Ukrainian Journal of Physical Optics
ISSUE 1
2024

OPTICAL SOLITONS FOR THE DISPERSIVE CONCATENATION MODEL BY LAPLACE-ADOMIAN DECOMPOSITION

O. GONZÁLEZ-GAXIOLA[1], ANJAN BISWAS[2,3,4,5], YAKUP YILDIRIM[6,7], ANWAR JAAFAR MOHAMAD JAWAD[8]

[1] Applied Mathematics and Systems Department, Universidad Autónoma Metropolitana-Cuajimalpa, Vasco de Quiroga 4871, 05348 Mexico City, Mexico
[2] Department of Mathematics and Physics, Grambling State University, Grambling, LA 71245-2715, USA
[3] Mathematical Modeling and Applied Computation (MMAC) Research Group, Center of Modern Mathematical Sciences and their Applications (CMMSA) Department of Mathematics, King Abdulaziz University, Jeddah–21589, Saudi Arabia
[4] Department of Applied Sciences, Cross–Border Faculty of Humanities, Economics and Engineering, Dunarea de Jos University of Galati, 111 Domneasca Street, Galati–800201, Romania
[5] Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa–0204, Pretoria, South Africa
[6] Department of Computer Engineering, Biruni University, 34010 Istanbul, Turkey
[7] Department of Mathematics, Near East University, 99138 Nicosia, Cyprus
[8] Department of Computer Technical Engineering, Al–Rafidain University College, 10064 Baghdad, Iraq

Received: 08.11.2023

Abstract. This work numerically studies bright and dark optical solitons that emerge from the dispersive concatenation model, having the Kerr law of nonlinear refractive index, using the Laplace-Adomian decomposition scheme. The simulations, surface plots, and contour plots are presented. The error measure is observed to be infinitesimally small.

Keywords: solitons, Schrödinger equation, concatenation model, Adomian polynomials
UDC: 535.32
DOI: 10.3116/16091833/Ukr.J.Phys.Opt.2024.01094

1. Introduction

About a decade ago, the concept of the concatenation model was conceived. It is the conjoining of the three most frequently studied models in optoelectronics. They are the nonlinear Schrödinger's equation (NLSE), the Sasa-Satsuma equation (SSE), and the Lakshmanan-Porsezian-Daniel (LPD) [1-3]. Subsequently, the concept of the dispersive concatenation model emerged. This is obtained from the Schrödinger-Hirota equation (SHE), the LPD model, and the fifth-order NLSE, thus making it into a dispersive version of the concatenation model that is true to its name [4-6]. Both models were later extensively studied, and their multiple features have been recovered. These include their conservation laws, the numerical simulation of the solitons with the concatenation model by the Laplace-Adomian decomposition method (LADM), recovering the quiescent solitons for nonlinear chromatic dispersion (DC) using Lie symmetry and several other integration schemes; the study of the solitons in the presence of white noise; addressing the model with Lie symmetry and several other features [7-11]. The current paper addresses the dispersive concatenation model numerically by the LADM that would illustrate the bright and dark solitons. The surface plots and contour plots of bright and dark optical solitons are presented. The error plots are also given for both kinds of solitons, and the error measure for both forms of solitons is of the order of 10^{-6}, which is immeasurably small.

2. Model of concatenation with power-law nonlinearity

The dispersive concatenation model takes into account four renowned nonlinear models. They are the SHE, LPD, and NLSE of the fifth order. This is the first time it has been written as [12]:

$$\begin{aligned} & iq_t + aq_{xx} + b|q|^2 q - i\delta_1(\sigma_1 q_{xxx} + \sigma_2 |q|^2 q_x) \\ & +\delta_2\left[\sigma_3 q_{xxxx} + \sigma_4 |q|^2 q_{xx} + \sigma_5 |q|^4 q + \sigma_6 |q_x|^2 q + \sigma_7 (q_x)^2 q^* + \sigma_8 q_{xx}^* q^2\right] \\ & -i\delta_3\begin{bmatrix} \sigma_9 q_{xxxxx} + \sigma_{10}|q|^2 q_{xxx} + \sigma_{11} |q|^4 q_x \\ +\sigma_{12} q q_x q_{xx}^* + \sigma_{13} q^* q_x q_{xx} + \sigma_{14} q q_x^* q_{xx} + \sigma_{15}(q_x)^2 q_x^* \end{bmatrix} = 0. \end{aligned} \tag{1}$$

In Eq. (1), q is a complex-valued function representing the wave profile, while q^* is its complex conjugate qt gives the temporal dispersion, q_x is the spatial dispersion, q_{xx}, q_{xxx} and q_{xxxx} correspond to the higher-order dispersions, and $i^2 = -1$. The first part represents the evolution in time in a linear manner. Constants a and b are the CD and self-phase modulation (SPM) coefficients, respectively. The coefficients σ_j with $j = 1,2,\ldots,15$ and δ_s with $s = 1,2,3$, are all real constants. We can observe in Eq. (1) the following:

- when $\delta_1 = \delta_2 = \delta_3 = 0$, Eq. (1) reduces to the standard NLSE;
- if, $\delta_1 \neq 0$ and $\delta_2 = \delta_3 = 0$, Eq. (1) reduced to SHE;
- for $\delta_1 = \delta_3 = 0$ with $\delta_2 \neq 0$, Eq. (1) yields the LPD equation;
- finally, when $\delta_1 = \delta_2 = 0$ but $\delta_3 \neq 0$, Eq. (1) is reduced to quintic-order NLSE.

Eq. (1) is thus a genuine concatenation of the well-known models that characterize soliton transmission's trans-continental and trans-oceanic dynamics. Using the Adomian decomposition technique in conjunction with the well-known Laplace transform, optical solitons for the model given by Eq. (1) will be presented for the first time. Various types of constraint requirements established for the system's structure can also guarantee the occurrence of solitons. In subsequent sections, particulars are listed and displayed.

