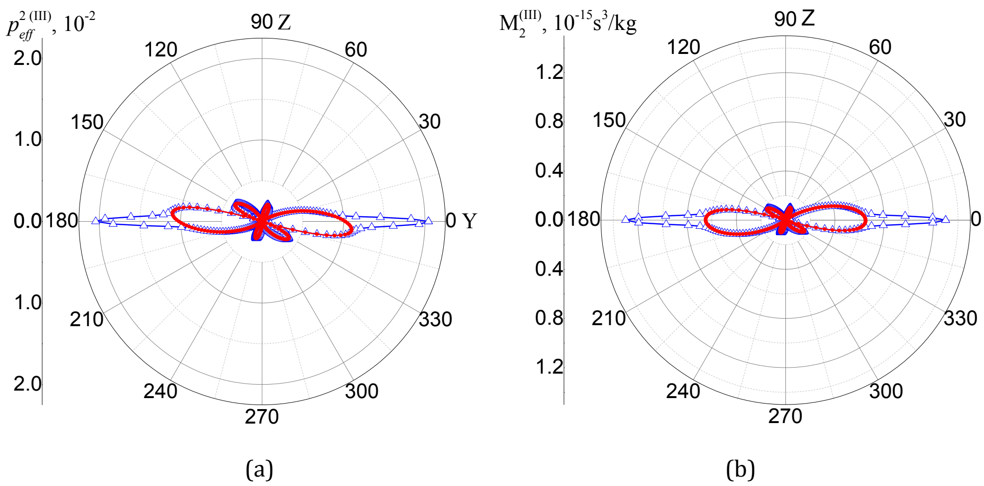
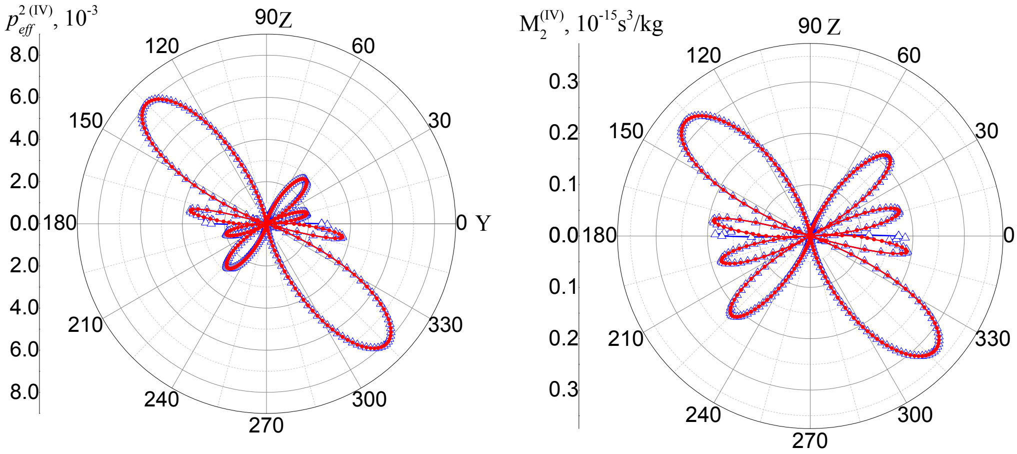


Fig. 1. Dependencies of effective EO coefficient (a) and AO figure of merit (b) on the angle θ with an accounting of ellipticity of incident and diffracted optical waves and ellipticity of AW (triangles) and with an accounting of ellipticity of incident optical wave and ellipticity of AW and neglecting of ellipticity of diffracted optical waves (circles) at the III type of AO interaction.

