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Highly Dispersive Solitons in Optical Couplers with Metamaterials Having Kerr Law Of Nonlinear Refractive Index

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Abstract. The present paper derives highly dispersive soliton solutions in a coupler from optical metamaterials with Kerr law of nonlinear refractive index. In nonlinear optics and photonics, these wave packets possess a unique property, allowing them to sustain their shape and velocity during propagation through nonlinear media. Optical couplers utilizing metamaterials are advanced devices tailored to enhance the coupling and transfer of optical signals between different optical components or systems by harnessing the unique characteristics of these specially engineered materials. The paper's novelty lies in considering highly dispersive optical solitons that can be studied in optical couplers constructed from metamaterials. This paper's primary goal is to integrate a governing model concerning highly dispersive solitons in optical couplers with the incorporation of metamaterials. Therefore, the unified Riccati equation expansion procedure and the enhanced Kudryashov's scheme are adopted. They are being applied to such a device for the first time. These yield a full spectrum of optical solitons as well as straddled solitons. The novelty lies in the joint application of the two procedures, which uncovers various solitons. Furthermore, one of the approaches uncovers an unexpected benefit by revealing straddled soliton solutions.

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1. Introduction

Optical solitons, the self-reinforcing wave packets that can maintain their shape and velocity while propagating through a nonlinear medium, are essential phenomena in nonlinear optics and photonics. Due to their unique properties, these solitons are of significant importance in theoretical research and practical applications. A multitude of scientific papers have been published, each delving into different aspects of solitons in optical systems, thereby illuminating their significance in various scenarios [1–24]. These papers cover a diverse range of topics related to optical solitons, including theoretical models, formation and dynamics, nonlinear effects, soliton propagation, stability and instability, soliton-based communication, solitons in nonlinear media, soliton interactions, experimental observations, and applications. By examining these various aspects of optical solitons, researchers and engineers can gain a deeper understanding of their behavior and leverage their unique properties for numerous technological advancements. The extensive body of research presented in these papers underscores the importance of optical solitons in advancing the field of nonlinear optics and photonics. Bright and singular solitons in a (2+1)-dimensional nonlinear Schrödinger equation with spatio-temporal dispersions are investigated in [1]. The authors explore the formation, properties, and dynamics of these solitons, highlighting their significance in optical systems with dispersive effects. A novel integration approach to analyze the perturbed Biswas-Milovic equation with Kudryashov's law of refractive index is introduced in [2]. The role of solitons in this perturbed optical system and its implications for optical signal transmission and control are explored. Novel soliton solutions for the conformable perturbed nonlinear Schrödinger equation are presented in [3]. The study reveals new types of solitons in this system, contributing to understanding their behavior and potential applications in optical signal processing. Asymmetrical and self-similar structures of optical breathers in the context of the Manakov system, considering photorefractive crystals and randomly birefringent fibers, are explored in [4]. The research examines the stability and interactions of these breathers, which are important for applications in optical communication and nonlinear optics. Optical vector lattice breathers within a two-component Rabi-coupled Gross-Pitaevskii system with variable coefficients are addressed in [5]. The investigation reveals the unique properties and potential applications of these vector lattice breathers in nonlinear optical systems. The interaction properties between rogue waves and breathers in the Manakov system, particularly arising from stationary self-focusing electromagnetic systems, are studied in [6]. The research highlights the complex dynamics of these interactions and their implications for optical system stability and control. The occurrence of a 'firewall' effect during the interactions between rogue waves and breathers in the Manakov system is examined in [7]. The study sheds light on the unexpected phenomena that can emerge during these interactions, providing insights into the robustness of optical systems. The oscillating collisions between three solitons in a dual-mode fiber coupler system are investigated in [8]. The study reveals the dynamics of soliton collisions and their potential applications in all-optical signal processing. The resonant behavior of solitons in a waveguide directional coupler system implemented in optical fibers is explored in [9]. The research demonstrates the importance of soliton resonances for efficient optical switching and signal manipulation. Soliton excitation in a coherently coupled nonlinear Schrödinger system with two orthogonally polarized components, implemented in optical fibers, is the focus of the paper [10]. The study reveals the potential of these solitons for optical communication and signal processing applications. Doubly periodic waves and bright and dark solitons in a coupled mono-mode step-index optical fiber system are explored in [11]. The investigation highlights the richness of soliton phenomena in this system and its relevance to optical communication and signal transmission.

Optical couplers with metamaterials refer to devices that use specially designed materials with unique properties to efficiently couple and transfer optical signals between different optical components or systems [12]. Metamaterials are artificially engineered materials that can manipulate light in unusual ways, such as bending or twisting light in opposite directions [13]. These materials are composed of subwavelength-sized structures designed to interact with light in specific ways, allowing them to exhibit unusual optical properties not found in naturally occurring materials. Optical couplers with metamaterials have many potential applications, including telecommunications, optical computing, and sensing. For example, these couplers can be used to couple light into and out of optical fibers, to connect optical components on a chip, or to direct and focus light in sensing applications [14]. Several types of optical couplers have been designed using metamaterials, including directional couplers, evanescent couplers, and plasmonic couplers. These couplers use different approaches to couple light between optical components, but they all rely on the unique optical properties of metamaterials to achieve efficient and precise coupling. The use of metamaterials in optical couplers can significantly improve the efficiency, speed, and performance of optical communication and sensing systems.

The study of optical couplers with metamaterials and metasurfaces is a trend that is gaining momentum during modern times. Therefore, it is imperative to look at solitons in couplers with metamaterials [15]. In order to be on the edge, it is important to look at highly dispersive (HD) solitons in optical couplers [16]. This concept of HD solitons emerged a couple of years ago when chromatic dispersion (CD) ran low. Low count chromatic dispersion in optical fiber refers to the situation where the chromatic dispersion of the fiber is relatively small. Low-count chromatic dispersion occurs when the difference in speed between the different wavelengths of light is relatively small. The depletion of CD compelled the inclusion of higher–order dispersion effects that would replenish this low count. These are CD, intermodal dispersion (IMD), fourth–order dispersion (4OD), third–order dispersion (3OD), sixth–order dispersion (6OD), and fifth–order dispersion (5OD). Such dispersive effects would naturally lead to different adverse consequences [17]. These would be the natural slow–down of the solitons and the presence of excessive soliton radiation. However, the current work ignores these detrimental effects and focuses on retrieving solitons to the model.

A wide variety of approaches are known during modern times [18]. We have come a long way since the days of Inverse Scattering Transform, the only successful algorithm that was applicable. Two integration schemes are adopted in the paper to retrieve these HD solitons. They are the enhanced Kudryashov's approach and the unified Riccati equation scheme, which are two mathematical techniques used to solve nonlinear differential equations. The enhanced Kudryashov's mechanism is an extension of the standard Kudryashov's method, a well-known method for solving nonlinear differential equations. The idea of the enhanced technique is to address an auxiliary function that holds a linear differential equation. This auxiliary function is then used to construct an exact solution of the original nonlinear differential equation. The enhanced approach is particularly useful when the standard Kudryashov's method fails to provide an exact solution. The unified Riccati equation scheme is another powerful technique for solving nonlinear differential equations. It is based on the observation that many nonlinear differential equations can be transformed into a Riccati equation, which is a type of first-order nonlinear differential equation. The unified Riccati equation scheme involves using a series of transformations to convert the original nonlinear differential equation into a Riccati equation. Once the equation is in this form, it can be solved using known methods for Riccati equations. Both the unified Riccati equation scheme and the enhanced Kudryashov's approach are valuable tools for solving nonlinear differential equations, particularly when other methods fail to provide an exact solution. These techniques have been applied to a full spectrum of problems in physics, engineering, and other fields. One obvious shortcoming of the Riccati equation expansion is that it fails to retrieve the much–needed bright solitons. Therefore, the enhanced Kudryashov's scheme comes to the rescue, as always.