3. The solitons in the governing model

The bright soliton solution to Eq. (1), which was recently investigated utilizing the enhanced Kudryashov's method and the Riccati equation expansion approach in [12], is given by

$$q(x,t) = \left[A_1 + B_1 \operatorname{sech}(x - vt)\right] e^{i(-\kappa x + \omega t + \theta_0)}, \tag{2}$$

where the soliton frequency is denoted by ω, the wavenumber by κ, the phase constant by θ_0, and the velocity of the soliton by v. Moreover,

$$A_1 = \pm\frac{1}{2}\sqrt{\frac{-(\Delta_1 + 2\Delta_2) + 2\Delta_3}{\Delta_4}}, \tag{3}$$

$$B_1 = \sqrt{\frac{(\Delta_1 + 6\Delta_2)}{2\Delta_4}}, \tag{4}$$

with Δ_1, Δ_2, Δ_3, and Δ_4 connected with the coefficients of the Eq. (1) as follows:

$$\Delta_1 = \frac{1}{\sigma_9}\left(\sigma_{12} + \sigma_{13} + \sigma_{14}\right), \tag{5}$$

$$\Delta_2 = \frac{\sigma_{10}}{\sigma_9}, \tag{6}$$

$$\Delta_3 = \frac{1}{\sigma_9 \delta_3}\begin{pmatrix} \delta_3 \kappa^2 \sigma_{12} - 3\delta_3 \kappa^2 \sigma_{10} - 3\delta_3 \kappa^2 \sigma_{13} + \delta_3 \kappa^2 \sigma_{14} \\ +\delta_3 \kappa^2 \sigma_{15} + 2\delta_2 \kappa \sigma_4 + 2\delta_2 \kappa \sigma_7 - 2\delta_2 \kappa \sigma_8 + \delta_1 \sigma_2 \end{pmatrix}, \tag{7}$$

$$\Delta_4 = \frac{\sigma_{11}}{\sigma_9}. \tag{8}$$

The constraints that are required for the existence of bright solitons are:

$$\Delta_4\left(\Delta_1 + 6\Delta_2\right) > 0, \tag{9}$$

and

$$\Delta_4\left[\left(\Delta_1 + 2\Delta_2\right) + 2\Delta_3\right] < 0. \tag{10}$$

As shown in [12], the dark solitons for the model given by Eq. (1) have also been found. They are given by:

$$q(x,t) = \left[A_2 + B_2 \tanh\left[\frac{\Delta}{2}(x - vt)\right]\right] \times e^{i(-\kappa x + \omega t + \theta_0)}, \tag{11}$$

where $\Delta \in \mathbb{R}$ and

$$A_2 = \pm \frac{1}{4\Delta_4}\sqrt{2\Delta_4\left[\Delta^2\left(\Delta_1 + 2\Delta_2\right) - 4\Delta_3\right]}, \tag{12}$$

$$B_2 = \pm \frac{\Delta}{4\Delta_4}\sqrt{-2\Delta_4\left(\Delta_1 + 6\Delta_2\right)}. \tag{13}$$

The constraints that are required for the existence of dark solitons are:

$$\Delta_4\left[\Delta^2\left(\Delta_1 + 2\Delta_2\right) - 4\Delta_3\right] > 0, \tag{14}$$

and

$$\Delta_4\left(\Delta_1 + 6\Delta_2\right) < 0. \tag{15}$$

4. Methodology brief overview

In this part, we will provide a concise exposition of the widely used Adomian decomposition method and its enhanced version, achieved through integrating the approach with the Laplace transform [13, 14]. The proposed methodology will be employed to acquire bright solitons for the novel concatenation model with power-law nonlinearity given by Eq. (1). In general, using operators, we can write Eq. (1) as

$$D_t q(x,t) + Lq(x,t) + Nq(x,t) = 0, \tag{16}$$

subject to an initial condition

$$q(x,0) = f(x). \tag{17}$$

In the context of the operational Eq. (16), the operators involved act on a complex-valued function q as:

$$D_t q = i q_t, \tag{18}$$

$$Lq(x,t) = a q_{xx} - i\delta_1 \sigma_1 q_{xxx} + \delta_2 \sigma_3 q_{xxxx} - i\delta_3 \sigma_9 q_{xxxxx}, \tag{19}$$

$$Nq(x,t)=b|q|^2 q-i\delta_1\sigma_2 |q|^2 q_x$$
$$+\delta_2\left[\sigma_4|q|^2 q_{xx}+\sigma_5|q|^4 q+\sigma_6|q_x|^2 q+\sigma_7(q_x)^2q^*+\sigma_8 q^*_{xx}q^2\right] \tag{20}$$
$$-i\delta_3\left[\sigma_{10}|q|^2 q_{xxx}+\sigma_{11}|q|^4 q_x+\sigma_{12}qq_xq^*_{xx}+\sigma_{13}q^*q_xq_{xx}+\sigma_{14}qq^*_xq_{xx}+\sigma_{15}(q_x)^2q^*_x\right].$$

It is clear that the operator N is nonlinear. Consequently, according to the Adomian decomposition approach, it may be decomposed into a series:

$$Nq(x,t)=\sum_{k=0}^{\infty}M_k(q_0\ \ldots,q_k), \tag{21}$$

where each of the M_k is an Adomian polynomial [15]. Also, by the Adomian decomposition method we have

$$q(x,t)=\sum_{k=0}^{\infty}q_k(x\ t). \tag{22}$$

To conveniently represent the nonlinear operator denoted by Eq. (21), we may write it as

$$Nq(x,t)=\sum_{l=1}^{13}N_l q(x\ t), \tag{23}$$

where

$$\begin{gathered}
N_1q=b|q|^2 q,\quad N_2q=-i\delta_1\sigma_2|q|^2 q_x,\quad N_3q=\delta_2\sigma_4|q|^2 q_{xx},\quad N_4q=\delta_2\sigma_5|q|^4 q,\\
N_5q=\delta_2\sigma_6|q_x|^2 q,\quad N_6q=\delta_2\sigma_7(q_x)^2q^*,\quad N_7q=\delta_2\sigma_8q^*_{xx}q^2,\\
N_8q=-i\delta_3\sigma_{10}|q|^2 q_{xxx},\quad N_9q=-i\delta_3\sigma_{11}|q|^4 q_x,\quad N_{10}q=-i\delta_3\sigma_{12}qq_xq^*_{xx},\\
N_{11}q=-i\delta_3\sigma_{13}q^*q_xq_{xx},\quad N_{12}q=-i\delta_3\sigma_{14}qq^*_xq_{xx},\quad N_{13}q=-i\delta_3\sigma_{15}(q_x)^2q^*_x,
\end{gathered} \tag{24}$$

and all nonlinear components $N_1,\ldots,N_{13}$ can be decomposed into an infinite series of Adomian polynomials given by:

$$N_lq=\sum_{k=0}^{\infty}M_k^l(q_0,q_1,\ldots,q_k),\quad l=1,2,\ldots,13. \tag{25}$$

