# Optical solitons and group invariants for Chen-Lee-Liu equation with time-dependent chromatic dispersion and nonlinearity by Lie symmetry

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**Abstract.** This paper uses the classical Lie symmetry method for optical solitons to study the Chen-Lee-Liu equation. We are able to establish symmetries that convert the model into a set of ordinary differential equations and obtain solutions of the reduced equations through various methods.

**Keywords**: solitons; Chen-Lee-Liu equation; Lie symmetry analysis

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## 1. Introduction

The theory of optical solitons provides technology across the globe in the field of fiber-optic communication systems. There are various models that govern the dynamics of soliton propagation, including birefringent fibers, optical fibers, photonic-crystal fiber (PCF), metamaterials, and others. Soliton solutions play a vital role in the telecommunications industry. The Nonlinear Schrödinger's equation (NLSE) is one of the most commonly studied equations. In recent years, many authors have analyzed various models for different aspects. Some of the available models are the Lakshmanan-Porsezian-Daniel model [1], the Gerdjikov-Ivanov equation [2], the Kaup-Newell equation [3], the Schrödinger-Hirota model [4], the complex Ginzburg-Landau equation [5], the Radhakrishnan-Kundu-Lakshmanan equation [6], and several others. Several papers are readily available with such models and their corresponding soliton dynamics.

In nonlinear optics, three types of derivative NLSEs are being studied. The Chen-Lee-Liu (CLL) equation [7] is one such model in the family of NLSEs that was first introduced in 1979 and has gained importance ever since. There have been several studies regarding this equation in the past [8, 9]. Several mathematical algorithms [10, 11] have already been successfully implemented to address such models using well-known techniques, including the Adomian decomposition method [12], the semi-inverse variational principle [13], the Darboux transformation [14], and the chirped W-shaped optical solitons [15].

In the context of optical fiber, the generation of combined solitary waves for the CLL equation was first reported by Li et al. [16] in 2000. The rogue wave solutions with this model were reported by Zhang et al. [17]. Triki et al. [8] retrieved chirped singular solitons using the traveling wave approach for the CLL equation. The work in this paper focuses on handling the propagation of an optical pulse modeled by a family of the following CLL equation [18]-[21], which is also known as the generalized CLL equation with variable coefficients, as described below

$$iz_t + f(t) z_{xx} + ig(t) |z|^2 z_x = 0, \tag{1}$$

where $z = z(x,t)$, the dependent variable, represents the complex-valued function to be determined, $z_t$ gives the temporal dispersion, $z_x$ is the spatial dispersion, $z_{xx}$ corresponds to the higher-order dispersion, $f(t)$ and $g(t)$ are arbitrary functions of time. This equation is a generalization of the standard CLL equation, as presented below

$$iz_t + fz_{xx} + ig |z|^2 z_x = 0, \tag{2}$$

where $f$ and $g$ are constants. Eq. (2) transforms into the regular CLL equation when $f = g = 1$. In optical fiber, the term $g$ represents the coefficient of self-steepening nonlinearity, while the coefficient $f$ gives the group velocity dispersion. In the upcoming sections of this manuscript, Lie symmetry analysis [22]-[23] will be presented to find optical solitons and group invariant solutions of Eq. (1).

## 2. Classical Lie symmetry analysis

The study of differential equations with symmetry analysis has been gaining popularity recently. In this paper, we explain how to derive the symmetries, symmetry reductions, and group invariant solutions of the variable coefficient CLL equation using the Lie classical method [24]-[28] algorithmic way. To determine the symmetry analysis admitted by Eq. (1), let us consider

$$z(x,t) = u(x,t) + iv(x,t), \tag{3}$$

which splits Eq. (1) into its real and imaginary portions as:

$$-v_t + f(t) u_{xx} - g(t)\left(u^2 + v^2\right) v_x = 0 \text{ and } u_t + f(t) v_{xx} + g(t)\left(u^2 + v^2\right) u_x = 0. \tag{4}$$