This collective count of the two algorithms successfully allows us a phenomenal variety of solitons for the model, which is in optical twin-core couplers with metamaterials. They are optical singular, dark, and bright solitons. Optical solitons are self-sustained wave packets that maintain their shape and velocity during propagation in a medium. Bright solitons are described by a localized region of high intensity surrounded by a region of lower intensity. The intensity profile of a bright soliton is shaped like a bell curve. Bright solitons arise when a self-focusing nonlinear effect in the medium balances out the diffraction or dispersion that would otherwise cause the wave to spread out over time. Dark solitons, on the other hand, are characterized by a localized region of low intensity surrounded by a region of higher intensity. The intensity profile of a dark soliton is shaped like an inverted bell curve. Dark solitons arise when a self-defocusing nonlinear effect in the medium balances out the diffraction or dispersion that would otherwise cause the wave to spread out over time. Singular solitons are a special type of soliton that have a discontinuity in their phase profile. They can be thought of as a combination of bright and dark solitons, where the intensity profile has a bell curve on one side of the discontinuity and an inverted bell curve on the other side. Singular solitons arise in situations where the nonlinearity is so strong that it leads to a breakdown of the usual balance between nonlinearity, dispersion, and diffraction. All three types of solitons are important in the study of nonlinear optics, and they have applications in a variety of fields such as optical communication, optical computing, and quantum information processing. The parameter restrictions, also known as constraints, naturally emerged from the schemes that are also presented. These constraints provide us the solitons.

The paper aims to integrate the models described by (1) and (2) that address the highly dispersive solitons in optical twin-core couplers with metamaterials. The novelty of the two procedures that are collectively implemented is that a plethora of solitons are revealed. In addition, straddled soliton solutions are revealed from one of the two approaches, which comes out as a bonus. The details of the solitons' derivation are addressed after skimming through the model and exposing the mathematical preliminaries.

2. Governing model

The governing model for locating highly dispersive solitons in optical twin-core couplers made up of metamaterials, which is addressed for the first time in the current paper, is given below

$$\begin{aligned} & iq_t + ia_{11}q_x + a_{12}q_{xx} + ia_{13}q_{xxx} + a_{14}q_{xxxx} + ia_{15}q_{xxxxx} + a_{16}q_{xxxxxx} + b_1|q|^2 q \\ & = \alpha_1\left(|q|^2 q\right)_{xx} + \beta_1|q|^2 q_{xx} + \gamma_1 q^2 q_{xx}^* + \delta_1 r + i\left[\lambda_1\left(|q|^2 q\right)_x + \mu_1\left(|q|^2\right)_x q + \theta_1|q|^2 q_x\right], \end{aligned} \tag{1}$$

and

$$\begin{aligned} & ir_t + ia_{21}r_x + a_{22}r_{xx} + ia_{23}r_{xxx} + a_{24}r_{xxxx} + ia_{25}r_{xxxxx} + a_{26}r_{xxxxxx} + b_2|r|^2 r \\ & = \alpha_2\left(|r|^2 r\right)_{xx} + \beta_2|r|^2 r_{xx} + \gamma_2 r^2 r_{xx}^* + \delta_2 q + i\left[\lambda_2\left(|r|^2 r\right)_x + \mu_2\left(|r|^2\right)_x r + \theta_2|r|^2 r_x\right], \end{aligned} \tag{2}$$

where the first terms represent the linear temporal evolution, where $i=\sqrt{-1}$. The constants, $(j=1,2, k=1,2,...,6)$ account for the IMD, CD, 3OD, 4OD, 5OD and 6OD in sequence. b_j yields Kerr–law nonlinearity, while γ_j, β_j and α_j give the optical metamaterial coefficients. δ_j comes from the coupling. λ_j arises from self–steepening terms, while μ_j and θ_j stem from self–frequency shift. $q=q(x,t)$ and $r=r(x,t)$ are complex-valued functions that describe the wave profiles in the respective cores of the optical fibers, while $q^*(x,t)$ and $r^*(x,t)$ are complex–conjugates. It must be noted that such higher-order dispersive terms, especially with 3OD are yet to be experimentally demonstrated, especially in optical metamaterials. Thus, The model is purely theoretical to gain a mathematical and physical insight of the soliton dynamics in such specialized forms of optical couplers.

In the model, linear-temporal evolution in optics refers to changes in the amplitude and phase of an optical signal as it propagates through a medium over time in a linear regime. This means that the material's response to the optical signal is proportional to the signal's amplitude and is not affected by nonlinear effects such as stimulated Raman scattering, four-wave mixing, or self-phase modulation. Inter-modal dispersion refers to the temporal spreading of a pulse of light due to the different propagation times of different modes of a multimode optical fiber. Inter-modal dispersion can occur when a pulse of light contains multiple frequency components, which travel at different velocities through the fiber and can experience different path lengths due to the different modes of the fiber. This causes the pulse to spread over time, leading to a loss of temporal coherence and reduced bandwidth. Chromatic dispersion is an important factor in nonlinear optics, as it can significantly affect the propagation and behavior of light pulses in nonlinear materials. Chromatic dispersion can form optical solitons, which are self-sustained, localized waves that can propagate through the material without spreading. The balance between nonlinearity and chromatic dispersion allows for the formation and propagation of solitons. Higher-order dispersions in optics refer to the effects of the third, fourth, and higher-order derivatives of the refractive index with respect to the frequency of light propagation through a material. These effects can become significant for short optical pulses with large spectral bandwidths and can limit the performance of optical communication systems. Kerr nonlinearity is one of the most important and widely studied nonlinear optical effects in which the refractive index of a material varies with the intensity of the incident electromagnetic field. The Kerr effect can have a number of important implications for the propagation of light through a material. For example, the Kerr nonlinearity can cause self-focusing, where a beam of light focuses on itself as it travels through the material. This can lead to the formation of optical solitons, which are self-sustaining pulses of light that propagate without spreading or changing shape. The Kerr nonlinearity can also cause self-phase modulation (SPM), where different frequency components of a pulse experience different phase shifts as they propagate through the material. This can lead to spectral broadening and the generation of new frequencies. Self-steepening is a nonlinear optical effect that occurs when an ultrafast optical pulse propagates through a medium with a nonlinear response. It is a higher-order nonlinear effect that arises from the temporal variation of the intensity of the pulse. Self-steepening leads to a temporal change in the pulse's shape, causing the pulse's leading edge to become steeper and the trailing edge to become broader. This results in a change in the pulse duration and the formation of chirped pulses. Self-steepening is related to the third-order dispersion effect in optics, which is caused by the frequency-dependent variation of the refractive index. Self-frequency shift is a nonlinear optical effect that occurs when an intense laser pulse propagates through a material with a nonlinear response. A phenomenon known as self-frequency shift results in the central frequency of a pulse being displaced towards longer or shorter wavelengths. This effect is due to the interaction between the intense laser pulse and the material's nonlinear optical response. Specifically, the pulse's intensity alters the material's refractive index, leading to a corresponding shift in the pulse's central frequency. The extent and direction of the shift are contingent on both the material's properties and the pulse's intensity.