M_k^l represents the Adomian polynomials for each $l=1,2,\ldots,13$ in Eq. (25), which can be calculated using the formulas established in [16], i.e.

$$M_k^l(q_0,q_1,\ldots,q_n)=\begin{cases}N_l(q_0), & k=0\\[2ex] \dfrac{1}{k}\sum_{i=0}^{k-1}(i+1)q_{i+1}\dfrac{\partial}{\partial q_0}M^l_{n-1-i}, & k=1,2,3,\ldots\end{cases} \tag{26}$$

In this context, the symbol $\mathcal{L}$ will be used to represent the Laplace transform, while $\mathcal{L}^{-1}$ will represent its inverse operator. Next, we apply the Laplace transform $\mathcal{L}$ to both sides of the operational Eq. (16) to obtain

$$\mathcal{L}\{D_tq(x,t)+Lq(x,t)+Nq(x,t)\}=0. \tag{27}$$

By utilizing the initial condition, which is obtained from the initial profiles of the solitons f, we acquire

$$\mathcal{L}\{q(x,t)\}=\frac{1}{s}f(x)-\frac{1}{s}\left(\mathcal{L}\{Lq(x,t)\}+\mathcal{L}\{Nq(x,t)\}\right). \tag{28}$$

By substituting the Eqs. (21, 22, 25) into Eq. (28), we get

$$\mathcal{L}\left\{\sum_{k=0}^{\infty} q_k(x\ t)\right\} = \frac{1}{s}f(x) - \frac{1}{s}\left(\mathcal{L}\left\{L\left(\sum_{k=0}^{\infty} q_k(x\ t)\right)\right\} + \mathcal{L}\left\{\sum_{j=1}^{8}\sum_{k=0}^{\infty} M_k^j(q_0,\ldots,q_k)\right\}\right). \tag{29}$$

By equating both sides of Eq. (29), we can calculate the Laplace transform of each individual component of the solution, that is

$$\mathcal{L}\{q_0(x,t)\} = \frac{1}{s}f(x). \tag{30}$$

The recursive relations can be written as follows for all values of m that are greater than one:

$$\mathcal{L}\{q_m(x,t)\} = -\frac{1}{s}\left(\mathcal{L}\{Lq_{m-1}(x,t)\} + \mathcal{L}\left\{\sum_{j=1}^{8},\sum_{m=0}^{\infty} M_{m-1}^j(q_0\ \ldots,q_{m-1})\right\}\right). \tag{31}$$

In order to calculate Adomian polynomials, we will focus on the nonlinear operators N_j acting on the function q described in Eq. (24). By applying the formula (26), for example, for $n=1$, we may get the following results:

$$M_0^1 = bq_0^2q_0^*,$$

$$M_1^1 = b\left(q_1^*q_0^2 + 2q_0^*q_1q_0\right),$$

$$M_2^1 = b\left(q_2^*q_0^2 + q_0^*q_1^2 + 2q_0^*q_2q_0 + 2q_1^*q_1q_0\right),$$

$$M_3^1 = b\left(q_3^*q_0^2 + q_1^*q_1^2 + 2q_2^*q_1q_0 + 2q_1^*q_2q_0 + 2q_0^*q_3q_0 + 2q_0^*q_1q_2\right),$$

$$M_4^1 = b\left(q_4^*q_0^2 + q_2^*q_1^2 + q_0^*q_2^2 + 2q_3^*q_1q_0 + 2q_2^*q_2q_0 + 2q_1^*q_3q_0 + 2q_0^*q_4q_0 + 2q_1^*q_1q_2 + 2q_0^*q_1q_3\right),$$

$$\vdots$$

$$M_0^2 = -i\delta_1\sigma_2q_0q_0^*q_{0x},\ M_1^2 = -i\delta_1\sigma_2\left(q_1q_0^*q_{0x} + q_0q_1^*q_{0x} + q_0q_0^*q_{1x}\right),$$

$$M_2^2 = -i\delta_1\sigma_2\left(q_2q_0^*q_{0x} + q_1q_1^*q_{0x} + q_0q_2^*q_{0x} + q_1q_0^*q_{1x} + q_0q_1^*q_{1x} + q_0q_0^*q_{2x}\right),$$

$$M_3^2 = -i\delta_1\sigma_2(q_3q_0^*q_{0x} + q_2q_1^*q_{0x} + q_1q_2^*q_{0x} + q_0q_3^*q_{0x} + q_2q_0^*q_{1x} + q_1q_1^*q_{1x} + q_0q_2^*q_{1x} + q_1q_0^*q_{2x} + q_0q_1^*q_{2x} + q_0q_0^*q_{3x}),$$

$$M_4^2 = -i\delta_1\sigma_2(q_4q_0^*q_{0x} + q_3q_1^*q_{0x} + q_2q_2^*q_{0x} + q_1q_3^*q_{0x} + q_0q_4^*q_{0x} + q_3q_0^*q_{1x} + q_2q_1^*q_{1x} + q_1q_2^*q_{1x} + q_0q_3^*q_{1x} + q_2q_0^*q_{2x} + q_1q_1^*q_{2x} + q_0q_2^*q_{2x} + q_1q_0^*q_{3x} + q_0q_1^*q_{3x} + q_0q_0^*q_{4x}),$$

$$\vdots$$

$$M_0^3 = \delta_2\sigma_4q_0^*q_0q_{0xx},\ M_1^3 = \delta_2\sigma_4\left(q_1^*q_0q_{0xx} + q_0^*q_1q_{0xx} + q_0^*q_0q_{1xx}\right),$$

$$M_2^3 = \delta_2\sigma_4\left(q_2^*q_0q_{0xx} + q_1^*q_1q_{0xx} + q_0^*q_2q_{0xx} + q_1^*q_0q_{1xx} + q_0^*q_1q_{1xx} + q_0^*q_0q_{2xx}\right),$$

$$M_3^3 = \delta_2\sigma_4(q_3^*q_0q_{0xx} + q_2^*q_1q_{0xx} + q_1^*q_2q_{0xx} + q_0^*q_3q_{0xx} + q_2^*q_0q_{1xx} + q_1^*q_1q_{1xx} + q_0^*q_2q_{1xx} + q_1^*q_0q_{2xx} + q_0^*q_1q_{2xx} + q_0^*q_0q_{3xx}),$$