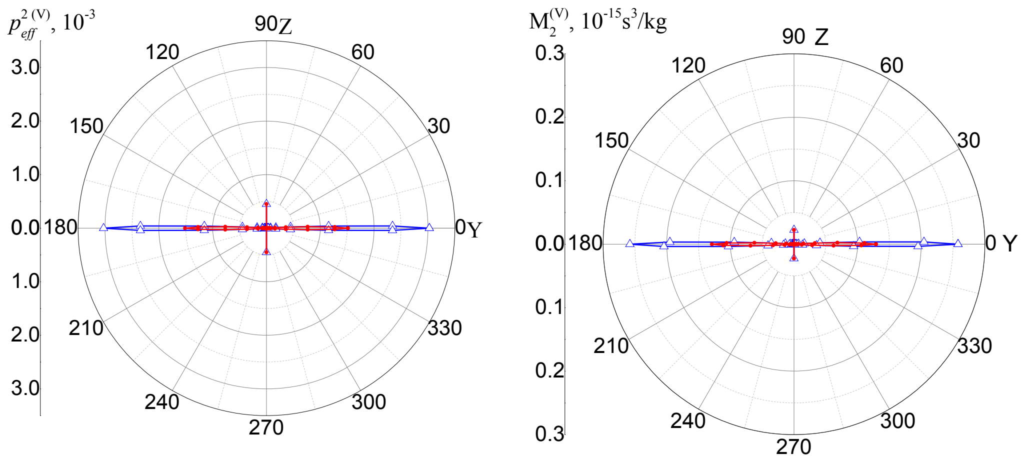
multiplication of the square of the ellipticity of incident or diffracted and incident optical waves. However, at $\theta = 90\,\text{deg}$, where the ellipticity of AW approaches its maximum value (i.e., unity), the ellipticity of optical waves approaches zero. This is also the reason, why the ellipticity of the incident wave has a small effect on the effective EO coefficient and AO figure of merit.

In the IV type of AO interaction (Fig. 2), the behavior of the effective EO coefficient and the AO figure of merit is similar to those in the III type of interaction. However, the maximal value of the AO figure of merit ($0.32\times10^{-15}\,s^3/kg$) is smaller than at the III type of interaction. In addition, the peak of the AO figure of merit caused by the ellipticity of the diffracted optical wave is small (up to $0.18\times10^{-15}\,s^3/kg$).



(a) (b)

Fig. 2. Dependencies of effective EO coefficient (a) and AO figure of merit (b) on the angle θ with an accounting of ellipticity of incident and diffracted optical waves and ellipticity of AW (triangles) and with an accounting of ellipticity of incident optical wave and ellipticity of AW and neglecting of ellipticity of diffracted optical waves (circles) at the IV type of AO interaction.



(a) (b)

Fig. 3. Dependencies of effective EO coefficient (a) and AO figure of merit (b) on the angle θ with an accounting of ellipticity of incident and diffracted optical waves and ellipticity of AW (triangles) and with an accounting of ellipticity of incident optical wave and ellipticity of AW and neglecting of ellipticity of diffracted optical waves (circles) at the V type of AO interaction.

The V type of AO interaction with PT_2 AW is accompanied by the appearance of a peak caused by the AW's ellipticity propagated along the *Z* axis (Fig. 3). This peak is quite narrow, which can be explained by the rapid decrease of the ellipticity of AW at the departure of its direction of propagation from the acoustic axis. Nonetheless, the acoustic activity, which is manifested in the ellipticity of eigenwaves, leads to an increase in effective EO coefficient and AO figure of merit in the case when the ellipticity of the excited AW is the same as the ellipticity of eigen AW. The ellipticity of the incident optical wave increases the AO figure of merit up to 0.13×10^{-15} s^3/kg. In contrast, the ellipticity of both incident and diffracted optical waves causes a further increase of the AO figure of merit up to 0.26×10^{-15} s^3/kg. The last fact concerns the influence of the last term in Eq. (26), which is proportional to $\chi_i^2\chi_d^2$.

At the VI type of AO interaction with PT_2 AW the ellipticity of this AW cause the peak like dependence of efficient EO coefficient and AO figure of merit (Fig. 4) in the case when this wave propagates along the acoustic axis. The AO figure of merit increases almost from zero up to 0.16×10^{-15} s^3/kg. The influence of ellipticity of both incident and diffracted optical waves is the same as at the V type of interaction. The accounting of the ellipticity of both optical waves causes the increase in the AO figure of merit up to 0.26×10^{-15} s^3/kg.