To locate the classical symmetries, we will now consider the Lie group of continuous transformations as follows:

$$\begin{aligned} &\hat{u} = u + \epsilon\eta(x,t,u,v) + o\left(\epsilon^2\right), \quad \hat{v} = v + \epsilon\phi(x,t,u,v) + o\left(\epsilon^2\right), \\ &\hat{x} = x + \epsilon\xi(x,t,u,v) + o\left(\epsilon^2\right), \quad \hat{t} = t + \epsilon\tau(x,t,u,v) + o\left(\epsilon^2\right), \end{aligned} \tag{5}$$

which remains the system (4) invariant under this one-parameter transformation. This leads to an overdetermined linear system of equations for the infinitesimals $\eta(x,t,u,v)$, $\phi(x,t,u,v)$, $\xi(x,t,u,v)$, and $\tau(x,t,u,v)$. Thus, the invariance condition of Eq. (1) yields:

$$-\phi^t + f(t)\eta^{xx} + \tau f'(t)u_{xx} - g(t)\left(u^2+v^2\right)\phi^x$$
$$-g(t)(2u\eta + 2v\phi)v_x - \left(u^2+v^2\right)\tau g'(t)v_x = 0$$
and (6)
$$\eta^t + f(t)\phi^{xx} + \tau f'(t)v_{xx} + g(t)\left(u^2+v^2\right)\eta^x$$
$$+g(t)(2u\eta + 2v\phi)u_x + \left(u^2+v^2\right)\tau g'(t)u_x = 0.$$

By substituting the values of the infinitesimal $\eta^t$, $\phi^t$, $\eta^x$, $\phi^x$, $\eta^{xx}$, and $\phi^{xx}$ into (6) and equating the same power of various differentials to zero, we obtain the desired system of overdetermined partial differential equations (PDEs). Solving this overdetermined system of PDEs yields:

$$\eta = c_1u + c_2v, \quad \phi = -c_2u + c_1v, \quad \xi = c_3x + c_4 \text{ and } \tau = \frac{2c_3}{f(t)}\int f(t)\,dt + \frac{c_5}{f(t)}, \tag{7}$$

where $c_1$, $c_2$, $c_3$, $c_4$, and $c_5$ are arbitrary constants, and $f(t)$ and $g(t)$ are governed by the following specific conditions:

$$\tau g'(t) + 2(c_1 - c_3)g(t) + g(t)\tau_t = 0, \text{ and } \tau f'(t) - 2c_3 f(t) + f(t)\tau_t = 0. \tag{8}$$

The associated infinitesimal generators are presented as follows:

$$G_1 = u\frac{\partial}{\partial u} + v\frac{\partial}{\partial v}, \quad G_2 = v\frac{\partial}{\partial u} - u\frac{\partial}{\partial v}, G_3 = x\frac{\partial}{\partial x} + \frac{2}{f(t)}\int f(t)\,dt\frac{\partial}{\partial t}, G_4 = \frac{\partial}{\partial x} \text{ and } G_5 = \frac{1}{f(t)}\frac{\partial}{\partial t}.$$
(9)

The vector fields form a Lie algebra provided by (9) as:

$$\begin{aligned}
&[G_1,G_2]=0, \quad [G_1,G_3]=0, \quad [G_1,G_4]=0, \quad [G_1,G_5]=0,\\
&[G_2,G_3]=0, \quad [G_2,G_4]=0, \quad [G_2,G_5]=0,\\
&[G_3,G_4]=-G_4, \quad [G_3,G_5]=-2G_5,\\
&[G_4,G_5]=0.
\end{aligned} \tag{10}$$

## 3. Reductions and exact solutions of variable coefficient CLL equation

In this section, our main goal is to derive the exact solutions of Eq. (1) through the reduced equations. The similarity variables and forms can be obtained by using characteristic equations as:

$$\frac{dx}{\xi} = \frac{dt}{\tau} = \frac{du}{\eta} = \frac{dv}{\phi}, \tag{11}$$

To achieve symmetry reductions and exact solutions, we will discuss the following four cases of vector fields: 1. $G_3$; 2. $G_4 + G_5$; 3. $G_5$ and 4. $G_2 + \lambda G_4 + G_5$, where $\lambda$ is an arbitrary real number different from zero.

### *3.1. Reduction under* $G_3$

Solving Eq. (11) leads to the similarity variables in the following form:

$$z(x,t) = U(\psi)e^{iV(\psi)}, \quad \psi = \frac{x}{\left(\int f(t)dt\right)^{1/2}}, \quad g(t) = \frac{f(t)}{2\left(\int f(t)\,dt\right)^{1/2}}. \tag{12}$$

Treating $U(\psi)$ and $V(\psi)$ as new dependent variables with a new independent variable $\psi$, and using Eq. (12) in Eq. (1), we obtain a special nonlinear ordinary differential equation as follows:

$$\frac{\psi}{2}U(\psi)V'(\psi)+U''(\psi)-U(\psi)V'(\psi)^2-\frac{1}{2}U^3(\psi)V'(\psi)=0$$

and (13)

$$-\frac{\psi}{2}U'(\psi)+U(\psi)V''(\psi)+2U'(\psi)V'(\psi)+\frac{1}{2}U'(\psi)U^2(\psi)=0.$$

We are only able to retrieve a constant solution for the studied equation here.