$$M_4^3 = \delta_2\sigma_4(q_4^*q_0q_{0xx} + q_3^*q_1q_{0xx} + q_2^*q_2q_{0xx} + q_1^*q_3q_{0xx} + q_0^*q_4q_{0xx} + q_3^*q_0q_{1xx} + q_2^*q_1q_{1xx} + q_1^*q_2q_{1xx} + q_0^*q_3q_{1xx} + q_2^*q_0q_{2xx} + q_1^*q_1q_{2xx} + q_0^*q_2q_{2xx} + q_1^*q_0q_{3xx} + q_0^*q_1q_{3xx} + q_0^*q_0q_{4xx}),$$

$$\vdots$$

$$M_0^4=\delta_2\sigma_5 q_0^{*2}q_0^3,$$

$$M_1^4=\delta_2\sigma_5\left(2q_0^*q_1^*q_0^3+3q_0^{*2}q_1q_0^2\right),$$

$$M_2^4=\delta_2\sigma_5\left(q_1^{*2}q_0^3+2q_0^*q_2^*q_0^3+6q_0^*q_1^*q_1q_0^2+3q_0^{*2}q_2q_0^2+3q_0^{*2}q_1^2q_0\right),$$

$$M_3^4=\delta_2\sigma_5(2q_1^*q_2^*q_0^3+2q_0^*q_3^*q_0^3+3q_1^{*2}q_1q_0^2+6q_0^*q_2^*q_1q_0^2+6q_0^*q_1^*q_2q_0^2+3q_0^{*2}q_3q_0^2$$
$$+6q_0^*q_1^*q_1^2q_0+6q_0^{*2}q_1q_2q_0+q_0^{*2}q_1^3),$$

$$M_4^4=\delta_2\sigma_5(q_2^{*2}q_0^3+2q_1^*q_3^*q_0^3+2q_0^*q_4^*q_0^3+6q_1^*q_2^*q_1q_0^2+6q_0^*q_3^*q_1q_0^2+3q_1^{*2}q_2q_0^2+6q_0^*q_2^*q_2q_0^2$$
$$+6q_0^*q_1^*q_3q_0^2+3q_0^{*2}q_4q_0^2+3q_1^{*2}q_1^2q_0+6q_0^*q_2^*q_1^2q_0+3q_0^{*2}q_2^2q_0+12q_0^*q_1^*q_1q_2q_0+6q_0^{*2}q_1q_3q_0$$
$$+2q_0^*q_1^*q_1^3+3q_0^{*2}q_1^2q_2),$$

$$\vdots$$

$$M_0^5=\delta_2\sigma_6 q_0q_{0x}^*q_{0x},$$

$$M_1^5=\delta_2\sigma_6\left(q_0q_{0x}^*q_{1x}+q_0q_{1x}^*q_{0x}+q_1q_{0x}^*q_{0x}\right),$$

$$M_2^5=\delta_2\sigma_6\left(q_0q_{0x}^*q_{2x}+q_0q_{1x}^*q_{1x}+q_0q_{2x}^*q_{0x}+q_1q_{0x}^*q_{1x}+q_1q_{1x}^*q_{0x}+q_2q_{0x}^*q_{0x}\right),$$

$$M_3^5=\delta_2\sigma_6(q_0q_{0x}^*q_{3x}+q_0q_{1x}^*q_{2x}+q_0q_{2x}^*q_{1x}+q_0q_{3x}^*q_{0x}+q_1q_{0x}^*q_{2x}$$
$$+q_1q_{1x}^*q_{1x}+q_1q_{2x}^*q_{0x}+q_2q_{0x}^*q_{1x}+q_2q_{1x}^*q_{0x}+q_3q_{0x}^*q_{0x}),$$

$$M_4^5=\delta_2\sigma_6(q_0q_{0x}^*q_{4x}+q_0q_{1x}^*q_{3x}+q_0q_{2x}^*q_{2x}+q_0q_{3x}^*q_{1x}+q_0q_{4x}^*q_{0x}+q_1q_{0x}^*q_{3x}+q_1q_{1x}^*q_{2x}$$
$$+q_1q_{2x}^*q_{1x}+q_1q_{3x}^*q_{0x}+q_2q_{0x}^*q_{2x}+q_2q_{1x}^*q_{1x}+q_2q_{2x}^*q_{0x}+q_3q_{0x}^*q_{1x}+q_3q_{1x}^*q_{0x}+q_4q_{0x}^*q_{0x}),$$

$$\vdots$$

$$M_0^6=\delta_2\sigma_7 q_{0x}^2q_0^*,$$

$$M_1^6=\delta_2\sigma_7\left(q_1^*q_{0x}^2+2q_0^*q_{1x}q_{0x}\right),$$

$$M_2^6=\delta_2\sigma_7\left(q_2^*q_{0x}^2+q_0^*q_{1x}^2+2q_0^*q_{2x}q_{0x}+2q_1^*q_{1x}q_{0x}\right),$$

$$M_3^6=\delta_2\sigma_7\left(q_3^*q_{0x}^2+q_1^*q_{1x}^2+2q_2^*q_{1x}q_{0x}+2q_1^*q_{2x}q_{0x}+2q_0^*q_{3x}q_{0x}+2q_0^*q_{1x}q_{2x}\right),$$

$$M_4^6=\delta_2\sigma_7(q_4^*q_{0x}^2+q_2^*q_{1x}^2+q_0^*q_{2x}^2+2q_3^*q_{1x}q_{0x}+2q_2^*q_{2x}q_{0x}+2q_1^*q_{3x}q_{0x}+2q_0^*q_{4x}q_{0x}$$
$$+2q_1^*q_{1x}q_{2x}+2q_0^*q_{1x}q_{3x}),$$

$$\vdots$$

$$M_0^7=\delta_2\sigma_8 q_0^2q_{0xx}^*,$$

$$M_1^7=\delta_2\sigma_8\left(q_0^2q_{1xx}^*+2q_0q_1q_{0xx}^*\right),$$

$$M_2^7=\delta_2\sigma_8\left(q_1^2q_{0xx}^*+2q_0q_2q_{0xx}^*+2q_0q_1q_{1xx}^*+q_0^2q_{2xx}^*\right),$$

$$M_3^7=\delta_2\sigma_8\left(q_1^2q_{1xx}^*+2q_1q_2q_{0xx}^*+2q_0q_3q_{0xx}^*+2q_0q_2q_{1xx}^*+2q_0q_1q_{2xx}^*+q_0^2q_{3xx}^*\right),$$