It has been pointed out that optical couplers are prevalent for short distances [19]. Therefore the solutions are valid even though there is soliton radiation due to the higher order dispersion terms. The soliton velocity will also slow down for a short distance, and therefore, the solutions remain effective for its propagation purposes. Optical solitons propagate through an optical fiber over long distances, while the optical coupler is essentially meant for a short distance. Therefore, such a medium does not quite correspond to the conditions for the formation and propagation of an optical soliton. If, however, a soliton from an optical fiber enters such a system, then, since the properties of the optical fiber and coupler are different, the soliton undergoes distortions, and its propagation mode is not quite a soliton any longer. In addition, due to the small size of an optical coupler, the influence of boundary conditions becomes significant, which additionally distorts the soliton propagation regime, which is not considered in the work just to focus on the soliton solution of the given model. The main content of the work, therefore, consists in solving equations (1) and (2), and so their particular solutions are identified. The fact that the solutions of these equations have some localization is not proof that these are optical solitons in the conventional sense. Therefore, the solutions obtained are not quite significant, and consequently, the results come with significant shortcomings. To reiterate, the main idea of the current study is to secure soliton solutions to the model so that they can be applied to address the dynamics in dual-core couplers, single-core couplers, as well as in optical switching. The issues of soliton radiation, shedding of energy, and soliton velocity slow-down have been tacitly ignored. It must be noted that nonlinear optical couplers play a crucial role in optical communication links by enabling rapid switching and signal coupling, rendering them highly advantageous devices. Nonlinear optical couplers find diverse applications as intensity-dependent switches and limiters, and they are indispensable for multiplexing two incoming streams into a single fiber and demultiplexing a single-bit stream. These couplers can be fabricated using various materials and structures, such as semiconductor material, dual-core and single-core fibers, and optical metamaterials. Additionally, solitons can travel through each core of the coupler, and the transfer of energy between the cores is facilitated by the evanescent fields' overlap.

Recent studies have focused on soliton pulse propagation characteristics in materials with negative real dielectric permittivity and magnetic permeability. These unique materials, commonly called metamaterials, are not naturally occurring but are synthesized through material processing techniques. Fabricating these complex materials through engineered processes has captured significant interest in the research community [20]. With their remarkable electromagnetic properties, metamaterials represent a novel class of artificially engineered materials. Their dielectric permittivity and magnetic permeability can be precisely controlled, allowing for tailored linear and nonlinear parameters. These exceptional characteristics provide an avenue for the experimental manipulation of optical solitons. Metamaterials are a subject of recent interest as their response to electromagnetic waves can be tailored to have desired optical beam/pulse propagation merely with an appropriate design. Naturally, soliton formation and propagation in optical metamaterials is a subject of recent interest for the optics community.

3. Mathematical analysis

The wave profiles in (1) and (2) stand as

$$\begin{aligned} q(x,t) &= g_1(\xi)\exp[iZ(x,t)], \\ r(x,t) &= g_2(\xi)\exp[iZ(x,t)], \end{aligned} \tag{3}$$

where

$$\xi = x - vt,\ Z\left(x,t\right) = -\kappa x + \omega t + \theta_0. \tag{4}$$

Here $g_j\left(\xi\right)$ comes out as the amplitude components of the solitons, v stands as the soliton velocity, κ enables us the soliton wave number, θ_0 arises from the phase constant and ω comes from the soliton frequency. Putting (3) and (4) into (1) and (2) secures the auxiliary equations

$$\begin{aligned} & a_{16}g_1^{(6)} + (a_{14} - 5a_{15}\kappa - 15a_{16}\kappa^2)g_1^{(4)} - \left(\beta_1 + \gamma_1 + 3\alpha_1\right)g_1^2 g_1'' - 6\alpha_1 g_1 g_1'^2 \\ & +(a_{12} + 3a_{13}\kappa - 6a_{14}\kappa^2 - 10a_{15}\kappa^3 + 15a_{16}\kappa^4)g_1'' \\ & +(-\omega + a_{11}\kappa - a_{16}\kappa^6 - a_{12}\kappa^2 - a_{13}\kappa^3 + a_{14}\kappa^4 + a_{15}\kappa^5)g_1 \\ & -\delta_1 g_2 + \left[b_1 - \kappa\left(\lambda_1 + \theta_1\right) + \kappa^2\left(\beta_1 + \alpha_1 + \gamma_1\right)\right]g_1^3 = 0, \end{aligned} \tag{5}$$

$$\begin{aligned} & a_{26}g_2^{(6)} + (a_{24} - 5a_{25}\kappa - 15a_{26}\kappa^2)g_2^{(4)} - \left(\beta_2 + \gamma_2 + 3\alpha_2\right)g_2^2 g_2'' - 6\alpha_2 g_2 g_2'^2 \\ & +(a_{22} + 3a_{23}\kappa - 6a_{24}\kappa^2 - 10a_{25}\kappa^3 + 15a_{26}\kappa^4)g_2'' \\ & +(-\omega + a_{21}\kappa - a_{26}\kappa^6 - a_{22}\kappa^2 - a_{23}\kappa^3 + a_{24}\kappa^4 + a_{25}\kappa^5)g_2 \\ & -\delta_2 g_1 + \left[b_2 - \kappa\left(\lambda_2 + \theta_2\right) + \kappa^2\left(\beta_2 + \alpha_2 + \gamma_2\right)\right]g_2^3 = 0, \end{aligned} \tag{6}$$

$$\begin{aligned} & \left(a_{15} - 6a_{16}\kappa\right)g_1^{(5)} + \left(a_{13} - 4a_{14}\kappa - 10a_{15}\kappa^2 + 20a_{16}\kappa^3\right)g_1''' \\ & +\left[2\kappa\left(\beta_1 - \gamma_1 + 3\alpha_1\right) - \left(3\lambda_1 + 2\mu_1 + \theta_1\right)\right]g_1^2\, g_1' \\ & +\left(-v + a_{11} - 2a_{12}\kappa - 3a_{13}\kappa^2 + 4a_{14}\kappa^3 + 5a_{15}\kappa^4 - 6a_{16}\kappa^5\right)g_1' = 0, \end{aligned} \tag{7}$$

and

$$\begin{aligned} & \left(a_{25} - 6a_{26}\kappa\right)g_2^{(5)} + \left(a_{23} - 4a_{24}\kappa - 10a_{25}\kappa^2 + 20a_{26}\kappa^3\right)g_2''' \\ & +\left[2\kappa\left(\beta_2 - \gamma_2 + 3\alpha_2\right) - \left(3\lambda_2 + 2\mu_2 + \theta_2\right)\right]g_2^2\, g_2' \\ & +\left(-v + a_{21} - 2a_{22}\kappa - 3a_{23}\kappa^2 + 4a_{24}\kappa^3 + 5a_{25}\kappa^4 - 6a_{26}\kappa^5\right)g_1' = 0. \end{aligned} \tag{8}$$

Eqs. (7) and (8) leave us with the soliton wave number

$$\kappa=\frac{a_{j5}}{6a_{j6}}, \tag{9}$$

the constraint conditions

$$a_{j3}-4a_{j4}\kappa-10a_{j5}\kappa^{2}+20a_{j6}\kappa^{3}=0, \tag{10}$$

$$2\kappa\left(3\alpha_{j}-\gamma_{j}+\beta_{j}\right)-\left(\theta_{j}+2\mu_{j}+3\lambda_{j}\right)=0, \tag{11}$$

and the soliton velocity

$$v=a_{j1}-2a_{j2}\kappa-3a_{j3}\kappa^{2}+4a_{j4}\kappa^{3}+5a_{j5}\kappa^{4}-6a_{j6}\kappa^{5}. \tag{12}$$

