# Optical solitons in magneto-optic waveguides for the concatenation model

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**Abstract.** The current paper focuses on the retrieval of solitons in magneto–optic waveguides for the concatenation model having Kerr law of nonlinear refractive index. The simplest equation approach as well as the extended simplest equation method collectively reveal a full spectrum of soliton solutions to the model. The parameter constraints guarantee the existence of such solitons.

**Keywords**: solitons, concatenation model, magneto-optic waveguide, simplest equation method

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## 1. Introduction

One of the most fascinating and interesting equations in nonlinear optics that was conceived during 2014 is the concatenation model [1, 2]. This is a conjunction of three of the pre–existing equations that describe the propagation of solitons through an optical fiber. They are the nonlinear Schrödinger's equation (NLSE), Lakshmanan–Porsezian–Daniel (LPD) model and the Sasa–Satsuma equation (SSE). The proposed concatenation model has recently achieved enormous popularity in the field and is relentlessly presenting a lasting impression in optics. A wide variety of features has been touched base with this model [3–11]. Some of the salient features that were addressed are the Painleve analysis, retrieval of solitons using the methods of undetermined coefficients, trial equation approach, Kudryashov's method, conservation laws, bifurcation analysis, quiescent solitons for nonlinear chromatic dispersion (CD), addressing internet bottleneck features. Indeed, this is one of the growing problems of Internet communications across the globe. The high demand for wired communications across the entire planet cannot be kept up because of the low bandwidth of optical fibers. This leads to slow communications across the globe which leads to an unfavorable situation in the telecommunication industry. The model was also addressed with the power law of self–phase modulation (SPM) and later it was extended with differential group delay whose soliton solutions were also determined by the aid of the method of undetermined coefficients.

The dynamics of solitons in magneto-optic waveguides have been extensively studied using a variety of models in the past [16-20]. Several soliton solutions have been recovered using a wide range of models. The conservation laws are also recovered for such models and various other aspects of such forms of waveguides have been touched base upon. It is now time to turn the page. The current paper now studies the concatenation model in magneto–optic waveguides for the first time. There are two integration schemes that will be implemented to recover the soliton solutions. They are the simplest equation method and the extended simplest equation approach. These algorithms would collectively lead to a full spectrum of optical soliton solutions. The model is first presented in magneto–optic waveguides. Subsequently, the two integration algorithms are revisited and recapitulated. Thereafter, the soliton solutions are derived and the parameter constraints, that naturally emerged from the integration schemes, are enlisted. The details are exhibited in the rest of the paper.

## 2. Mathematical model

The expression of the concatenation model for polarization-preserving fibers can be summarized as follows:

$$\begin{aligned} & i\phi_t + a\phi_{zz} + b|\phi|^2\phi + ic_2\left[\sigma_7\phi_{zzz} + \sigma_8|\phi|^2\phi_z + \sigma_9\phi^2\phi_z^*\right] \\ & + c_1\left[\sigma_1\phi_{zzzz} + \sigma_2(\phi_z)^2\phi^* + \sigma_3|\phi_z|^2\phi + \sigma_4|\phi|^2\phi_{zz} + \sigma_5\phi^2\phi_{zz}^* + \sigma_6|\phi|^4\phi\right] = 0, \\ & i=\sqrt{-1}, \end{aligned} \tag{1}$$

In Eq. (1), variable $\phi(z,t)$ represents the complex wave profile, where $z$ corresponds to the spatial component along the length of the fiber and $t$ represents the temporal variable. Here, $a$ represents the CD, $b$ and $\sigma_6$ represent the SPM coefficients. Next, $\sigma_1$ and $\sigma_7$ are the coefficients of third–order dispersion (3OD) and fourth–order dispersion (4OD), respectively. Finally, the coefficients $\sigma_2$, $\sigma_3$, $\sigma_8$ and $\sigma_9$ imply the additional nonlinear effects, while the coefficients $\sigma_4$ and $\sigma_5$ give the nonlinear dispersive effects.

Eq. (1) represents the concatenation model derived from the combination of three extensively studied models in the field of fiber optics, namely the NLSE, LPD equation, and SSE. It is important to highlight that specific parameter values yield each individual model within the concatenation model. When $c_1 = c_2 = 0$, the NLSE is obtained. If only $c_1 = 0$, the SSE is recovered, and if only $c_2 = 0$, the LPD model is obtained. Hence, Eq. (1) represents the concatenated form that incorporates these three widely studied models from nonlinear fiber optics.

For magneto–optic waveguides, Eq. (1) is split into two separate components and the coupled model reads:

$$
\begin{aligned}
& iu_t + a_1 u_{zz} + \left(b_1 |u|^2 + c_1 |v|^2\right) u \\
& + c_{11} \left\{ d_1 u_{zzzz} + \left[ e_1 (u_z)^2 + f_1 (v_z)^2 \right] u^* + \left( g_1 |u_z|^2 + h_1 |v_z|^2 \right) u + \left[ k_1 |u|^2 + l_1 |v|^2 \right] u_{zz} \right. \\
& \left. + \left( m_1 u^2 + n_1 v^2 \right) u_{zz}^* + \left[ \alpha_1 |u|^4 + \beta_1 |v|^4 \right] u \right\} \\
& + i c_{12} \left[ \gamma_1 u_{zzz} + \left( \zeta_1 |u|^2 + \varsigma_1 |v|^2 \right) u_z + \left( \varepsilon_1 u^2 + s_1 v^2 \right) u_z^* \right] = \delta_1 v,
\end{aligned} \tag{2}
$$