$$M_4^7=\delta_2\sigma_8(q_1^2q_{2xx}^*+2q_1q_2q_{1xx}^*+q_2^2q_{0xx}^*+2q_1q_3q_{0xx}^*+2q_0q_4q_{0xx}^*$$
$$+2q_0q_3q_{1xx}^*+2q_0q_2q_{2xx}^*+2q_0q_1q_{3xx}^*+q_0^2q_{4xx}^*),$$

$$\vdots$$

$$M_0^8=-i\delta_3\sigma_{10}q_0q_0^*q_{0xxx},$$

$$M_1^8=-i\delta_3\sigma_{10}\left(q_0q_0^*q_{1xxx}+q_0q_1^*q_{0xxx}+q_1q_0^*q_{0xxx}\right),$$

$$M_2^8=-i\delta_3\sigma_{10}\left(q_0q_0^*q_{2xxx}+q_0q_1^*q_{1xxx}+q_0q_2^*q_{0xxx}+q_1q_0^*q_{1xxx}+q_1q_1^*q_{0xxx}+q_2q_0^*q_{0xxx}\right),$$

$$M_3^8=-i\delta_3\sigma_{10}(q_0q_0^*q_{3xxx}+q_0q_1^*q_{2xxx}+q_0q_2^*q_{1xxx}+q_0q_3^*q_{0xxx}+q_1q_0^*q_{2xxx}+q_1q_1^*q_{1xxx}$$
$$+q_1q_2^*q_{0xxx}+q_2q_0^*q_{1xxx}+q_2q_1^*q_{0xxx}+q_3q_0^*q_{0xxx}),$$
$$M_4^8=-i\delta_3\sigma_{10}(q_0q_0^*q_{4xxx}+q_0q_1^*q_{3xxx}+q_0q_2^*q_{2xxx}+q_0q_3^*q_{1xxx}+q_0q_4^*q_{0xxx}$$
$$+q_1q_0^*q_{3xxx}+q_1q_1^*q_{2xxx}+q_1q_2^*q_{1xxx}+q_1q_3^*q_{0xxx}+q_2q_0^*q_{2xxx}$$
$$+q_2q_1^*q_{1xxx}+q_2q_2^*q_{0xxx}+q_3q_0^*q_{1xxx}+q_3q_1^*q_{0xxx}+q_4q_0^*q_{0xxx}),$$
$$\vdots$$
$$M_0^9=-i\delta_3\sigma_{11}q_0^2q_0^{*2}q_{0x},$$
$$M_1^9=-i\delta_3\sigma_{11}\left(q_1^2q_0^{*2}q_{0x}+q_0^2q_1^{*2}q_{0x}+q_0^2q_0^{*2}q_{1x}\right),$$
$$M_2^9=-i\delta_3\sigma_{11}\left(q_2^2q_0^{*2}q_{0x}+q_1^2q_1^{*2}q_{0x}+q_0^2q_2^{*2}q_{0x}+q_1^2q_0^{*2}q_{1x}+q_0^2q_1^{*2}q_{1x}+q_0^2q_0^{*2}q_{2x}\right),$$
$$M_3^9=-i\delta_3\sigma_{11}(q_3^2q_0^{*2}q_{0x}+q_2^2q_1^{*2}q_{0x}+q_1^2q_2^{*2}q_{0x}+q_0^2q_3^{*2}q_{0x}+q_2^2q_0^{*2}q_{1x}+q_1^2q_1^{*2}q_{1x}+q_0^2q_2^{*2}q_{1x}$$
$$+q_1^2q_0^{*2}q_{2x}+q_0^2q_1^{*2}q_{2x}+q_0^2q_0^{*2}q_{3x}),$$
$$M_4^9=-i\delta_3\sigma_{11}(q_4^2q_0^{*2}q_{0x}+q_3^2q_1^{*2}q_{0x}+q_2^2q_2^{*2}q_{0x}+q_1^2q_3^{*2}q_{0x}+q_0^2q_4^{*2}q_{0x}+q_3^2q_0^{*2}q_{1x}+q_2^2q_1^{*2}q_{1x}$$
$$+q_1^2q_2^{*2}q_{1x}+q_0^2q_3^{*2}q_{1x}+q_2^2q_0^{*2}q_{2x}+q_1^2q_1^{*2}q_{2x}+q_0^2q_2^{*2}q_{2x}+q_1^2q_0^{*2}q_{3x}+q_0^2q_1^{*2}q_{3x}+q_0^2q_0^{*2}q_{4x}),$$
$$\vdots$$
$$M_0^{10}=-i\delta_3\sigma_{12}q_0q_{0xx}^*q_{0x},$$
$$M_1^{10}=-i\delta_3\sigma_{12}\left(q_0q_{0xx}^*q_{1x}+q_0q_{1xx}^*q_{0x}+q_1q_{0xx}^*q_{0x}\right),$$
$$M_2^{10}=-i\delta_3\sigma_{12}\left(q_0q_{0xx}^*q_{2x}+q_0q_{1xx}^*q_{1x}+q_0q_{2xx}^*q_{0x}+q_1q_{0xx}^*q_{1x}+q_1q_{1xx}^*q_{0x}+q_2q_{0xx}^*q_{0x}\right),$$
$$M_3^{10}=-i\delta_3\sigma_{12}(q_0q_{0xx}^*q_{3x}+q_0q_{1xx}^*q_{2x}+q_0q_{2xx}^*q_{1x}+q_0q_{3xx}^*q_{0x}+q_1q_{0xx}^*q_{2x}$$
$$+q_1q_{1xx}^*q_{1x}+q_1q_{2xx}^*q_{0x}+q_2q_{0x}^*q_{1xx}+q_2q_{1xx}^*q_{0x}+q_3q_{0xx}^*q_{0x}),$$
$$M_4^{10}=-i\delta_3\sigma_{12}(q_0q_{0xx}^*q_{4x}+q_0q_{1xx}^*q_{3x}+q_0q_{2xx}^*q_{2x}+q_0q_{3xx}^*q_{1x}$$
$$+q_0q_{4xx}^*q_{0x}+q_1q_{0xx}^*q_{3x}+q_1q_{1xx}^*q_{2x}+q_1q_{2xx}^*q_{1x}+q_1q_{3xx}^*q_{0x}$$
$$+q_2q_{0xx}^*q_{2x}+q_2q_{1xx}^*q_{1x}+q_2q_{2xx}^*q_{0x}+q_3q_{0xx}^*q_{1x}+q_3q_{1xx}^*q_{0x}+q_4q_{0xx}^*q_{0x}),$$
$$\vdots$$
$$M_0^{11}=-i\delta_3\sigma_{13}q_0^*q_{0x}q_{0xx},$$
$$M_1^{11}=-i\delta_3\sigma_{13}\left(q_1^*q_{0x}q_{0xx}+q_0^*q_{1x}q_{0xx}+q_0^*q_{0x}q_{1xx}\right),$$
$$M_2^{11}=-i\delta_3\sigma_{13}\left(q_2^*q_{0x}q_{0xx}+q_1^*q_{1x}q_{0xx}+q_0^*q_{2x}q_{0xx}+q_1^*q_{0x}q_{1xx}+q_0^*q_{1x}q_{1xx}+q_0^*q_{0x}q_{2xx}\right),$$
$$M_3^{11}=-i\delta_3\sigma_{13}(q_3^*q_{0x}q_{0xx}+q_2^*q_{1x}q_{0xx}+q_1^*q_{2x}q_{0xx}+q_0^*q_{3x}q_{0xx}+q_2^*q_{0x}q_{1xx}$$
$$+q_1^*q_{1x}q_{1xx}+q_0^*q_{2x}q_{1xx}+q_1^*q_{0x}q_{2xx}+q_0^*q_{1x}q_{2xx}+q_0^*q_{0x}q_{3xx}),$$
$$M_4^{11}=-\delta_3\sigma_{13}(q_4^*q_{0x}q_{0xx}+q_3^*q_{1x}q_{0xx}+q_2^*q_{2x}q_{0xx}+q_1^*q_{3x}q_{0xx}+q_0^*q_{4x}q_{0xx}$$
$$+q_3^*q_{0x}q_{1xx}+q_2^*q_{1x}q_{1xx}+q_1^*q_{2x}q_{1xx}+q_0^*q_{3x}q_{1xx}+q_2^*q_{0x}q_{2xx}+q_1^*q_{1x}q_{2xx}$$
$$+q_0^*q_{2x}q_{2xx}+q_1^*q_{0x}q_{3xx}+q_0^*q_{1x}q_{3xx}+q_0^*q_{0x}q_{4xx}),$$
$$\vdots$$
$$M_0^{12}=-i\delta_3\sigma_{14}q_{0x}^*q_0q_{0xx},$$
$$M_1^{12}=-i\delta_3\sigma_{14}\left(q_{1x}^*q_0q_{0xx}+q_{0x}^*q_1q_{0xx}+q_{0x}^*q_0q_{1xx}\right),$$
$$M_2^{12}=-i\delta_3\sigma_{14}\left(q_{2x}^*q_0q_{0xx}+q_{1x}^*q_1q_{0xx}+q_{0x}^*q_2q_{0xx}+q_{1x}^*q_0q_{1xx}+q_{0x}^*q_1q_{1xx}+q_{0x}^*q_0q_{2xx}\right),$$
$$M_3^{12}=-i\delta_3\sigma_{14}(q_{3x}^*q_0q_{0xx}+q_{2x}^*q_1q_{0xx}+q_{1x}^*q_2q_{0xx}+q_{0x}^*q_3q_{0xx}+q_{2x}^*q_0q_{1xx}$$
$$+q_{1x}^*q_1q_{1xx}+q_{0x}^*q_2q_{1xx}+q_{1x}^*q_0q_{2xx}+q_{0x}^*q_1q_{2xx}+q_{0x}^*q_0q_{3xx}),$$