Eqs. (5) and (6) are also indicated below

$$a_{16}g_{1}^{(6)}+A_{1}g_{1}^{(4)}+B_{1}g_{1}''+C_{1}g_{1}^{2}g_{1}''-6\alpha_{1}g_{1}g_{1}'^{2}+\Delta_{1}g_{1}-\delta_{1}g_{2}+E_{1}g_{1}^{3}=0, \tag{13}$$

and

$$a_{26}g_{2}^{(6)}+A_{2}g_{2}^{(4)}+B_{2}g_{2}''+C_{2}g_{2}^{2}g_{2}''-6\alpha_{2}g_{2}g_{2}'^{2}+\Delta_{2}g_{2}-\delta_{2}g_{1}+E_{2}g_{2}^{3}=0, \tag{14}$$

where, for $j=1,2$, we get

$$\left.\begin{aligned}
&A_{j}=a_{j4}-5a_{j5}\kappa-15a_{j6}\kappa^{2},\\
&B_{j}=a_{j2}+3a_{j3}\kappa-6a_{j4}\kappa^{2}-10a_{j5}\kappa^{3}+15a_{j6}\kappa^{4},\\
&C_{j}=-\left(\beta_{j}+\gamma_{j}+3\alpha_{j}\right),\\
&\Delta_{j}=-\omega+a_{j1}\kappa-a_{j2}\kappa^{2}-a_{j3}\kappa^{3}+a_{j4}\kappa^{4}+a_{j5}\kappa^{5}-a_{j6}\kappa^{6},\\
&E_{j}=b_{j}-\kappa\left(\lambda_{j}+\theta_{j}\right)+\kappa^{2}\left(\beta_{j}+\alpha_{j}+\gamma_{j}\right).
\end{aligned}\right\} \tag{15}$$

4. Highly dispersive solitons

In optical fibers, highly dispersive solitons are a type of soliton that arises due to the interplay between the nonlinearity and dispersion of the fiber. Optical fibers are used to transmit information over long distances by encoding the information in the form of optical pulses. However, the pulses tend to spread out over time and distance due to the dispersion of the fiber, making it difficult to maintain their shape and intensity. Highly dispersive solitons can be used to overcome this problem [21]. They are formed when the nonlinear effects in the fiber balance the dispersion, resulting in a stable pulse that maintains its shape and intensity as it propagates through the fiber. This allows the pulse to travel over long distances without significant distortion. One of the important properties of highly dispersive solitons is their frequency chirp, which is caused by the varying group velocity dispersion (GVD) along the fiber. The frequency chirp can be either positive or negative depending on the sign of the GVD and can be controlled by adjusting the fiber parameters. This property has important applications in optical communication and signal processing, including dispersion compensation, frequency conversion, and pulse compression. Highly dispersive solitons can be generated in various ways, such as through Raman scattering, optical modulation instability, and self-phase modulation. They have been addressed in fiber-optic communication and are a key component in many modern communication systems.

In this section, a wide range of solitons are recovered by using powerful and prolific integration structures. Solitons are self-reinforcing waves that maintain their shape and speed as they travel through a medium. Different types of solitons can be classified based on their amplitude, phase, and polarization properties. Four common types of solitons are singular, dark, bright, and straddled solitons. Bright solitons have a higher intensity than the background medium. They arise due to the balance between nonlinear effects and dispersion, which causes the pulse to self-focus and maintain its shape. Bright solitons are common in optical fibers and are used in optical communication systems. Dark solitons are solitons that have a lower intensity than the background medium. They arise due to the balance between nonlinear effects and dispersion, which causes the pulse to self-defocus and maintain its shape. Dark solitons are also common in optoelectronics and are addressed in signal processing and data storage. Singular solitons, also known as envelope solitons, are solitons that have a zero amplitude at some point along their trajectory. They arise due to the balance between nonlinear effects and dispersion, which causes the pulse to break into two parts, one with positive amplitude and the other with negative amplitude. Singular solitons are important in studying nonlinear wave phenomena and have applications in optical communication and signal processing. Straddled solitons are solitons that have a non-zero amplitude at the zero-dispersion point of the medium. They arise due to the interplay between dispersion, nonlinearity, and third-order dispersion. Straddled solitons have applications in high-speed optical communication and pulse compression. Different types of solitons have different properties and applications, and understanding their behavior is important for the development of advanced optical communication and signal processing technologies.

4.1. Unified Riccati equation expansion procedure

The unified Riccati equation expansion procedure is a mathematical technique used to address a phenomenal variety of model equations [22]. It is based on the idea of transforming a given differential equation into a Riccati equation, which is a type of first-order nonlinear ordinary differential equation. The procedure then involves expanding the solution of the Riccati equation into a power series and searching for the solutions for the coefficients of the series. The procedure has been considered with a full spectrum of model equations, including the Korteweg-de Vries equation, the Benjamin-Bona-Mahony equation, and the Burgers equation, among others. It has also been used to study a range of physical phenomena, such as soliton dynamics and wave propagation in optical fibers. One advantage of the procedure is that it provides a systematic and unified approach to solving nonlinear differential equations, which allows for a wide range of equations to be solved using a single technique. Additionally, the power series solution obtained using the procedure can often be truncated to obtain a good approximation of the exact solution, which makes it a useful tool for numerical simulations and modeling. The procedure is a powerful mathematical tool that provides a systematic and unified approach to solving nonlinear differential equations. It has been used to study a range of physical phenomena and has applications in a variety of fields, including optics, fluid dynamics, and plasma physics.

Eqs. (13) and (14) satisfy the formal solutions

$$g_1(\xi)=\sum_{l=0}^{N}\Gamma_l Q^l(\xi),\ \Gamma_N \neq 0, \tag{16}$$

and

$$g_2(\xi)=\sum_{l=0}^{M}\Omega_l Q^l(\xi),\ \Omega_M \neq 0, \tag{17}$$

with the aid of the Riccati equation

$$Q'(\xi)=h_0+h_1Q(\xi)+h_2Q^2(\xi),\ h_2\neq 0, \tag{18}$$

where $Q(\xi)$ holds the explicit solutions

$$Q(\xi)=-\frac{h_1}{2h_2}-\frac{\sqrt{h_1^2-4h_0h_2}}{2h_2}\left[\frac{r_1\tanh\left(\frac{\xi}{2}\sqrt{h_1^2-4h_0h_2}\right)+r_2}{r_1+r_2\tanh\left(\frac{\xi}{2}\sqrt{h_1^2-4h_0h_2}\right)}\right], \tag{19}$$

if $h_1^2-4h_0h_2>0$ and $r_1^2+r_2^2\neq 0$,

$$Q(\xi)=-\frac{h_1}{2h_2}+\frac{\sqrt{-\left(h_1^2-4h_0h_2\right)}}{2h_2}\left[\frac{r_3\tan\left(\frac{\xi}{2}\sqrt{-\left(h_1^2-4h_0h_2\right)}\right)-r_4}{r_3+r_4\tan\left(\frac{\xi}{2}\sqrt{-\left(h_1^2-4h_0h_2\right)}\right)}\right], \tag{20}$$

if $h_1^2-4h_0h_2<0$ and $r_3^2+r_4^2\neq 0$,

and

$$Q(\xi)=-\frac{h_1}{2h_2}+\frac{1}{h_2\xi+r_5},\ \text{if}\quad h_1^2-4h_0h_2=0. \tag{21}$$