and

$$
\begin{aligned}
& iv_t + a_2 v_{zz} + \left(b_2 |v|^2 + c_2 |u|^2\right) v \\
& + c_{21} \left\{ d_2 v_{zzzz} + \left[ e_2 (v_z)^2 + f_2 (u_z)^2 \right] v^* + \left( g_2 |v_z|^2 + h_2 |u_z|^2 \right) v + \left[ k_2 |v|^2 + l_2 |u|^2 \right] v_{zz} \right. \\
& \left. + \left( m_2 v^2 + n_2 u^2 \right) v_{zz}^* + \left[ \alpha_2 |v|^4 + \beta_2 |u|^4 \right] v \right\} \\
& + i c_{22} \left[ \gamma_2 v_{zzz} + \left( \zeta_2 |v|^2 + \varsigma_2 |u|^2 \right) v_z + \left( \varepsilon_2 v^2 + s_2 u^2 \right) v_z^* \right] = \delta_2 u.
\end{aligned} \tag{3}
$$

The constraints $a_j$, $b_j$, $c_j$, $c_{j1}$, $d_j$, $e_j$, $f_j$, $g_j$, $h_j$, $k_j$, $l_j$, $m_j$, $n_j$, $\alpha_j$, $\beta_j$, $c_{2j}$, $\gamma_j$, $\zeta_j$, $\varsigma_j$, $\varepsilon_j$, $s_j$ and $\delta_j$, $(j=1,2)$ are parameters, while, $u(z,t)$, $v(z,t)$ are the complex wave profiles. Then, the coefficients $a_j$, $\gamma_j$ and $d_j$ are CD, 3OD and 4OD terms along the two components, respectively. Next, $b_j$ and $\alpha_j$ are the SPM terms, while $c_j$ and $\beta_j$ are the cross–phase modulation effect. Also, the coefficients $e_j$, $f_j$, $g_j$, $h_j, \zeta_j, \varsigma_j$, $\varepsilon_j$ and $s_j$, give the nonlinear dispersive effects and the remaining coefficients give the effect of additional dispersions, while the coefficients $k_j$, $l_j$, $m_j$ and $n_j$ give the nonlinear dispersive effects. Finally, $\delta_j$, $(j=1,2)$ are the coefficients of magneto–optic effects.

The primary objective of this article is to utilize two methods in order to identify the dark, bright, and singular soliton solutions, for Eqs. (2) and (3). The structure of this article can be outlined as follows: Section–2 provides a preliminary analysis. Sections–3 and 4 present the solutions of Eqs. (2) and (3) utilizing the two methods stated above. Finally, Section–5, pens down a few conclusive words.

## 3. Preliminary analysis

In this section, we will suppose that Eqs. (2) and (3) possess the following solutions:

$$
\begin{aligned}
u(z,t) &= F_1(\zeta)\exp\left[iH(z,t)\right], \\
v(z,t) &= F_2(\zeta)\exp\left[iH(z,t)\right],
\end{aligned} \tag{4}
$$

and

$$
\begin{aligned}
\zeta &= z - ct, \\
H(z,t) &= -\kappa z + \Omega t + \varsigma_0.
\end{aligned} \tag{5}
$$

Assuming that $c, \kappa, \Omega$ and $\varsigma_0$ are all non-zero parameters, where $c$ represents the soliton's velocity, $\kappa$ denotes its wave number, $\Omega$ represents its frequency, and $\varsigma_0$ is the phase constant, we have real functions $F_1(\zeta), F_2(\zeta)$ and $H(z,t)$ that represent the amplitude and phase components of the soliton, respectively. Eqs. (2) and (3) may be changed to Eqs. (4) and (5) by isolating their real $\Re_j$ and imaginary $\Im_j$ portions. From this, we can conclude that:

$$\begin{aligned}
&\Re_1: c_{11}d_1F_1^{(4)} + \left(a_1 + 3c_{12}\gamma_1\kappa - 6c_{11}d_1\kappa^2\right)F_1'' + c_{11}\left[\left(k_1+m_1\right)F_1^2F_1'' + \left(l_1+n_1\right)F_2^2F_1''\right] \\
&+c_{11}\left[\left(e_1+g_1\right)F_1'^2F_1 + \left(f_1+h_1\right)F_2'^2F_1\right] - \left(\Omega + a_1\kappa^2 - c_{11}d_1\kappa^4 + c_{12}\gamma_1\kappa^3\right)F_1 - \delta_1F_2 \\
&+\left[b_1 + c_{11}\left(g_1 - k_1 - m_1 - e_1\right)\kappa^2 + c_{12}\left(\zeta_1 - \varepsilon_1\right)\kappa\right]F_1^3 \\
&+\left[c_1 - c_{11}\left(f_1 - h_1 + l_1 + n_1\right)\kappa^2 + c_{12}\left(\varsigma_1 - s_1\right)\kappa\right]F_1F_2^2 + \alpha_1c_{11}F_1^5 + c_{11}\beta_1F_1F_2^4 = 0,
\end{aligned} \tag{6}$$