$$M_4^{12} = -\delta_3\sigma_{14}(q_{4x}^* q_0 q_{0xx} + q_{3x}^* q_1 q_{0xx} + q_{2x}^* q_2 q_{0xx} + q_{1x}^* q_3 q_{0xx} + q_{0x}^* q_4 q_{0xx}$$
$$+q_{3x}^* q_0 q_{1xx} + q_{2x}^* q_1 q_{1xx} + q_{1x}^* q_2 q_{1xx} + q_{0x}^* q_3 q_{1xx} + q_{2x}^* q_0 q_{2xx} + q_{1x}^* q_1 q_{2xx}$$
$$+q_{0x}^* q_2 q_{2xx} + q_{1x}^* q_0 q_{3xx} + q_{0x}^* q_1 q_{3xx} + q_{0x}^* q_0 q_{4xx}),$$
$$\vdots$$
$$M_0^{13} = -i\delta_3\sigma_{15} q_{0x}^2 q_{0x}^*,$$
$$M_1^{13} = -i\delta_3\sigma_{15}\left(q_{1x}^* q_{0x}^2 + 2q_{0x}^* q_{1x} q_{0x}\right),$$
$$M_2^{13} = -i\delta_3\sigma_{15}\left(q_{2x}^* q_{0x}^2 + q_{0x}^* q_{1x}^2 + 2q_{0x}^* q_{2x} q_{0x} + 2q_{1x}^* q_{1x} q_{0x}\right),$$
$$M_3^{13} = -i\delta_3\sigma_{15}\left(q_{3x}^* q_{0x}^2 + q_{1x}^* q_{1x}^2 + 2q_{2x}^* q_{1x} q_{0x} + 2q_{1x}^* q_{2x} q_{0x} + 2q_{0x}^* q_{3x} q_{0x} + 2q_{0x}^* q_{1x} q_{2x}\right),$$
$$M_4^{13} = -i\delta_3\sigma_{15}(q_{4x}^* q_{0x}^2 + q_{2x}^* q_{1x}^2 + q_{0x}^* q_{2x}^2 + 2q_{3x}^* q_{1x} q_{0x} + 2q_{2x}^* q_{2x} q_{0x}$$
$$+2q_{1x}^* q_{3x} q_{0x} + 2q_{0x}^* q_{4x} q_{0x} + 2q_{1x}^* q_{1x} q_{2x} + 2q_{0x}^* q_{1x} q_{3x}),$$
$$\vdots$$

and similarly for a variety of other Adomian polynomials.