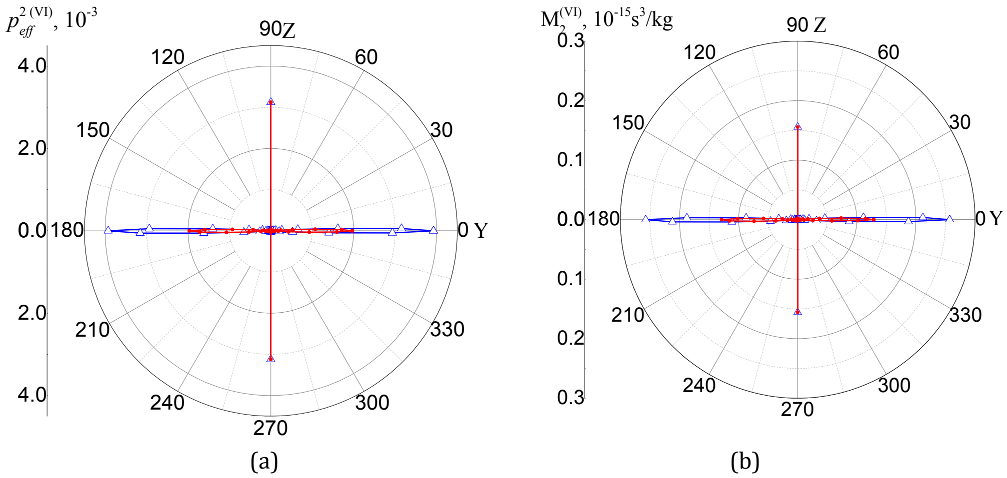


Fig. 4. Dependencies of effective EO coefficient (a) and AO figure of merit (b) on the angle θ with an accounting of ellipticity of incident and diffracted optical waves and ellipticity of AW (triangles) and with an accounting of ellipticity of incident optical wave and ellipticity of AW and neglecting of ellipticity of diffracted optical waves (circles) at the VI type of AO interaction.

3.2. Anisotropic diffraction

Now, let us consider anisotropic AO diffraction on the QT QWs. The results presented in Figs. 5,6 were obtained for three angles of an incident of the optical wave, i.e., 87, 89, and 90 deg. Thus, the incident optical wave propagates near the optical axis. In Fig. 5 the dependencies of effective EO coefficient and AO figure of merit on the diffraction angle are presented for VIII type of AO interaction. The peak of the effective EO coefficient and AO figure of merit caused by the ellipticity of optical waves increases with the approaching incident angle of an optical wave to 90 deg. This process is accompanied by the splitting of the peak. It should be noted that peak-like dependence of effective EO coefficient and AO figure of merit is observed in the vicinity of diffraction angles equal to 0 and 180 deg. These angles correspond to the collinear type of AO diffraction. Exactly at this type of diffraction,