$$\begin{aligned}
&\Re_2: c_{21}d_2F_2^{(4)} + \left(a_2 + 3c_{22}\gamma_2\kappa - 6c_{21}d_2\kappa^2\right)F_2'' \\
&+c_{21}\left[\left(k_2+m_2\right)F_2^2F_2'' + \left(l_2+n_2\right)F_1^2F_2''\right] \\
&+c_{21}\left[\left(e_2+g_2\right)F_2'^2F_2 + \left(f_2+h_2\right)F_1'^2F_2\right] \\
&-\left(\Omega + a_2\kappa^2 - c_{21}d_2\kappa^4 + c_{22}\gamma_2\kappa^3\right)F_2 - \delta_2F_1 \\
&+\left[b_2 + c_{21}\left(g_2 - k_2 - m_2 - e_2\right)\kappa^2 + c_{22}\left(\zeta_2 - \varepsilon_2\right)\kappa\right]F_2^3 \\
&+\left[c_2 - c_{21}\left(f_2 - h_2 + l_2 + n_2\right)\kappa^2 + c_{22}\left(\varsigma_2 - s_2\right)\kappa\right]F_2F_1^2 \\
&+\alpha_2c_{21}F_2^5 + c_{21}\beta_2F_2F_1^4 = 0,
\end{aligned} \tag{7}$$

and

$$\begin{aligned}
&\Im_1: \left(c_{12}\gamma_1 - 4c_{11}d_1\kappa\right)F_1''' + \left[2c_{11}\left(m_1 - e_1 - k_1\right)\kappa + c_{12}\left(\zeta_1 + \varepsilon_1\right)\right]F_1^2F_1' \\
&-2c_{11}f_1\kappa F_1F_2F_2' + \left[2c_{11}\left(n_1 - l_1\right)\kappa + c_{12}\left(s_1 + \varsigma_1\right)\right]F_1'F_2^2 \\
&-\left(c + 2a_1\kappa - 4c_{11}d_1\kappa^3 + 3c_{12}\gamma_1\kappa^2\right)F_1' = 0,
\end{aligned} \tag{8}$$

$$\begin{aligned}
&\Im_2: \left(c_{22}\gamma_2 - 4c_{21}d_2\kappa\right)F_2''' \\
&+\left[2c_{21}\left(m_2 - e_2 - k_2\right)\kappa + c_{22}\left(\zeta_2 + \varepsilon_2\right)\right]F_2^2F_2' - 2c_{21}f_2\kappa F_2F_1F_1' \\
&+\left[2c_{21}\left(n_2 - l_2\right)\kappa + c_{22}\left(s_2 + \varsigma_2\right)\right]F_2'F_1^2 \\
&-\left(c + 2a_2\kappa - 4c_{21}d_2\kappa^3 + 3c_{22}\gamma_2\kappa^2\right)F_2' = 0.
\end{aligned} \tag{9}$$

Set

$$F_2(\zeta) = AF_1(\zeta), \tag{10}$$

provided $A \neq 0, 1$. Now, Eqs. (6)–(9) become

$$\begin{aligned}
&\Re_1: c_{11}d_1F_1^{(4)} + \left(a_1 + 3c_{12}\gamma_1\kappa - 6c_{11}d_1\kappa^2\right)F_1'' \\
&+c_{11}\left[k_1 + m_1 + A^2\left(l_1 + n_1\right)\right]F_1^2F_1'' \\
&+c_{11}\left[e_1 + g_1 + A^2\left(f_1 + h_1\right)\right]F_1'^2F_1 \\
&-\left(\Omega + a_1\kappa^2 - c_{11}d_1\kappa^4 + c_{12}\gamma_1\kappa^3 + \delta_1A\right)F_1 \\
&+\left(b_1 + c_{11}\left[g_1 - k_1 - m_1 - e_1 - A^2\left(f_1 - h_1 + l_1 + n_1\right)\right]\kappa^2\right. \\
&\left.+c_{12}\left[\zeta_1 - \varepsilon_1 + A^2\left(\varsigma_1 - s_1\right)\right]\kappa + A^2c_1\right)F_1^3 \\
&+c_{11}\left(\alpha_1 + \beta_1A^4\right)F_1^5 = 0,
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\Re_2 : c_{21}d_2AF_1^{(4)} + A\left(a_2 + 3c_{22}\gamma_2\kappa - 6c_{21}d_2\kappa^2\right)F_1'' \\
&+Ac_{21}\left[A^2\left(k_2+m_2\right)+l_2+n_2\right]F_1^2F_1'' \\
&+Ac_{21}\left[A^2\left(e_2+g_2\right)+f_2+h_2\right]F_1'^2F_1 \\
&-\left[A\left(\Omega+a_2\kappa^2-c_{21}d_2\kappa^4+c_{22}\gamma_2\kappa^3\right)+\delta_2\right]F_1 \\
&+A\left(c_2+A^2b_2-c_{21}\left[f_2-h_2+l_2+n_2-A^2\left(g_2-k_2-m_2-e_2\right)\right]\kappa^2\right. \\
&\left.+c_{22}\left[\varsigma_2-s_2+A^2\left(\zeta_2-\varepsilon_2\right)\right]\kappa\right)F_1^3 \\
&+c_{21}A\left(\alpha_2A^4+\beta_2\right)F_1^5=0,
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
&\Im_1 : \left(c_{12}\gamma_1 - 4c_{11}d_1\kappa\right)F_1''' \\
&+\left(2c_{11}\left[m_1-e_1-k_1+A^2\left(n_1-l_1-f_1\right)\right]\kappa + c_{12}\left[\zeta_1+\varepsilon_1+A^2\left(s_1+\varsigma_1\right)\right]\right)F_1^2F_1' \\
&-\left(c+2a_1\kappa-4c_{11}d_1\kappa^3+3c_{12}\gamma_1\kappa^2\right)F_1'=0,
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\Im_2 : \left(c_{22}\gamma_2 - 4c_{21}d_2\kappa\right)F_1''' \\
&+\left(2c_{21}\left[n_2-l_2-f_2+A^2\left(m_2-e_2-k_2\right)\right]\kappa + c_{22}\left[s_2+\varsigma_2+A^2\left(\zeta_2+\varepsilon_2\right)\right]\right)F_1^2F_1' \\
&-\left(c+2a_2\kappa-4c_{21}d_2\kappa^3+3c_{22}\gamma_2\kappa^2\right)F_1'=0,
\end{aligned} \tag{14}$$