Eventually, when contemplating the inverse Laplace transform $\mathcal{L}^{-1}$, the components q_0, q_1, q_2, and so forth, are subsequently ascertained through an iterative procedure, which is given as:

$$\begin{cases} q_0(x,t) = f(x), \\ q_1(x,t) = -\mathcal{L}^{-1}\Big(\frac{1}{s}\mathcal{L}\{Rq_0(x,t)\} + \frac{1}{s}\Big[\mathcal{L}\Big\{\sum_{j=1}^{13} P_0^j(q_0)\Big\}\Big]\Big), \\ q_2(x,t) = -\mathcal{L}^{-1}\Big(\frac{1}{s}\mathcal{L}\{Rq_1(x,t)\} + \frac{1}{s}\Big[\mathcal{L}\Big\{\sum_{j=1}^{13} P_1^j(q_0,q_1)\Big\}\Big]\Big), \\ \vdots \\ q_m(x,t) = -\mathcal{L}^{-1}\Big(\frac{1}{s}\mathcal{L}\{Rq_{m-1}(x,t)\} + \frac{1}{s}\Big[\mathcal{L}\Big\{\sum_{j=1}^{13} P_{m-1}^j(q_0,\ldots,q_{m-1})\Big\}\Big]\Big), \quad m \geq 1. \end{cases} \tag{32}$$

where q_0 is referred to as the zeroth component, which is taken as the initial condition in this method. Within the context of the Laplace-Adomian decomposition approach, the solution functions q are generated as

$$q(x\ t) = \sum_{k=0}^{\infty} q_k(x,t). \tag{33}$$

5. Numerical and graphic results

An approximation level of N steps will be used to obtain solutions for system (1) under some parameter sets and initial conditions in order to demonstrate the efficacy, utility, and precision of LADM in solving directly applicable mathematical models.

5.1. Simulations of bright solitons

Example 1: In this particular example, the simulation will be conducted by taking into account equation (1) with the subsequent collection of coefficients:

$$\begin{cases} a = 0.1,\ b = -0.4,\ \delta_1 = -6.4,\ \delta_2 = 2.2,\ \delta_3 = 0.8, \\ \sigma_1 = 2.3,\ \sigma_2 = 0.4,\ \sigma_3 = 1.5,\ \sigma_4 = 5.5, \sigma_5 = -1.1, \\ \sigma_6 = 0.2,\ \sigma_7 = -5.3,\ \sigma_8 = 3.1,\ \sigma_9 = 0.9,\ \sigma_{10} = 0.6, \\ \sigma_{11} = 3.3,\ \sigma_{12} = 0.2,\ \sigma_{13} = 1.6 \end{cases} \tag{34}$$

and with initial condition:

$$f(x) = \left(2.33 + 3.46\operatorname{sech}(x)\right)e^{i[-0.88x+0.55]}.$$

Fig. 1 illustrates the error committed in this numerical simulation, the two-dimensional density plot, and the graphical achievements of the three-dimensional profile evolution for $|q|^2$ in a number of $N = 16$ steps.

Example 2. In this particular example, the simulation will be conducted by taking into account Eq. (1) with the subsequent collection of coefficients:

$$\begin{cases} a = 0.5,\ b = 9.1,\ \delta_1 = 8.4,\ \delta_2 = 4.0,\ \delta_3 = 0.9, \\ \sigma_1 = 0.1,\ \sigma_2 = -7.2,\ \sigma_3 = -1.0,\ \sigma_4 = -2.1, \sigma_5 = 5.5, \\ \sigma_6 = 7.4, \sigma_7 = 0.3,\ \sigma_8 = -9.5,\ \sigma_9 = 3.3,\ \sigma_{10} = -5.1, \\ \sigma_{11} = 0.5,\ \sigma_{12} = 0.6,\ \sigma_{13} = 5.8 \end{cases} \tag{35}$$

and with initial condition:

$$f(x) = \left(-6.33 - 3.46\operatorname{sech}(x)\right)e^{i[-0.34x+0.95]}.$$

Fig. 2 illustrates the error committed in this numerical simulation, the two-dimensional density plot, and the graphical achievements of the three-dimensional profile evolution for $|q|^2$ in a number of $N = 16$ steps.

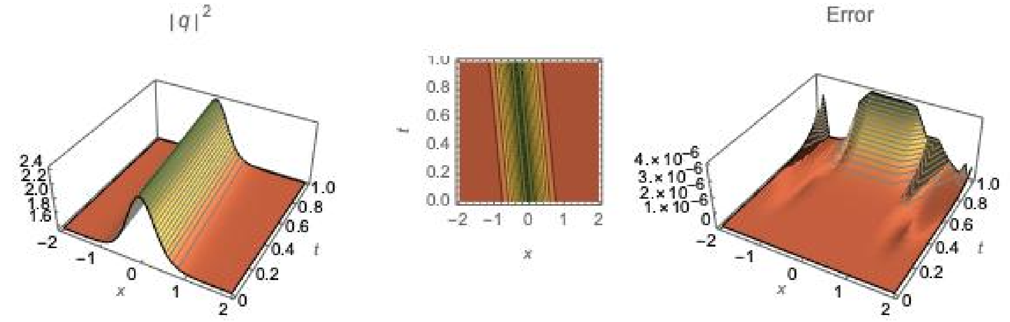

Fig. 1. 3D optical bright soliton solution of Eq. (1) (left); 2D density graphs represent bright soliton evolution (center); the absolute error in the simulation for a total of $N = 16$ steps, using the parameter values presented in example 1 (right).

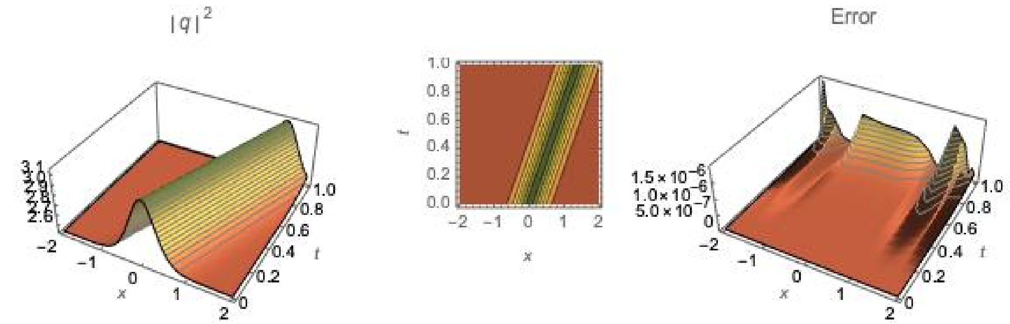

Fig. 2. 3D optical bright soliton solution of Eq. (1) (left); 2D density graphs represent bright soliton evolution (center); the absolute error in the simulation for a total of $N = 16$ steps, using the parameter values presented in example 2 (right).