Here Γ_l, Ω_l, h_0, h_1, h_2, r_1, r_2, r_3, r_4 and r_5 are constants, while N and M arise from the balancing method in Eqs. (13) and (14). Therefore, Eqs. (16) and (17) are structured as

$$g_1(\xi)=\Gamma_0+\Gamma_1Q(\xi)+\Gamma_2Q^2(\xi)+\Gamma_3Q^3(\xi),\ \Gamma_3\neq 0, \tag{22}$$

and

$$g_2(\xi)=\Omega_0+\Omega_1Q(\xi)+\Omega_2Q^2(\xi)+\Omega_3Q^3(\xi),\ \Omega_3\neq 0. \tag{23}$$

Substituting Eqs. (22) and (23) along with (18) into Eqs. (13) and (14) paves way to the results:

$$\begin{gathered}\Gamma_0=\Gamma_1=\Gamma_2=0,\ \Gamma_3=\pm 15h_2^3\sqrt{\frac{22a_{16}}{E_1}},\ \Omega_0=\Omega_1=\Omega_2=0,\\ \Omega_3=\pm 15h_2^3\sqrt{\frac{22a_{26}}{E_2}},\ h_0=\frac{31E_1}{165\alpha_1h_2},\ h_1=0,\ h_2=h_2,\end{gathered} \tag{24}$$

and

$$\begin{gathered}A_1=-\frac{217a_{16}E_1}{12\alpha_1},\ A_2=-\frac{217a_{26}E_1}{12\alpha_1},\ B_1=\frac{114359E_1^2a_{16}}{2475\alpha_1^2},\ B_2=\frac{114359E_1^2a_{26}}{2475\alpha_1^2},\\ C_1=\frac{9}{2}\alpha_1,\ C_2=\frac{9\alpha_1E_2}{2E_1},\ \alpha_2=\frac{\alpha_1E_2}{E_1},\ \delta_1=\frac{1815\Delta_1\alpha_1^3-59582a_{16}E_1^3}{1815\alpha_1^3}\sqrt{\frac{a_{16}E_2}{a_{26}E_1}},\\ \delta_2=\frac{1815\Delta_2\alpha_1^3-59582a_{26}E_1^3}{1815\alpha_1^3}\sqrt{\frac{a_{26}E_1}{a_{16}E_2}},\ a_{16}E_1>0,\ a_{26}E_2>0,\end{gathered} \tag{25}$$

where E_1, E_2, Δ_1 and Δ_2 come from Eq. (15).

Case-1. Setting $a_{16}\alpha_1<0$, $E_1\alpha_1<0$, $a_{26}E_2>0$ and $h_1^2-4h_0h_2>0$, then inserting Eq. (24) along with (19) into Eqs. (22) and (23) enables us the dark-singular straddled solitons

$$q(x,t)=\pm\frac{31E_1}{22\alpha_1}\sqrt{-\frac{248a_{16}}{15\alpha_1}}\left[\frac{r_1\tanh\left(\frac{1}{2}\sqrt{-\frac{124E_1}{165\alpha_1}}(x-vt)\right)+r_2}{r_1+r_2\tanh\left(\frac{1}{2}\sqrt{-\frac{124E_1}{165\alpha_1}}(x-vt)\right)}\right]^3 e^{i(-\kappa x+\omega t+\theta_0)}, \quad (26)$$

and

$$r(x,t)=\pm\frac{31E_1}{22\alpha_1}\sqrt{-\frac{248a_{26}E_1}{15\alpha_1E_2}}\left[\frac{r_1\tanh\left(\frac{1}{2}\sqrt{-\frac{124E_1}{165\alpha_1}}(x-vt)\right)+r_2}{r_1+r_2\tanh\left(\frac{1}{2}\sqrt{-\frac{124E_1}{165\alpha_1}}(x-vt)\right)}\right]^3 e^{i(-\kappa x+\omega t+\theta_0)}. \quad (27)$$

Taking $r_1\neq 0$ and $r_2=0$ retrieves the dark solitons

$$q(x,t)=\pm\frac{31E_1}{22\alpha_1}\sqrt{-\frac{248a_{16}}{15\alpha_1}}\tanh^3\left(\frac{1}{2}\sqrt{-\frac{124E_1}{165\alpha_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)}, \quad (28)$$

and

$$r(x,t)=\pm\frac{31E_1}{22\alpha_1}\sqrt{-\frac{248a_{26}E_1}{15\alpha_1E_2}}\tanh^3\left(\frac{1}{2}\sqrt{-\frac{124E_1}{165\alpha_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)}, \quad (29)$$

while, setting $r_1=0$ and $r_2\neq 0$ enables us the singular wave fields

$$q(x,t)=\pm\frac{31E_1}{22\alpha_1}\sqrt{-\frac{248a_{16}}{15\alpha_1}}\coth^3\left(\frac{1}{2}\sqrt{-\frac{124E_1}{165\alpha_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)}, \quad (30)$$

and

$$r(x,t)=\pm\frac{31E_1}{22\alpha_1}\sqrt{-\frac{248a_{26}E_1}{15\alpha_1E_2}}\coth^3\left(\frac{1}{2}\sqrt{-\frac{124E_1}{165\alpha_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)}. \quad (31)$$

Fig. 1 exhibits the profile of dark waveforms for $b_j=2$, $\lambda_j=1$, $\theta_j=1$, $\beta_j=1$, $\gamma_j=1$, $a_{j1}=1$, $a_{j2}=1$, $a_{j3}=44$, $a_{j4}=1$, $a_{j5}=6$, $a_{j6}=1$ and $\alpha_j=-1$.

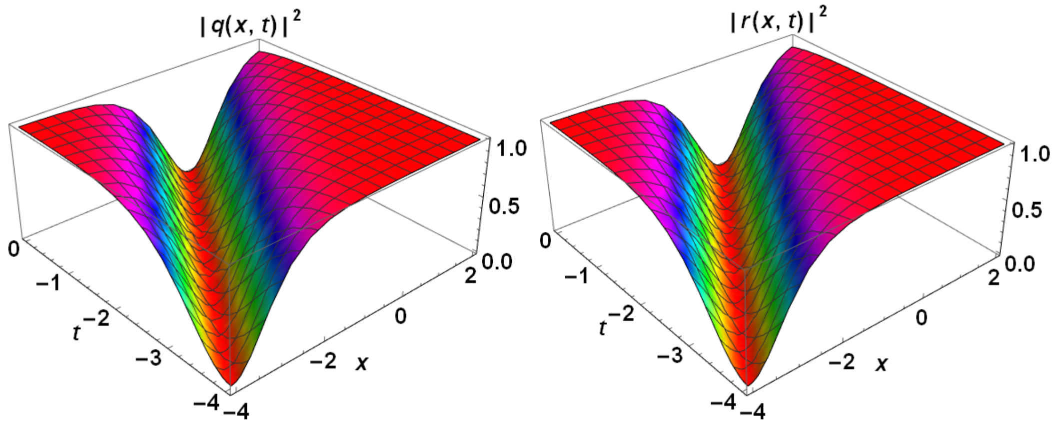

Fig. 1. Surface plots of highly dispersive dark optical solitons given by Eqs. (28) and (29) in optical couplers with metamaterials.