By equating the coefficients of the linearly independent functions in Eqs. (13,14) to zero, we obtain:

$$\kappa = \frac{c_{j2}\gamma_j}{4c_{j1}d_j}, \quad j=1,2, \tag{15}$$

$$c = 4c_{j1}d_j\kappa^3 - 2a_j\kappa - 3c_{j2}\gamma_j\kappa^2,\ j=1,2, \tag{16}$$

$$\begin{aligned}
&2c_{11}\left[m_1-e_1-k_1+A^2\left(n_1-l_1-f_1\right)\right]\kappa + c_{12}\left[\zeta_1+\varepsilon_1+A^2\left(s_1+\varsigma_1\right)\right]=0, \\
&2c_{21}\left[n_2-l_2-f_2+A^2\left(m_2-e_2-k_2\right)\right]\kappa + c_{22}\left[s_2+\varsigma_2+A^2\left(\zeta_2+\varepsilon_2\right)\right]=0.
\end{aligned} \tag{17}$$

Eqs. (11,12) exhibit identical forms, under the following constraint conditions:

$$\begin{aligned}
&c_{11}d_1 = c_{21}d_2A, \left(a_1+3c_{12}\gamma_1\kappa-6c_{11}d_1\kappa^2\right) = A\left(a_2+3c_{22}\gamma_2\kappa-6c_{21}d_2\kappa^2\right), \\
&c_{11}\left[k_1+m_1+A^2\left(l_1+n_1\right)\right] = Ac_{21}\left[A^2\left(k_2+m_2\right)+l_2+n_2\right], \\
&c_{11}\left(\alpha_1+\beta_1A^4\right) = c_{21}A\left(\alpha_2A^4+\beta_2\right), \\
&c_{11}\left[e_1+g_1+A^2\left(f_1+h_1\right)\right] = Ac_{21}\left[A^2\left(e_2+g_2\right)+f_2+h_2\right], \\
&\Omega+a_1\kappa^2-c_{11}d_1\kappa^4+c_{12}\gamma_1\kappa^3+\delta_1A = A\left(\Omega+a_2\kappa^2-c_{21}d_2\kappa^4+c_{22}\gamma_2\kappa^3\right)+\delta_2, \\
&A\left(c_2+A^2b_2-c_{21}\begin{bmatrix} f_2-h_2+l_2+n_2 \\ -A^2\left(g_2-k_2-m_2-e_2\right)\end{bmatrix}\kappa^2 + c_{22}\left[\varsigma_2-s_2+A^2\left(\zeta_2-\varepsilon_2\right)\right]\kappa\right) \\
&=\begin{pmatrix} b_1+c_{11}\left[g_1-k_1-m_1-e_1-A^2\left(f_1-h_1+l_1+n_1\right)\right]\kappa^2 \\ +c_{12}\left[\zeta_1-\varepsilon_1+A^2\left(\varsigma_1-s_1\right)\right]\kappa+A^2c_1\end{pmatrix}.
\end{aligned} \tag{18}$$

From Eqs. (15–18), one derives the following:

$$\gamma_1=\frac{c_{22}\gamma_2 A}{c_{12}},\ d_2=\frac{2a_1+3c_{22}\gamma_2 A\kappa}{4Ac_{21}\kappa^2},\ a_2=\frac{a_1}{A},\ d_1=\frac{2a_1+3c_{22}\gamma_2 A\kappa}{4c_{11}\kappa^2},$$

$$m_1=\frac{2c_{11}\left[e_1+k_1-A^2\left(n_1-l_1-f_1\right)\right]\kappa-c_{12}\left[\zeta_1+\varepsilon_1+A^2\left(s_1+\varsigma_1\right)\right]}{2c_{11}},$$

$$n_2=\frac{2c_{21}\left[l_2+f_2-A^2\left(m_2-e_2-k_2\right)\right]\kappa-c_{22}\left[s_2+\varsigma_2+A^2\left(\zeta_2+\varepsilon_2\right)\right]}{2c_{21}},$$

$$\zeta_1=\frac{\begin{pmatrix}2c_{11}\left[\left(k_1+A^2l_1\right)(1+\kappa)+\left(e_1+A^2f_1\right)\kappa+A^2n_1(1-\kappa)\right]\\ -2Ac_{21}\left[\left(l_2+k_2A^2\right)(1+\kappa)+\kappa\left(f_2+e_2A^2\right)+m_2(1-\kappa)A^2\right]\end{pmatrix}}{c_{12}} \tag{19}$$

$$+\frac{Ac_{22}\left[s_2+\varsigma_2+A^2\left(\zeta_2+\varepsilon_2\right)\right]-c_{12}\left[\varepsilon_1+A^2\left(s_1+\varsigma_1\right)\right]}{c_{12}},\ \beta_1=\frac{c_{21}A\left(\alpha_2A^4+\beta_2\right)-c_{11}\alpha_1}{c_{11}A^4},$$

$$e_1=\frac{Ac_{21}\left[A^2\left(e_2+g_2\right)+f_2+h_2\right]-c_{11}\left[g_1+A^2\left(f_1+h_1\right)\right]}{c_{11}},\ \Omega=\frac{\delta_2-\delta_1 A}{1-A}.$$