5.2. Simulations of dark solitons

Example 3. In this particular example, the simulation will be conducted by taking into account equation (1) with the subsequent collection of coefficients:

$$\begin{cases} a=3.2,\ b=6.1,\ \delta_1=0.6,\ \delta_2=3.3,\ \delta_3=4.5, \\ \sigma_1=0.3,\ \sigma_2=-7.8,\ \sigma_3=-9.2,\ \sigma_4=-8.2, \sigma_5=7.2, \\ \sigma_6=6.6,\ \sigma_7=-2.7,\ \sigma_8=7.2,\ \sigma_9=5.5,\ \sigma_{10}=4.2, \\ \sigma_{11}=0.1,\ \sigma_{12}=0.5,\ \sigma_{13}=4.4 \end{cases} \tag{36}$$

and with initial condition:

$$f(x)=\left(2.1+1.25\tanh(4.5x)\right)e^{i[2.05x-5.3]}.$$

Fig. 3 illustrates the error committed in this numerical simulation, the two-dimensional density plot, and the graphical achievements of the three-dimensional profile evolution for $|q|^2$ in a number of $N=16$ steps.

Example 4. In this particular example, the simulation will be conducted by taking into account Eq. (1) with the subsequent collection of coefficients:

$$\begin{cases} a=9.1,\ b=-0.5,\ \delta_1=7.3,\ \delta_2=1.4,\ \delta_3=8.8, \\ \sigma_1=-5.4,\ \sigma_2=-0.3,\ \sigma_3=-0.9,\ \sigma_4=4.2, \sigma_5=-0.4, \\ \sigma_6=5.1,\ \sigma_7=8.2,\ \sigma_8=0.6,\ \sigma_9=-2.3,\ \sigma_{10}=0.1, \\ \sigma_{11}=0.9,\ \sigma_{12}=5.7,\ \sigma_{13}=0.5 \end{cases} \tag{37}$$

and with initial condition:

$$f(x)=\left(2.5+0.5\tanh(4.1x)\right)e^{i[2.05x-5.3]}.$$

Fig. 4 illustrates the error committed in this numerical simulation, the two-dimensional density plot, and the graphical achievements of the three-dimensional profile evolution for $|q|^2$ in a number of $N=16$ steps.

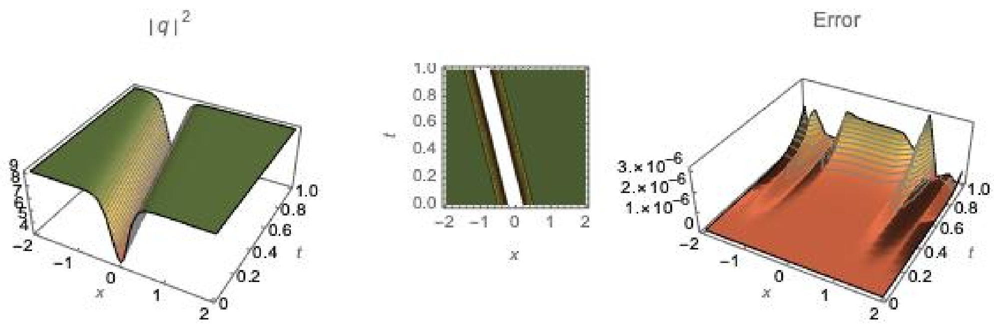

Fig. 3. 3D optical dark soliton solution of Eq. (1) (left); 2D density graphs represent dark soliton evolution (center); the absolute error in the simulation for a total of $N=16$ steps, using the parameter values presented in example 3 (right).

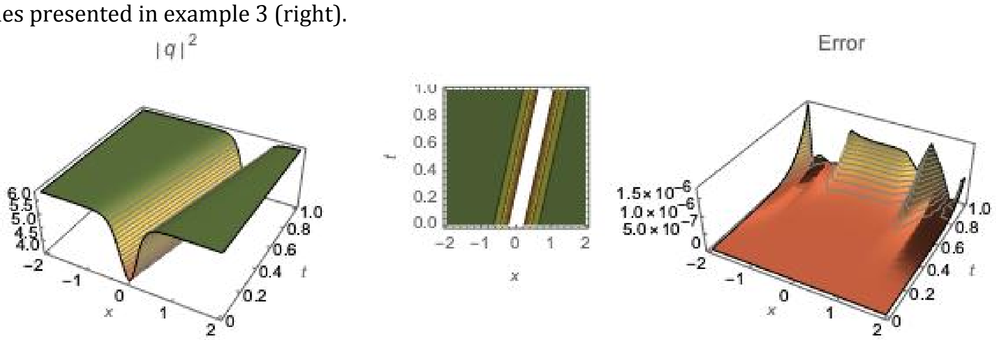

Fig. 4. 3D optical dark soliton solution of Eq. (1) (left); 2D density graphs represent dark soliton evolution (center); the absolute error in the simulation for a total of $N=16$ steps, using the parameter values presented in example 4 (right).

6. Conclusions

In the frame of the dispersive concatenation model, the current paper, using numerical simulations, revealed the bright and dark 1-soliton solutions of the dispersive concatenation model studied with the Kerr law of nonlinearity. The infinitesimally small error measure was particularly noticeable. These results are in the same spirit as previously reported ones from the concatenation model. The current paper stands on a strong footing to expand the work further in the same spirit. Later, the work will be extended to the concatenation and dispersive concatenation models with the power-law of nonlinearity. Additionally, the model will be numerically addressed for additional forms of optoelectronic devices using LADM, and they are fibers with differential group delay and dispersion-flattened fibers. The results of those research activities will be disseminated with time.

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doi: 10.3116/16091833/Ukr.J.Phys.Opt.2024.01094

Анотація. *У цій роботі чисельно досліджені світлі та темні оптичні солітони, які виникають із моделі дисперсійної конкатенації. В моделі закладено закон Керра для нелінійного показника заломлення з використанням схем розкладання Лапласа-Адоміана. В роботі представлено результати моделювання, поверхнями та двомірними графіками. Показано, що похибка є нескінченно малою.*

Ключові слова*: солітони, рівняння Шредінгера, конкатенаційна модель, поліноми Адоміана.*