Case-2. Taking $a_{16}\alpha_1>0$, $E_1\alpha_1>0$, $a_{26}E_2>0$ and $h_1^2-4h_0h_2<0$, then putting Eq. (24) together with (20) into Eqs. (22) and (23) leaves us with the combo periodic solutions

$$q(x,t)=\pm\frac{31E_1}{22\alpha_1}\sqrt{\frac{248a_{16}}{15\alpha_1}}\left[\frac{r_3\tan\left(\frac{1}{2}\sqrt{\frac{124E_1}{165\alpha_1}}(x-vt)\right)-r_4}{r_3+r_4\tan\left(\frac{1}{2}\sqrt{\frac{124E_1}{165\alpha_1}}(x-vt)\right)}\right]^3 e^{i(-\kappa x+\omega t+\theta_0)}, \tag{32}$$

and

$$r(x,t)=\pm\frac{31E_1}{22\alpha_1}\sqrt{\frac{248a_{26}E_1}{15\alpha_1E_2}}\left[\frac{r_3\tan\left(\frac{1}{2}\sqrt{\frac{124E_1}{165\alpha_1}}(x-vt)\right)-r_4}{r_3+r_4\tan\left(\frac{1}{2}\sqrt{\frac{124E_1}{165\alpha_1}}(x-vt)\right)}\right]^3 e^{i(-\kappa x+\omega t+\theta_0)}. \tag{33}$$

Setting $r_3\neq 0$ and $r_4=0$ yields the periodic solutions

$$q(x,t)=\pm\frac{31E_1}{22\alpha_1}\sqrt{\frac{248a_{16}}{15\alpha_1}}\tan^3\left(\frac{1}{2}\sqrt{\frac{124E_1}{165\alpha_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)}, \tag{34}$$

and

$$r(x,t)=\pm\frac{31E_1}{22\alpha_1}\sqrt{\frac{248a_{26}E_1}{15\alpha_1E_2}}\tan^3\left(\frac{1}{2}\sqrt{\frac{124E_1}{165\alpha_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)}, \tag{35}$$

while, assuming $r_3=0$ and $r_4\neq 0$ provides us the periodic solutions

$$q(x,t)=\pm\frac{31E_1}{22\alpha_1}\sqrt{\frac{248a_{16}}{15\alpha_1}}\cot^3\left(\frac{1}{2}\sqrt{\frac{124E_1}{165\alpha_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)}, \tag{36}$$

and

$$r(x,t)=\pm\frac{31E_1}{22\alpha_1}\sqrt{\frac{248a_{26}E_1}{15\alpha_1E_2}}\cot^3\left(\frac{1}{2}\sqrt{\frac{124E_1}{165\alpha_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)}. \tag{37}$$

Case-3. When $a_{16}E_1<0$, $a_{26}E_2<0$ and $h_0=\frac{h_1^2}{4h_2}$, we arrive the results:

$$\begin{aligned}&\Gamma_0=\Gamma_1=\Gamma_2=0,\ \Gamma_3=\pm 24h_2^3\sqrt{-\frac{35a_{16}}{E_1}},\\&\Omega_0=\Omega_1=\Omega_2=0,\\&\Omega_3=\pm 24h_2^3\sqrt{-\frac{35a_{26}}{E_2}},\ h_0=0,\ h_1=0,\ h_2=h_2,\end{aligned} \tag{38}$$

and

$$\begin{aligned}&B_2=0,\ B_1=0,\ A_2=0,\ A_1=0,\\&C_1=\frac{9}{2}\alpha_1,\ C_2=\frac{9}{2}\alpha_2,\\&\delta_1=\Delta_1\sqrt{\frac{a_{16}E_2}{a_{26}E_1}},\ \delta_2=\Delta_2\sqrt{\frac{a_{26}E_1}{a_{16}E_2}}.\end{aligned} \tag{39}$$

Plugging Eq. (38) with the aid of (21) into Eqs. (22) and (23) allows us the rational solutions

$$q(x,t)=\pm 24\sqrt{-\frac{35a_{16}}{E_1}}\left(\frac{h_2}{h_2(x-vt)+r_5}\right)^3 e^{i(-\kappa x+\omega t+\theta_0)}, \tag{40}$$

and

$$r(x,t)=\pm 24\sqrt{-\frac{35a_{26}}{E_2}}\left(\frac{h_2}{h_2(x-vt)+r_5}\right)^3 e^{i(-\kappa x+\omega t+\theta_0)}. \tag{41}$$

4.2. Enhanced Kudryashov's scheme

The enhanced Kudryashov's approach is a mathematical technique addressed to secure exact solutions to model equations [23]. It is an extension of the standard Kudryashov's method, which involves assuming a trial solution for the differential equation and then using it to derive a set of equations that can be solved to obtain the exact solution. The enhanced approach involves incorporating additional terms into the trial solution, allowing a wider range of equations to be solved. This is achieved by introducing an auxiliary function, which is used to generate additional terms in the trial solution. These additional terms can be used to approximate more complex nonlinearities and provide more accurate solutions. The enhanced Kudryashov's mechanism has been considered to address a variety of model equations, including the nonlinear Schrödinger equation, the modified Korteweg-de Vries equation, and the Korteweg-de Vries equation. It has also been used to study various physical phenomena, such as soliton dynamics and wave propagation in optical fibers. The enhanced Kudryashov's approach allows us a methodology for solving a wide range of model equations. It allows for more accurate solutions to be obtained and can be used to study complex physical phenomena in various fields.

Eqs. (13) and (14) also admit the formal solutions

$$g_1(\xi)=\sum_{l=0}^{N} G_j R^j(\xi) \quad G_N \neq 0, \tag{42}$$

and

$$g_2(\xi)=\sum_{l=0}^{M} H_j R^j(\xi) \quad H_M \neq 0, \tag{43}$$

along with the auxiliary equation

$$R'^2(\xi)=R^2(\xi)\left[1-\Upsilon R^{2p}(\xi)\right]\ln^2 a,\ 0<a\neq 1, \tag{44}$$

where $R(\xi)$ permits the exact solution

$$R(\xi)=\left[\frac{4\Pi}{\left(4\Pi^2+\Upsilon\right)\cosh(p\xi\ln a)+\left(4\Pi^2-\Upsilon\right)\sinh(p\xi\ln a)}\right]^{\frac{1}{p}}. \tag{45}$$

Here G_j, Υ, Π, p and H_j are constants. By the application the balancing technique in Eqs. (13) and (14), we arrive

$$N=3p \quad \text{and} \quad M=3p. \tag{46}$$

Case-1. Taking $p=1$ changes Eqs. (42) and (43) to

$$g_1(\xi)=G_0+G_1R(\xi)+G_2R^2(\xi)+G_3R^3(\xi),\ G_3\neq 0, \tag{47}$$

and

$$g_2(\xi)=H_0+H_1R(\xi)+H_2R^2(\xi)+H_3R^3(\xi),\ H_3\neq 0. \tag{48}$$

Substituting Eqs. (47) and (48) along with (44) into Eqs. (13) and (14) gives the results:

$$\begin{aligned}&G_0=G_1=G_2=0,\ G_3=\pm 24\Upsilon\left(\ln^3 a\right)\sqrt{\frac{70\Upsilon a_{16}}{2E_1-27\alpha_1\ln^2 a}},\\&H_0=H_1=H_2=0,\ H_3=\pm 24\Upsilon\left(\ln^3 a\right)\sqrt{\frac{70\Upsilon a_{26}}{2E_2-27\alpha_2\ln^2 a}},\end{aligned} \tag{49}$$

and

$$\begin{aligned}&A_1=-83a_{16}\ln^2 a,\ A_2=-83a_{26}\ln^2 a,\ B_1=1891a_{16}\ln^4 a,\ B_2=1891a_{26}\ln^4 a,\\&C_1=\frac{9}{2}\alpha_1,\ C_2=\frac{9}{2}\alpha_2,\ \delta_1=\left(\Delta_1+11025a_{16}\ln^6 a\right)\sqrt{\frac{a_{16}\left(2E_2-27\alpha_2\ln^2 a\right)}{a_{26}\left(2E_1-27\alpha_1\ln^2 a\right)}},\\&\delta_2=\left(\Delta_2+11025a_{26}\ln^6 a\right)\sqrt{\frac{a_{26}\left(2E_1-27\alpha_1\ln^2 a\right)}{a_{16}\left(2E_2-27\alpha_2\ln^2 a\right)}},\\&\Upsilon a_{16}\left(2E_1-27\alpha_1\ln^2 a\right)>0,\ \Upsilon a_{26}\left(2E_2-27\alpha_2\ln^2 a\right)>0,\\&a_{16}a_{26}\left(2E_2-27\alpha_2\ln^2 a\right)\left(2E_1-27\alpha_1\ln^2 a\right)>0,\end{aligned} \tag{50}$$

where E_1, E_2, Δ_1 and Δ_2 stem from Eq. (15).