We can express Eq. (11) in an alternative form as follows:

$$F_1^{(4)}+\lambda_1F_1''+\lambda_2F_1'^2F_1+\lambda_3F_1^2F_1''+\lambda_4F_1+\lambda_5F_1^3+\lambda_6F_1^5=0, \tag{20}$$

where

$$\lambda_1=\frac{\left(a_1+3c_{12}\gamma_1\kappa-6c_{11}d_1\kappa^2\right)}{c_{11}d_1},\ \lambda_2=\frac{c_{11}\left[e_1+g_1+A^2\left(f_1+h_1\right)\right]}{c_{11}d_1},$$

$$\lambda_3=\frac{c_{11}\left[k_1+m_1+A^2\left(l_1+n_1\right)\right]}{c_{11}d_1},\ \lambda_4\frac{-\Omega-a_1\kappa^2+c_{11}d_1\kappa^4-c_{12}\gamma_1\kappa^3-\delta_1A}{c_{11}d_1},$$

$$\lambda_5=\frac{b_1+c_{11}\left[g_1-k_1-m_1-e_1-A^2\left(f_1-h_1+l_1+n_1\right)\right]\kappa^2}{c_{11}d_1} \tag{21}$$

$$+\frac{c_{12}\left[\zeta_1-\varepsilon_1+A^2\left(\varsigma_1-s_1\right)\right]\kappa+A^2c_1}{c_{11}d_1},$$

$$\lambda_6=\frac{c_{11}\left(\alpha_1+\beta_1A^4\right)}{c_{11}d_1}.$$

In the upcoming sections, we will employ the following approaches to solve Eq. (20).

## 4. Simplest equation method

Eq. (20) permits the exact solution:

$$F_1(\zeta)=A_0+A_1Z(\zeta),\qquad A_1\neq 0, \tag{22}$$

where $A_0$ and $A_1$ are constants to be detected, while the function $Z(\zeta)$ satisfies Bernoulli's equation

$$Z'(\zeta)=\tau_1Z(\zeta)+\tau_2Z^2(\zeta), \tag{23}$$

or the Riccati equation

$$Z'(\zeta)=\sigma+Z^2(\zeta), \tag{24}$$

where $\tau_1$, $\tau_2$ and $\sigma$ are constants to be determined later. Eq. (23) has the following solutions:

$$Z(\zeta)=\frac{\tau_1\exp\left[\tau_1(\zeta+\zeta_0)\right]}{1-\tau_2\exp\left[\tau_1(\zeta+\zeta_0)\right]},\quad \text{if}\ \ \tau_1>0,\ \ \tau_2<0, \tag{25}$$

and

$$Z(\zeta)=-\frac{\tau_1\exp\left[\tau_1(\zeta+\zeta_0)\right]}{1+\tau_2\exp\left[\tau_1(\zeta+\zeta_0)\right]},\quad \text{if}\ \ \tau_1<0,\ \ \tau_2>0, \tag{26}$$

where $\zeta_0$ is a constant of integration, while Eq. (24) has the following three types of solutions:

**<u>Type – 1</u>**: $\sigma<0$,

$$Z(\zeta)=-\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}\zeta\right), \tag{27}$$

or

$$Z(\zeta)=-\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\zeta\right). \tag{28}$$

**<u>Type – 2</u>**: $\sigma>0$,

$$Z(\zeta)=\sqrt{\sigma}\tan\left(\sqrt{\sigma}\zeta\right), \tag{29}$$

or

$$Z(\zeta)=-\sqrt{\sigma}\cot\left(\sqrt{\sigma}\zeta\right). \tag{30}$$

**<u>Type – 3</u>**: $\sigma=0$,

$$Z(\zeta)=-\frac{1}{\zeta+K}, \tag{31}$$

where $K$ is a constant of integration.

### *4.1. Bernoulli's equation*

Inserting (22) along with the Bernoulli Eq. (23) into Eq. (20), gathering all the coefficients of each power $Z^s(\zeta),(s=0,1,..,5)$, and setting these coefficients to zero, one procures the results as

$$A_0=\frac{30\tau_1}{\sqrt{-15(2\lambda_2+3\lambda_3)}},\quad A_1=\frac{30\tau_2}{\sqrt{-15(2\lambda_2+3\lambda_3)}}, \tag{32}$$

and

$$\lambda_1=-5\tau_1^2,\ \ \lambda_4=4\tau_1^4,\ \ \lambda_5=\frac{1}{3}\tau_1^2(\lambda_2+3\lambda_3),\ \ \lambda_6=\frac{1}{300}(2\lambda_2+3\lambda_3)(4\lambda_3+\lambda_2), \tag{33}$$

provided

$$(2\lambda_2+3\lambda_3)<0.$$

(I) If $\tau_1>0$, $\tau_2<0$, we have

$$u(z,t)=\frac{30\tau_1}{\sqrt{-15(2\lambda_2+3\lambda_3)}}\left[1+\frac{\tau_2\exp\left[\tau_1(z-ct+\zeta_0)\right]}{1-\tau_2\exp\left[\tau_1(z-ct+\zeta_0)\right]}\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}, \tag{34}$$

and

$$v(z,t)=\frac{30A\tau_1}{\sqrt{-15(2\lambda_2+3\lambda_3)}}\left[1+\frac{\tau_2\exp\left[\tau_1(z-ct+\zeta_0)\right]}{1-\tau_2\exp\left[\tau_1(z-ct+\zeta_0)\right]}\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}, \tag{35}$$