### *3.2. Reduction under* $G_4+G_5$

The similarity variables for the vector fields $G_4+G_5$ are given by:

$$z(x,t)=U(\psi)e^{iV(\psi)},\quad \psi=x-\int f(t)dt,\quad g(t)=c_6 f(t). \tag{14}$$

By substituting Eq. (14) into Eq. (1), Eq. (1) reduces to the following system of ordinary differential equations (ODEs):

$$U(\psi)V'(\psi)+U''(\psi)-U(\psi)V'(\psi)^2-c_6U^3(\psi)V'(\psi)=0, \tag{15}$$

and

$$-U'(\psi)+U(\psi)V''(\psi)+2U'(\psi)V'(\psi)+c_6U'(\psi)U^2(\psi)=0. \tag{16}$$

Multiplying Eq. (16) by $U(\psi)$ and integrating the resulting equation with respect to $\psi$ gives:

$$U^2(\psi)V'(\psi)-\frac{1}{2}U^2(\psi)+c_6\frac{1}{4}U^4(\psi)=0. \tag{17}$$

Using Eq. (17) leads to:

$$V'(\psi)=\frac{1}{2}-\frac{1}{4}c_6U^2(\psi)+B, \tag{18}$$

where $B$ is an arbitrary constant. By substituting Eq. (18) into Eq. (15) and multiplying it again with $U'(\psi)$ and integrating, we obtain:

$$-4c_6(1+B)U^4(\psi)+4\left(1-4B^2\right)U^2(\psi)+16(U'(\psi))^2+c_6^2U^6(\psi)=0. \tag{19}$$

By choosing $U^2(\psi)=Q(\psi)$ in Eq. (19), we conclude that:

$$c_6^2Q^4(\psi)-4c_6(1+B)Q^3(\psi)+4\left(1-4B^2\right)Q^2(\psi)+4(Q'(\psi))^2=0. \tag{20}$$

Upon solving Eq. (20), the solution of the variable coefficient CLL equation is given as:

$$\begin{aligned} z(x,t)=&\frac{\mathrm{e}^{\psi\sqrt{-1+4B^2}}\left(-1+4B^2\right)}{\left(\mathrm{e}^{C_1\sqrt{-1+4B^2}}\right)}\\ &\times\left(16c_6{}^2+\frac{\left(\mathrm{e}^{\psi\sqrt{-1+4B^2}}\right)^2}{\left(\mathrm{e}^{C_1\sqrt{-1+4B^2}}\right)^2}+64B^2-8\frac{\mathrm{e}^{\psi\sqrt{-1+4B^2}}}{\mathrm{e}^{C_1\sqrt{-1+4B^2}}}c_6B\right.\\ &\left.-8\frac{\mathrm{e}^{\psi\sqrt{-1+4B^2}}}{\mathrm{e}^{C_1\sqrt{-1+4B^2}}}c_6+32c_6{}^2B+16c_6{}^2B^2-16\right)^{-1} \end{aligned} \tag{21}$$

where $\psi$ is given by Eq. (14) and $C_1$ is an arbitrary constant of integration.

### *3.3. Reduction under* $G_5$

In this case, the similarity variables are given as follows:

$$\begin{aligned} &z(x,t) = U(\psi)e^{iV(\psi)}, \\ &\psi = x, \\ &g(t) = c_7 f(t), \end{aligned} \tag{22}$$

which presents the reduced form of Eq. (1) as:

$$U''(\psi) - U(\psi)V'(\psi)^2 - c_7 U^3(\psi)V'(\psi) = 0, \tag{23}$$

and

$$U(\psi)V''(\psi) + 2U'(\psi)V'(\psi) + c_7 U'(\psi)F^2(\psi) = 0. \tag{24}$$

In this case, due to the complexity of the above system of equations, we are unable to find non-trivial solutions for it.

### *3.4. Reduction under* $G_2 + \lambda G_4 + G_5$

The similarity variables are obtained by taking the characteristic equation as:

$$\begin{aligned} &z(x,t) = e^{-i\left(\int f(t)dt + V(\psi)\right)}U(\psi), \\ &\psi = x - \lambda\int f(t)\,dt, \qquad\qquad . \\ &g(t) = f(t) \end{aligned} \tag{25}$$

Hence, Eq. (1) turns out to be the following system of ODEs

$$U(\psi) - \lambda U(\psi)V'(\psi) + U''(\psi) - U(\psi)\left(V'(\psi)\right)^2 + U^3(\psi)V'(\psi) = 0\,, \tag{26}$$

and

$$-\lambda U'(\psi) - U(\psi)V''(\psi) - 2U'(\psi)V'(\psi) + U^2(\psi)U'(\psi) = 0\,. \tag{27}$$