Putting Eq. (49) along with (45) into Eqs. (47) and (48) gives the straddled profiles

$$\begin{aligned}q(x,t)=&\pm 24\Upsilon\left(\ln^3 a\right)\sqrt{\frac{70\Upsilon a_{16}}{2E_1-27\alpha_1\ln^2 a}}\\&\times\left[\frac{4\Pi}{\left(4\Pi^2+\Upsilon\right)\cosh\left[(x-vt)\ln a\right]+\left(4\Pi^2-\Upsilon\right)\sinh\left[(x-vt)\ln a\right]}\right]^3 e^{i(-\kappa x+\omega t+\theta_0)},\end{aligned} \tag{51}$$

and

$$\begin{aligned}r(x,t)=&\pm 24\Upsilon\left(\ln^3 a\right)\sqrt{\frac{70\Upsilon a_{26}}{2E_2-27\alpha_2\ln^2 a}}\\&\times\left[\frac{4\Pi}{\left(4\Pi^2+\Upsilon\right)\cosh\left[(x-vt)\ln a\right]+\left(4\Pi^2-\Upsilon\right)\sinh\left[(x-vt)\ln a\right]}\right]^3 e^{i(-\kappa x+\omega t+\theta_0)}.\end{aligned} \tag{52}$$

Assuming $\Upsilon=4\Pi^2$ simplifies Eqs. (51) and (52) to the bright wave profiles

$$q(x,t)=\pm 24\left(\ln^3 a\right)\sqrt{\frac{70a_{16}}{2E_1-27\alpha_1\ln^2 a}}\operatorname{sech}^3\left[(x-vt)\ln a\right]e^{i(-\kappa x+\omega t+\theta_0)}, \tag{53}$$

and

$$r(x,t)=\pm 24\left(\ln^3 a\right)\sqrt{\frac{70a_{26}}{2E_2-27\alpha_2\ln^2 a}}\operatorname{sech}^3\left[\left(x-vt\right)\ln a\right]e^{i\left(-\kappa x+\omega t+\theta_0\right)}, \tag{54}$$

while, taking $\Upsilon=-4\Pi^2$ translates Eqs. (51) and (52) to the singular wave profiles

$$q(x,t)=\pm 24\left(\ln^3 a\right)\sqrt{-\frac{70a_{16}}{2E_1-27\alpha_1\ln^2 a}}\operatorname{csch}^3\left[\left(x-vt\right)\ln a\right]e^{i\left(-\kappa x+\omega t+\theta_0\right)}, \tag{55}$$

and

$$r(x,t)=\pm 24\left(\ln^3 a\right)\sqrt{-\frac{70a_{26}}{2E_2-27\alpha_2\ln^2 a}}\operatorname{csch}^3\left[\left(x-vt\right)\ln a\right]e^{i\left(-\kappa x+\omega t+\theta_0\right)}. \tag{56}$$

Fig. 2 displays the profile of bright nonlinear waveforms for $a=e$, $b_j=2$, $\lambda_j=1$, $\theta_j=1$, $\beta_j=1$, $\gamma_j=1$, $a_{j1}=1$, $a_{j2}=1$, $a_{j3}=44$, $a_{j4}=1$, $a_{j5}=6$, $a_{j6}=1$ and $\alpha_j=-1$.

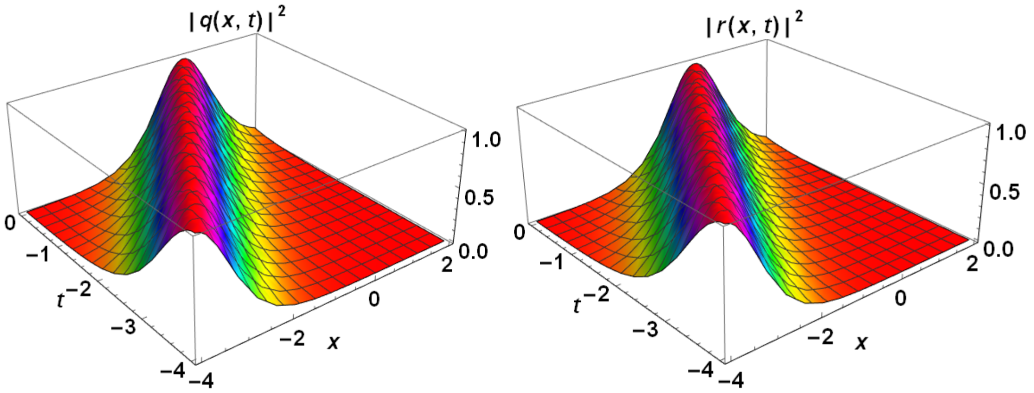

Fig. 2. Surface plots of highly dispersive bright optical solitons given by Eqs. (53) and (54) in optical couplers with metamaterials.

Case-2. Choosing $p=2$ decreases Eqs. (42) and (43) to

$$g_1(\xi)=G_0+G_1R(\xi)+G_2R^2(\xi)+G_3R^3(\xi)+G_4R^4(\xi)+G_5R^5(\xi)+G_6R^6(\xi),\quad G_6\neq 0, \tag{57}$$

and

$$g_2(\xi)=H_0+H_1R(\xi)+H_2R^2(\xi)+H_3R^3(\xi)+H_4R^4(\xi)+H_5R^5(\xi)+H_6R^6(\xi),\quad H_6\neq 0. \tag{58}$$

Plugging Eqs. (57) and (58) along with (44) into Eqs. (13) and (14) gives the results:

$$\begin{aligned}
&G_5=G_4=G_3=G_2=G_1=G_0=0,\\
&G_6=\pm 192\Upsilon\left(\ln^3 a\right)\sqrt{\frac{35\Upsilon a_{16}}{E_1-54\alpha_1\ln^2 a}},\\
&H_5=H_4=H_3=H_2=H_1=H_0=0,\\
&H_6=\pm 192\Upsilon\left(\ln^3 a\right)\sqrt{\frac{35\Upsilon a_{26}}{E_2-54\alpha_2\ln^2 a}},
\end{aligned} \tag{59}$$

and

$$
\begin{aligned}
&A_1 = -332 a_{16} \ln^2 a, \ \ A_2 = -332 a_{26} \ln^2 a,\\
&B_1 = 30256 a_{16} \ln^4 a, \ B_2 = 30256 a_{26} \ln^4 a,\\
&C_1 = \frac{9}{2}\alpha_1, \ C_2 = \frac{9}{2}\alpha_2,\\
&\delta_1 = \left(\Delta_1 + 705600 a_{16} \ln^6 a\right)\sqrt{\frac{a_{16}\left(E_2 - 54\alpha_2 \ln^2 a\right)}{a_{26}\left(E_1 - 54\alpha_1 \ln^2 a\right)}},\\
&\delta_2 = \left(\Delta_2 + 705600 a_{26} \ln^6 a\right)\sqrt{\frac{a_{26}\left(E_1 - 54\alpha_1 \ln^2 a\right)}{a_{16}\left(E_2 - 54\alpha_2 \ln^2 a\right)}},\\
&\Upsilon a_{16}\left(E_1 - 54\alpha_1 \ln^2 a\right) > 0,\\
&\Upsilon a_{26}\left(E_2 - 54\alpha_2 \ln^2 a\right) > 0,\\
&a_{16} a_{26}\left(E_2 - 54\alpha_2 \ln^2 a\right)\left(E_1 - 54\alpha_1 \ln^2 a\right) > 0.
\end{aligned}
\tag{60}
$$