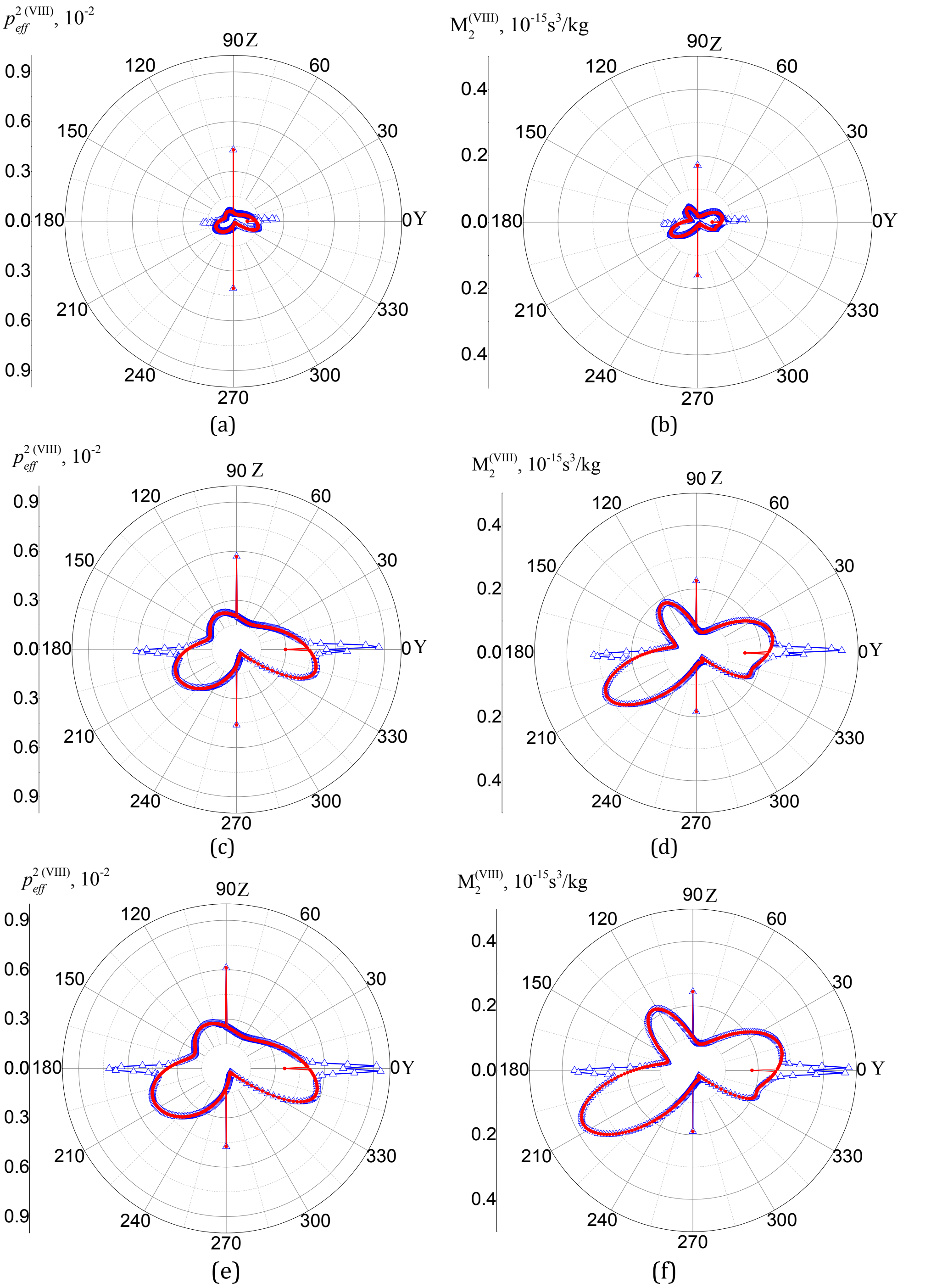


Fig. 5. Dependencies of effective EO coefficient (a, c, e) and AO figure of merit (b, d, f) on the diffraction angle with an accounting of ellipticity of incident and diffracted optical waves and ellipticity of AW (triangles) and with an accounting of ellipticity of incident optical wave and ellipticity of AW and neglecting of ellipticity of diffracted optical waves (circles) at the VIII type of AO interaction. Panels (a, b) correspond to the angle of incidence of optical wave equal to 87 deg; (c, d) - 89 deg; (e, f) - 90 deg.

the non-orthogonality of AW is equal to zero. In such conditions, some terms in Eq. (28) vanish. However, at a slight deviation from the exact conditions of collinear diffraction, the angle of non-orthogonality becomes non-zero, and all terms of Eq. 28 play the role of leading to the appearance of two peaks surrounding the gap. At the same time the peak caused by the ellipticity of AW increases too with approaching incident angle 90 deg. The maximal