Multiplying Eq. (27) by $U(\psi)$ and integrating once gives:

$$-U^2(\psi)V'(\psi) - \frac{1}{2}\lambda U^2(\psi) + \frac{1}{4}U^4(\psi) = 0\,. \tag{28}$$

Using Eq. (28) leads to:

$$V'(\psi) = -\frac{\lambda}{2} + \frac{1}{4}U^2(\psi) + M\,, \tag{29}$$

where $M$ is an arbitrary constant. Putting (29) into (26) and once more multiplying it with $U(\psi)$ and integrating, leads to:

$$4(M-\lambda)U^4(\psi) + \left(16 + 4\lambda^2 - 16M^2\right)U^2(\psi) + 16\left(U'(\psi)\right)^2 + U^6(\psi) = 0\,. \tag{30}$$

Substituting $U^2(\psi) = H(\psi)$ into Eq. (30) gives us:

$$H^4(\psi) + 4(M-\lambda)H^3(\psi) + 4\left(4 + \lambda^2 - 4M^2\right)H^2(\psi) + 4\left(H'(\psi)\right)^2 = 0. \tag{31}$$

Upon solving Eq. (31), the following solution of the variable coefficient CLL equation is obtained:

$$z(x,t)=\sqrt{C_4}\,\mathrm{e}^{-i\left(\int f(t)dt+\left(-\frac{\lambda}{2}+\frac{3}{8}C_4+\frac{1}{8}\sqrt{5C_4^2-16C_4\lambda+16\lambda^2+64}\right)\left(x-\lambda\int f(t)dt\right)\right)}. \tag{32}$$

where $C_4$ is an arbitrary positive constant of integration.

To obtain different solutions, we take the following form of $H(\psi)$:

$$H(\psi)=C_0\tanh(C_1\psi), \tag{33}$$

($C_0$ represents the difference between the two stable states of the shock wave) which leads to the dark optical soliton solution of the variable coefficient CLL equation as:

$$\begin{aligned}
&z(x,t)=U(\psi)e^{-i\left(\int f(t)dt+V(\psi)\right)},\\
&\psi=x-\lambda\int f(t)dt,\\
&U(\psi)=\frac{1}{5}\Bigg[20\lambda-5\sqrt{\lambda^2+20}-5\sqrt{-8\lambda\sqrt{\lambda^2+20}+17\lambda^2+20}\\
&\qquad\times\tanh\left(-\frac{1}{10}\sqrt{8\lambda\sqrt{\lambda^2+20}-17\lambda^2-20}\psi+C_2\right)\Bigg]^{1/2},
\end{aligned} \tag{34}$$

and

$$\begin{aligned}
V(\psi)=&-\frac{1}{10}\lambda\psi+\frac{3\psi}{20}\sqrt{\lambda^2+20}-\frac{1}{4}\frac{\sqrt{-8\lambda\sqrt{\lambda^2+20}+17\lambda^2+20}}{\sqrt{8\lambda\sqrt{\lambda^2+20}-17\lambda^2-20}}\\
&\times\ln\left|-\tanh\left(-\frac{1}{10}\sqrt{8\lambda\sqrt{\lambda^2+20}-17\lambda^2-20}\psi+C_2\right)-1\right|\\
&-\frac{1}{4}\frac{\sqrt{-8\lambda\sqrt{\lambda^2+20}+17\lambda^2+20}}{\sqrt{8\lambda\sqrt{\lambda^2+20}-17\lambda^2-20}}\\
&\times\ln\left|-\tanh\left(-\frac{1}{10}\sqrt{8\lambda\sqrt{\lambda^2+20}-17\lambda^2-20}\psi+C_2\right)+1\right|,
\end{aligned} \tag{35}$$

where $C_2$ is the constant of integration that represents the center position shift.

However, the discriminant $8\lambda\sqrt{\lambda^2+20}-17\lambda^2-20$ must be both positive and negative simultaneously for solitons. Therefore, we can set this discriminant to zero in order to obtain certain solitons.