Putting Eq. (59) along with (45) into Eqs. (57) and (58) enables us the straddled nonlinear waveforms

$$
q(x,t) = \pm 192\Upsilon\left(\ln^3 a\right)\sqrt{\frac{35\Upsilon a_{16}}{E_1 - 54\alpha_1 \ln^2 a}}\left[\frac{4\Pi}{\left(\begin{array}{l}\left(4\Pi^2 + \Upsilon\right)\cosh\left[2(x - vt)\ln a\right]\\ +\left(4\Pi^2 - \Upsilon\right)\sinh\left[2(x - vt)\ln a\right]\end{array}\right)}\right]^3 \times e^{i(-\kappa x + \omega t + \theta_0)},
\tag{61}
$$

and

$$
r(x,t) = \pm 192\Upsilon\left(\ln^3 a\right)\sqrt{\frac{35\Upsilon a_{26}}{E_2 - 54\alpha_2 \ln^2 a}}\left[\frac{4\Pi}{\left(\begin{array}{l}\left(4\Pi^2 + \Upsilon\right)\cosh\left[2(x - vt)\ln a\right]\\ +\left(4\Pi^2 - \Upsilon\right)\sinh\left[2(x - vt)\ln a\right]\end{array}\right)}\right]^3 \times e^{i(-\kappa x + \omega t + \theta_0)}.
\tag{62}
$$

Setting $\Upsilon = 4\Pi^2$ simplifies Eqs. (61) and (62) to the bright soliton wave profiles

$$
q(x,t) = \pm 192\left(\ln^3 a\right)\sqrt{\frac{35 a_{16}}{E_1 - 54\alpha_1 \ln^2 a}}\,\mathrm{sech}^3\left[2(x - vt)\ln a\right] e^{i(-\kappa x + \omega t + \theta_0)},
\tag{63}
$$

and

$$
r(x,t) = \pm 192\left(\ln^3 a\right)\sqrt{\frac{35 a_{26}}{E_2 - 54\alpha_2 \ln^2 a}}\,\mathrm{sech}^3\left[2(x - vt)\ln a\right] e^{i(-\kappa x + \omega t + \theta_0)},
\tag{64}
$$

while, taking $\Upsilon = -4\Pi^2$ decreases Eqs. (61) and (62) to the singular solitons

$$q(x,t) = \pm 192\left(\ln^3 a\right)\sqrt{-\frac{35a_{16}}{E_1 - 54\alpha_1 \ln^2 a}}\operatorname{csch}^3\left[2(x - vt)\ln a\right]e^{i(-\kappa x + \omega t + \theta_0)}, \tag{65}$$

and

$$r(x,t) = \pm 192\left(\ln^3 a\right)\sqrt{-\frac{35a_{26}}{E_2 - 54\alpha_2 \ln^2 a}}\operatorname{csch}^3\left[2(x - vt)\ln a\right]e^{i(-\kappa x + \omega t + \theta_0)}. \tag{66}$$

5. Conclusions

This paper retrieved a complete spectrum of highly dispersive soliton solutions in optical couplers with metamaterials. These are 1–soliton solutions that are recovered and exhibited. Two mathematical algorithms have made this retrieval possible. The Kudryashov's technique and the unified Riccati equation expansion procedure were at play in this work. The cons of using Riccati's equation expansion method is that it fails to acquire the much-needed bright soliton solution. Therefore, the enhanced Kudryashov's scheme is to the rescue that recovers the bright solitons, and thus, collectively, the two separate and independent integration approaches yielded a full spectrum of soliton solutions. Additionally, the enhanced Kudryashov's integration scheme secured the straddled solitons that come out as a byproduct of this scheme. The numerical simulations are also included with the aid of Mathematica throughout the parameter constraints, which indicate the existence criteria for these solitons. It must be noted that with six dispersion terms present in the model, there will be a substantial radiative effect from these solitons, and thus, it will lead to a shedding of energy. Consequently, there will be a substantial slow-down of solitons. These radiative effects cannot be captured from any of the two integration techniques employed in the paper. This is a major shortcoming of this couple of algorithms.

It must be noted that the results of the current paper are the outcome of theoretical modeling. There are quite a few results from the experimental standpoint in mode-locked lasers that have been reported earlier [24]. In future, these analytical results will be aligned with the experimental results and the outcome of such studies will be addressed with time.

The optoelectronics community can greatly benefit from the diverse range of soliton solutions available. The analytical results recovered in this paper fully display a complete spectrum of highly dispersive optical solitons together with straddled solitons, which are an additional set of solitons that naturally emerge from the enhanced Kudryashov's scheme. These analytical results lead to a floodgate of upcoming opportunities. An immediate next thought would be to address the conservation laws of the model. This would naturally lead to further additional studies, such as the analysis of the four–wave mixing effect, computation of the collision–induced timing jitter, and quasi–monochromatic dynamics of solitons. The numerical studies of the solitons would also be taken care of next. The implementation of the Laplace–Adomian decomposition algorithm, variational iteration procedure, and more would lead to a visual and numerical perspective of the solitons derived in this work. The results will be aligned with the output of pre–existing works [1–24] and disseminated in the future.

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Анотація. *У цій роботі отримані рішення високодисперсних солітонів в з'єднувачах, які виготовлені з оптичних метаматеріалів із законом Керра для нелінійного показника* *заломлення. У нелінійній оптиці та фотоніці ці хвильові пакети володіють унікальною властивістю, що дозволяє їм зберігати свою форму та швидкість під час поширення через нелінійні середовища. Оптичні з'єднувачі, що використовують метаматеріали, є передовими пристроями, спеціально розробленими для покращення зв'язку та передачі оптичних сигналів між різними оптичними компонентами або системами, використовуючи унікальні характеристики цих спеціально налаштованих матеріалів. Новизна статті полягає в розгляді високодисперсних оптичних солітонів в оптичному з'єднувачі, що виготовлений з метаматеріалів. Основною метою цієї статті є інтеграція керуючої моделі стосовно високодисперсних солітонів у оптичних з'єднувачах з використанням метаматеріалів. Для цього використовується єдина процедура розширення рівняння Рікаті та покращена схема Кудряшова. Вони застосовуються до такого пристрою вперше. Ці методи дозволяють отримати повний спектр оптичних солітонів, а також солітонів, що перекриваються. Новизна полягає в спільному застосуванні обох процедур, що розкриває багате розмаїття солітонів. Крім того, один з підходів виявляє несподівану перевагу - отримання рішення для солітонів, що перекриваються.*

Ключові слова: *солітони, дисперсія, з’єднувачі, рівняння Ріккаті, підхід Кудряшова.*