(II) If $\tau_1 < 0,\ \tau_2 > 0,$ we have

$$u(z,t)=\frac{30\tau_1}{\sqrt{-15(2\lambda_2+3\lambda_3)}}\left[1-\frac{\tau_2\exp[\tau_1(z-ct+\zeta_0)]}{1+\tau_2\exp[\tau_1(z-ct+\zeta_0)]}\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}, \tag{36}$$

and

$$v(z,t)=\frac{30A\tau_1}{\sqrt{-15(2\lambda_2+3\lambda_3)}}\left[1-\frac{\tau_2\exp[\tau_1(z-ct+\zeta_0)]}{1+\tau_2\exp[\tau_1(z-ct+\zeta_0)]}\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}. \tag{37}$$

In particular if $\tau_1 = 1,\ \tau_2 = -1$ or $\tau_1 = -1,\ \tau_2 = 1,$ we have the dark soliton solutions

$$u(z,t)=\frac{15}{\sqrt{-15(2\lambda_2+3\lambda_3)}}\left[1-\tanh\left(\frac{z-ct+\zeta_0}{2}\right)\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}, \tag{38}$$

and

$$v(z,t)=\frac{15A}{\sqrt{-15(2\lambda_2+3\lambda_3)}}\left[1-\tanh\left(\frac{z-ct+\zeta_0}{2}\right)\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}, \tag{39}$$

also

$$u(z,t)=-\frac{15}{\sqrt{-15(2\lambda_2+3\lambda_3)}}\left[1+\tanh\left(\frac{z-ct+\zeta_0}{2}\right)\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}, \tag{40}$$

and

$$v(z,t)=-\frac{15A}{\sqrt{-15(2\lambda_2+3\lambda_3)}}\left[1+\tanh\left(\frac{z-ct+\zeta_0}{2}\right)\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}. \tag{41}$$

### *4.2. Riccati's equation*

Plugging Eq. (22) along with the Riccati Eq. (24) into Eq. (20), gathering the coefficients of each power $Z^s(\zeta)$, $(s = 0,1,2,..,5)$, and setting each of these coefficients to zero, one gets the following results:

$$A_0=\sqrt{-\frac{\lambda_1\left(\lambda_2+\sqrt{\lambda_2^2+576\lambda_6}\right)}{120\lambda_6}},\quad A_1=\sqrt{\frac{\lambda_2+\sqrt{\lambda_2^2+576\lambda_6}}{6\lambda_6}},\quad \sigma=\frac{1}{20}\lambda_1, \tag{42}$$

and

$$\begin{aligned}\lambda_3&=-\frac{2\left(\lambda_2^2+\lambda_2\sqrt{\lambda_2^2+576\lambda_6}+180\lambda_6\right)}{3\left(\lambda_2+\sqrt{\lambda_2^2+576\lambda_6}\right)},\\ \lambda_4&=\frac{4}{25}\lambda_1^2,\\ \lambda_5&=\frac{\lambda_1\left(\lambda_2^2+\lambda_2\sqrt{\lambda_2^2+576\lambda_6}+360\lambda_6\right)}{15\left(\lambda_2+\sqrt{\lambda_2^2+576\lambda_6}\right)},\end{aligned} \tag{43}$$

provided

$$\left(\lambda_2^2+576\lambda_6\right)>0,\ \ \lambda_6\left(\lambda_2+\sqrt{\lambda_2^2+576\lambda_6}\right)>0 \text{ and } \lambda_1<0.$$

Now, one derives the following exact solutions to Eqs. (2) and (3):

(i) If $\lambda_1 < 0$ , we have the singular soliton solutions:

$$u(z,t) = \sqrt{-\frac{\lambda_1\left(\lambda_2 + \sqrt{\lambda_2^2 + 576\lambda_6}\right)}{120\lambda_6}}\left[1 - \coth\left(\sqrt{-\frac{\lambda_1}{20}}(z - ct)\right)\right]e^{i(-\kappa z + \Omega t + \varsigma_0)}, \tag{44}$$

and

$$v(z,t) = A\sqrt{-\frac{\lambda_1\left(\lambda_2 + \sqrt{\lambda_2^2 + 576\lambda_6}\right)}{120\lambda_6}}\left[1 - \coth\left(\sqrt{-\frac{\lambda_1}{20}}(z - ct)\right)\right]e^{i(-\kappa z + \Omega t + \varsigma_0)}, \tag{45}$$

and the dark soliton solutions:

$$u(z,t) = \sqrt{-\frac{\lambda_1\left(\lambda_2 + \sqrt{\lambda_2^2 + 576\lambda_6}\right)}{120\lambda_6}}\left[1 - \tanh\left(\sqrt{-\frac{\lambda_1}{20}}(z - ct)\right)\right]e^{i(-\kappa z + \Omega t + \varsigma_0)}, \tag{46}$$

and

$$v(z,t) = A\sqrt{-\frac{\lambda_1\left(\lambda_2 + \sqrt{\lambda_2^2 + 576\lambda_6}\right)}{120\lambda_6}}\left[1 - \tanh\left(\sqrt{-\frac{\lambda_1}{20}}(z - ct)\right)\right]e^{i(-\kappa z + \Omega t + \varsigma_0)}. \tag{47}$$

Finally, there are several other Type–2 and Type–3 solutions to Eqs. (2) and (3) that are excluded here for convenience.