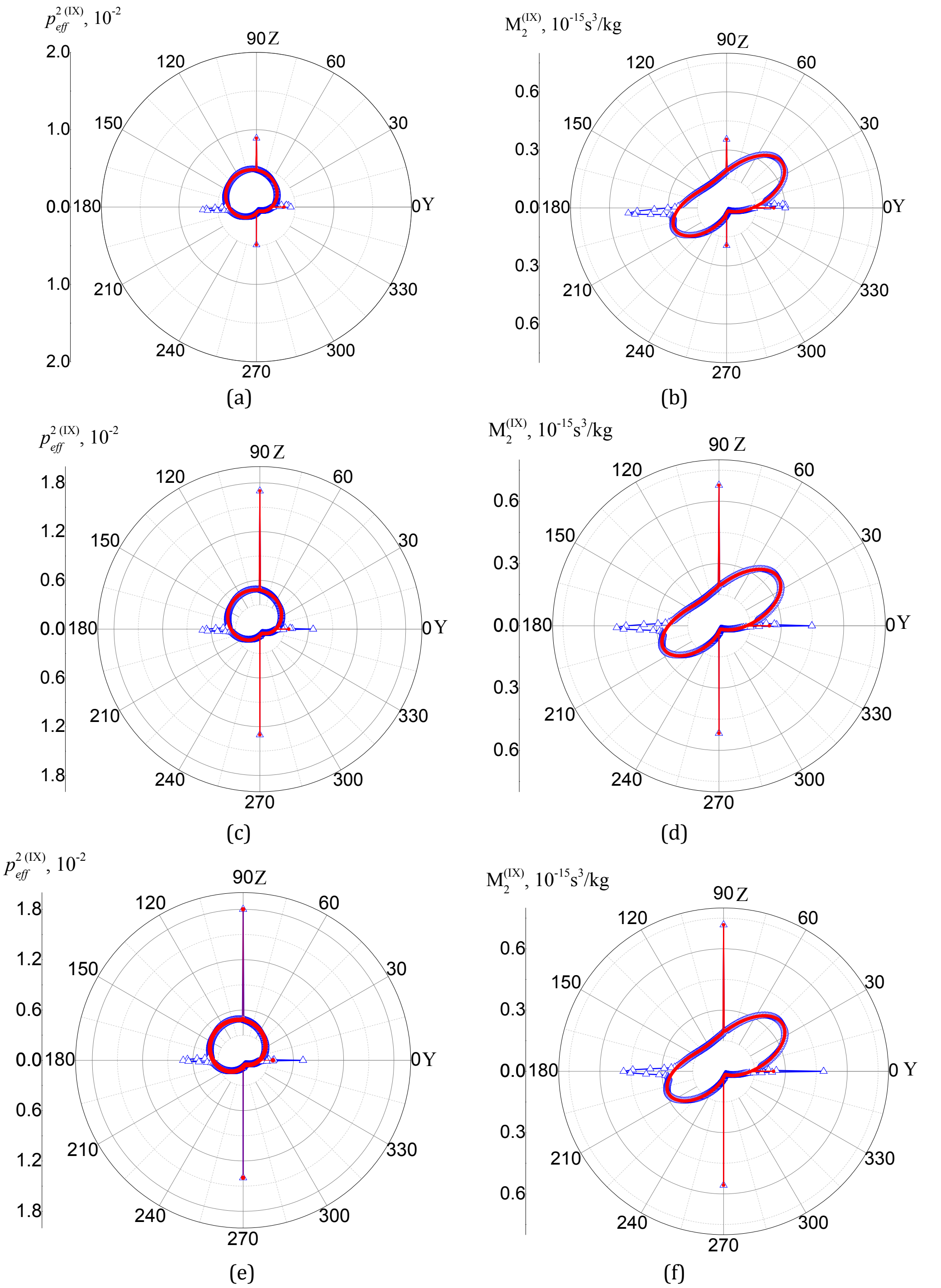


Fig. 6. Dependencies of effective EO coefficient (a, c, e) and AO figure of merit (b, d, f) on the diffraction angle with an accounting of ellipticity of incident and diffracted optical waves and ellipticity of AW (triangles) and with an accounting of ellipticity of incident optical wave and ellipticity of AW and neglecting of ellipticity of diffracted optical waves (circles) at the IX type of AO interaction. Panels (a, b) correspond to the angle of incidence of optical wave equal to 87 deg; (c, d) - 89 deg; (e, f) - 90 deg.

value of AO figure of merit caused by the ellipticity of eigen optical waves is equal to $0.48\times10^{-15}s^3/kg$, while the maximal value of the peak caused by the ellipticity of AW is equal to $0.24\times10^{-15}s^3/kg$ (Fig. 5f). It should be noted that at the absence of ellipticity of optical waves and AW the effective EO coefficient is equal to zero and AO diffraction cannot be realized.

The similar behavior of effective EO coefficient and AO figure of merit is observed at the IX type of AO interaction with PT_2 AW (Fig. 6). Approaching the incident angle of the optical wave to 90 deg leads to increases in peak caused by the ellipticity of optical eigenwaves and the ellipticity of AW. The peak related to the ellipticity of the optical wave is not split since the AW that participates in the AO interaction is the pure transverse AW. The peak value of AO figure of merit caused by the ellipticity of AW exceeds the peak value of the, caused by the ellipticity of optical waves. This value is equal to $0.72\times10^{-15}s^3/kg$, while in the case accounting of ellipticity of optical wave the peak value reach only $0.49\times10^{-15}s^3/kg$ (Fig. 6f). It should be noted that IX type of AO interaction can be realized at the negligibly small ellipticity of optical waves and AW (see Eq.(29)).

4. Conclusions