Let us take $H(\psi)$ to be of the form:

$$H(\psi)=C_0\coth(C_1\psi), \tag{36}$$

and the singular soliton solution appears as:

$$\begin{aligned}
&z(x,t)=U(\psi)e^{-i\left(\int f(t)dt+V(\psi)\right)},\\
&\psi=x-\lambda\int f(t)dt,\\
&U(\psi)=\frac{1}{5}\Bigg[20\lambda-5\sqrt{\lambda^2+20}-5\sqrt{-8\lambda\sqrt{\lambda^2+20}+17\lambda^2+20}\\
&\qquad\times\coth\left(-\frac{1}{10}\sqrt{8\lambda\sqrt{\lambda^2+20}-17\lambda^2-20}\psi+C_2\right)\Bigg]^{1/2},
\end{aligned} \tag{37}$$

and

$$
\begin{aligned}
V(\psi) = {} & -\frac{1}{10}\lambda\psi + \frac{3\psi}{20}\sqrt{\lambda^2+20} \\
& -\frac{1}{4}\frac{\sqrt{-8\lambda\sqrt{\lambda^2+20}+17\lambda^2+20}}{\sqrt{8\lambda\sqrt{\lambda^2+20}-17\lambda^2-20}} \\
& \times\ln\left|-\coth\left(-\frac{1}{10}\sqrt{8\lambda\sqrt{\lambda^2+20}-17\lambda^2-20}\psi+C_2\right)-1\right| \\
& -\frac{1}{4}\frac{\sqrt{-8\lambda\sqrt{\lambda^2+20}+17\lambda^2+20}}{\sqrt{8\lambda\sqrt{\lambda^2+20}-17\lambda^2-20}} \\
& \times\ln\left|-\coth\left(-\frac{1}{10}\sqrt{8\lambda\sqrt{\lambda^2+20}-17\lambda^2-20}\psi+C_2\right)+1\right|.
\end{aligned}
\tag{38}
$$

However, the discriminant $8\lambda\sqrt{\lambda^2+20}-17\lambda^2-20$ must be both positive and negative simultaneously for solitons. We can set this number to zero to obtain certain solutions.

Taking $H(\psi)$ to be of the form:

$$H(\psi) = C_0\operatorname{sech}(C_1\psi), \tag{39}$$

which leads to the bright soliton solution:

$$
\begin{aligned}
z(x,t) = {} & \left[2\sqrt{3\lambda^2-4}\operatorname{sech}\left(\left(x-\lambda\int f(t)dt\right)\sqrt{3\lambda^2-4}-C_1\right)\right]^{1/2} \\
& \times e^{-i\left(\int f(t)dt+\frac{\lambda}{2}\left(x-\lambda\int f(t)dt\right)+\frac{1}{2}\operatorname{rctan}\left(\sinh\left(\left(x-\lambda\int f(t)dt\right)\sqrt{3\lambda^2-4}-C_1\right)\right)\right)},
\end{aligned}
\tag{40}
$$

whereas the discriminant for bright solitons, given by Eq. (40), must be positive, which is expressed as:

$$3\lambda^2-4>0. \tag{41}$$

Assuming the following form of $H(\psi)$:

$$H(\psi) = C_0\operatorname{csch}(C_1\psi), \tag{42}$$

yields another kind of singular soliton solution as:

$$
\begin{aligned}
z(x,t) = {} & \left[-2\sqrt{-3\lambda^2+4}\operatorname{csch}\left(-\psi\sqrt{3\lambda^2-4}+C_1\right)\right]^{\frac{1}{2}} \\
& \times e^{-i\left(\int f(t)dt+\frac{1}{2}\lambda\psi+\frac{1}{2}\frac{\sqrt{-3\lambda^2+4}}{\sqrt{3\lambda^2-4}}\ln\left(-\tanh\left(-\frac{1}{2}\psi\sqrt{3\lambda^2-4}+\frac{C_1}{2}\right)\right)\right)}.
\end{aligned}
\tag{43}
$$

However, the discriminant $3\lambda^2-4$ must be both positive and negative simultaneously for solitary solitons, as given by Eq. (43). As a result, it can be concluded that the variable coefficient CLL equation does not contain any such form of singular soliton.

## 4. Conclusions

In this manuscript, we have successfully found the optical soliton solutions of the variable coefficient CLL equation with the aid of the Lie classical method. The reduced system of nonlinear ODEs obtained by symmetry reductions provides new bright soliton solutions by implementing the scheme of integration and gives us certain conditions under which other singular solitons can be obtained. The results of this paper can be useful for further investigation in future research. All the results have been verified using Maple software.

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***Анотація.*** *У цій статті вивчається рівняння Чена-Лі-Лю з використанням класичного методу симетрії Лі для оптичних солітонів. Продемонстровано можливість встановлення симетрій, які перетворюють модель на набір звичайних диференціальних рівнянь і отримання розв'язків скорочених рівнянь різними методами.*

***Ключові слова:*** *солітони, рівняння Чен-Лі-Лю, аналіз симетрії Лі.*