## 5. Extended simples equation

Eq. (20) assumes the explicit solution:

$$F_1(\zeta) = \chi_0 + \chi_1\left[\frac{\Theta'(\zeta)}{\Theta(\zeta)}\right] + B_0\left[\frac{1}{\Theta(\zeta)}\right], \tag{48}$$

where $\chi_0, \chi_1$ and $B_0$ are constants, $\chi_1^2 + B_0^2 \neq 0$ and the function $\Theta(\zeta)$ presumes the auxiliary equation

$$\Theta''(\zeta) + \delta\Theta(\zeta) = \upsilon_0, \tag{49}$$

where $\delta$ and $\upsilon_0$ are constants and $\Theta'(\zeta)$, $\Theta''(\zeta)$ are the first and second order derivatives respectively with respect to their corresponding independent variables. Now, we have the following three sorts:

<u>**Type – I**</u>: $\delta < 0$. In this case, we substitute Eq. (48) into Eq. (20) and use Eq. (49) together with the relation

$$\left(\frac{\Theta'(\zeta)}{\Theta(\zeta)}\right)^2 = L_1\left(\frac{1}{\Theta(\zeta)}\right)^2 - \delta + \frac{2\upsilon_0}{\Theta(\zeta)}, \tag{50}$$

where $L_1 = \delta\left(\rho_1^2 - \rho_2^2\right) - \frac{\upsilon_0^2}{\delta}$ , while $\rho_1$ and $\rho_2$ are constants, gives way to the results:

<u>Result – 1</u>:

$$\begin{gathered} \chi_0 = 0,\ \chi_1 = 0,\ B_0 = \sqrt{-\frac{60L_1}{2\lambda_2 + 3\lambda_3}},\ \lambda_1 = 5\delta,\ \lambda_4 = 4\delta^2, \\ \lambda_5 = \frac{\delta(4\lambda_2 + 3\lambda_3)L_1 + 3\upsilon_0^2(2\lambda_2 + 3\lambda_3)}{6L_1},\ \lambda_6 = \frac{1}{300}(2\lambda_2 + 3\lambda_3)(\lambda_2 + 4\lambda_3), \end{gathered} \tag{51}$$

provided $\left(2\lambda_2+3\lambda_3\right)L_1<0.$

Consequently, we obtain the following straddled bright–singular soliton solutions of Eqs. (2) and (3) as follows:

$$u(z,t)=\sqrt{-\frac{60L_1}{2\lambda_2+3\lambda_3}}\times\left[\frac{1}{\rho_1\cosh\left[\sqrt{-\delta}\left(z-ct\right)\right]+\rho_2\sinh\left[\sqrt{-\delta}\left(z-ct\right)\right]+\frac{\upsilon_0}{\delta}}\right]e^{i\left(-\kappa z+\Omega t+\varsigma_0\right)}, \tag{52}$$

and

$$v(z,t)=A\sqrt{-\frac{60L_1}{2\lambda_2+3\lambda_3}}\times\left[\frac{1}{\rho_1\cosh\left[\sqrt{-\delta}\left(z-ct\right)\right]+\rho_2\sinh\left[\sqrt{-\delta}\left(z-ct\right)\right]+\frac{\upsilon_0}{\delta}}\right]e^{i\left(-\kappa z+\Omega t+\varsigma_0\right)}. \tag{53}$$

In particular, if we set $\rho_1=0,\ \rho_2\neq 0$ and $\upsilon_0=0$ in Eq. (52) and (53), the singular solitons are as follows:

$$u(z,t)=\sqrt{\frac{60\delta}{2\lambda_2+3\lambda_3}}\operatorname{csch}\left[\sqrt{-\delta}\left(z-ct\right)\right]e^{i\left(-\kappa z+\Omega t+\varsigma_0\right)}, \tag{54}$$

and

$$v(z,t)=A\sqrt{\frac{60\delta}{2\lambda_2+3\lambda_3}}\operatorname{csch}\left[\sqrt{-\delta}\left(z-ct\right)\right]e^{i\left(-\kappa z+\Omega t+\varsigma_0\right)}, \tag{55}$$

provided $\left(2\lambda_2+3\lambda_3\right)<0,$ while if we set $\rho_1\neq 0, \rho_2=0$ and $\upsilon_0=0$ in Eq. (52) and (53), the bright solitons are as follows:

$$u(z,t)=\sqrt{-\frac{60\delta}{2\lambda_2+3\lambda_3}}\operatorname{sech}\left[\sqrt{-\delta}\left(z-ct\right)\right]e^{i\left(-\kappa z+\Omega t+\varsigma_0\right)}, \tag{56}$$

and

$$v(z,t)=A\sqrt{-\frac{60\delta}{2\lambda_2+3\lambda_3}}\operatorname{sech}\left[\sqrt{-\delta}\left(z-ct\right)\right]e^{i\left(-\kappa z+\Omega t+\varsigma_0\right)}, \tag{57}$$

provided $\left(2\lambda_2+3\lambda_3\right)>0.$

<u>Result – 2</u>:

$$\lambda_1=\frac{2\delta L_1+3\upsilon_0^2}{2L_1},\quad \lambda_4=-\frac{3\upsilon_0^2\left(\lambda_2-2\lambda_3\right)\left(4L_1\delta+3\upsilon_0^2\right)}{8L_1^2\left(\lambda_3+2\lambda_2\right)},$$
$$\lambda_5=-\frac{\left(\lambda_2-2\lambda_3\right)\left(2L_1\delta+3\upsilon_0^2\right)}{10L_1},\lambda_6=-\frac{1}{50}\left(\lambda_3+2\lambda_2\right)\left(\lambda_2-2\lambda_3\right), \tag{58}$$
$$\chi_0=\frac{\upsilon_0}{2L_1}\sqrt{-\frac{30L_1}{\left(\lambda_3+2\lambda_2\right)}},\ \chi_1=0, B_0=\sqrt{-\frac{30L_1}{\lambda_3+2\lambda_2}},$$