In the present work, we have shown that the ellipticity of the eigen AWs that are caused by the acoustic activity leads to the peak-like increase of the AO figure of merit whenever the AO interaction is carried out with the elliptical (circular) QT AWs that propagate in the directions close to the acoustic axes. The enhancement of efficiency of AO interaction appears due to the inclusion into the relations for effective EO coefficients, the terms that are proportional to the square of ellipticity of AWs. This effect is similar to the one described in our recent papers [1-4] and which is caused by the ellipticity of optical eigenwaves in optically active crystals. The effect of ellipticity of the QT AWs is analyzed on the example of quartz crystals for all types of AO interactions with QT AWs, and it has been shown that peak-like increasing of AO figure of merit is peculiar for all these types of interactions. In this paper, at the derivation of relations for effective EO coefficients, we have taken into account the ellipticity of diffracted optical waves, which has not been done in our recent works [1,2]. As a result, the obtained relations for effective EO coefficients and calculated values of AO figures of merit became more accurate.

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Анотація. *Дана робота присвячена аналізу впливу еліптичності акустичних хвиль, спричинених акустичною активністю, на ефективність акустооптичної взаємодії. Показано, що пікоподібне зростання ефективних пружнооптичних коефіцієнтів і коефіцієнта акустооптичної якості проявляється у випадку, коли акустооптична дифракція виникає на квазіпоперечних еліптичних (або циркулярних) акустичних хвилях, які поширюються в напрямках, близьких до акустичних осей. У даній роботі проведено аналіз на прикладі кристалів кварцу та показано, що підвищення коефіцієнта акустооптичної якості властиве всім типам акустооптичних взаємодій з квазіпоперечними акустичними хвилями. Показано, що врахування еліптичності дифрагованих оптичних хвиль при аналізі акустооптичної взаємодії в оптично активних кристалах дає більш коректні результати, ніж отримані в наших останніх роботах.*

Ключові слова: *акустична активність, акустооптична дифракція, коефіцієнт акустооптичної якості, еліптичність власних акустичних хвиль*