provided $L_1\left(\lambda_3+2\lambda_2\right)<0.$

Consequently, we obtain the following straddled bright–singular soliton solutions of Eqs. (2) and (3) as follows:

$$u(z,t)=\sqrt{-\frac{30L_1}{(\lambda_3+2\lambda_2)}}\times\left[\frac{\upsilon_0}{2L_1}+\frac{1}{\rho_1\cosh\left[\sqrt{-\delta}(z-ct)\right]+\rho_2\sinh\left[\sqrt{-\delta}(z-ct)\right]+\frac{\upsilon_0}{\delta}}\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}, \tag{59}$$

and

$$v(z,t)=A\sqrt{-\frac{30L_1}{(\lambda_3+2\lambda_2)}}\times\left[\frac{\upsilon_0}{2L_1}+\frac{1}{\rho_1\cosh\left[\sqrt{-\delta}(z-ct)\right]+\rho_2\sinh\left[\sqrt{-\delta}(z-ct)\right]+\frac{\upsilon_0}{\delta}}\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}. \tag{60}$$

In particular, if we set $\rho_1=0, \rho_2\neq 0$ and $\upsilon_0=0$ in (59) and (60), we see the singular soliton solutions:

$$u(z,t)=\sqrt{\frac{30\delta}{(\lambda_3+2\lambda_2)}}\operatorname{csch}\left[\sqrt{-\delta}(z-ct)\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}, \tag{61}$$

and

$$v(z,t)=A\sqrt{\frac{30\delta}{(\lambda_3+2\lambda_2)}}\operatorname{csch}\left[\sqrt{-\delta}(z-ct)\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}, \tag{62}$$

provided $(\lambda_3+2\lambda_2)<0,$ while if we set . $\rho_1\neq 0, \rho_2=0$ . and $\upsilon_0=0$ in Eq. (59) and (60), we recover bright soliton solutions:

$$u(z,t)=\sqrt{-\frac{30\delta}{(\lambda_3+2\lambda_2)}}\operatorname{sech}\left[\sqrt{-\delta}(z-ct)\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}, \tag{63}$$

and

$$v(z,t)=A\sqrt{-\frac{30\delta}{(\lambda_3+2\lambda_2)}}\operatorname{sech}\left[\sqrt{-\delta}(z-ct)\right]e^{i(-\kappa z+\Omega t+\varsigma_0)}, \tag{64}$$

provided $(\lambda_3+2\lambda_2)>0$.

<u>Result – 3</u>:

$$\chi_0=0,\ \chi_1=0,\ B_0=\sqrt{-\frac{2\delta\left(\rho_1^2-\rho_2^2\right)(10\delta-\lambda_1)}{\delta(\lambda_2+\lambda_3)-\lambda_5}},\ \upsilon_0=0,\ \lambda_4=-\delta(\delta-\lambda_1),$$
$$\lambda_6=-\frac{\left[\delta(\lambda_2+\lambda_3)-\lambda_5\right]\left[2\delta(\lambda_2-4\lambda_3)+\lambda_1(\lambda_2+2\lambda_3)-12\lambda_5\right]}{2(10\delta-\lambda_1)^2}, \tag{65}$$

provided $\left(\rho_1^2-\rho_2^2\right)(10\delta-\lambda_1)\left[\delta(\lambda_2+\lambda_3)-\lambda_5\right]>0.$

Consequently, we obtain the following straddled bright-singular soliton solutions of Eqs. (2) and (3) as follows:

$$u(z,t)=\sqrt{-\frac{2\delta\left(\rho_1^2-\rho_2^2\right)\left(10\delta-\lambda_1\right)}{\delta\left(\lambda_2+\lambda_3\right)-\lambda_5}}$$
$$\times\left[\frac{1}{\rho_1\cosh\left[\sqrt{-\delta}\left(z-ct\right)\right]+\rho_2\sinh\left[\sqrt{-\delta}\left(z-ct\right)\right]}\right]e^{i\left(-\kappa z+\Omega t+\varsigma_0\right)}, \tag{66}$$

and

$$v(z,t)=A\sqrt{-\frac{2\delta\left(\rho_1^2-\rho_2^2\right)\left(10\delta-\lambda_1\right)}{\delta\left(\lambda_2+\lambda_3\right)-\lambda_5}}$$
$$\times\left[\frac{1}{\rho_1\cosh\left[\sqrt{-\delta}\left(z-ct\right)\right]+\rho_2\sinh\left[\sqrt{-\delta}\left(z-ct\right)\right]}\right]e^{i\left(-\kappa z+\Omega t+\varsigma_0\right)}. \tag{67}$$

In particular, if we set $\rho_1=0,\ \rho_2\neq 0$ in (66) and (67), we arrive at the singular soliton solutions:

$$u(z,t)=\sqrt{\frac{2\delta\left(10\delta-\lambda_1\right)}{\delta\left(\lambda_2+\lambda_3\right)-\lambda_5}}\operatorname{csch}\left[\sqrt{-\delta}\left(z-ct\right)\right]e^{i\left(-\kappa z+\Omega t+\varsigma_0\right)}, \tag{68}$$

and

$$v(z,t)=A\sqrt{\frac{2\delta\left(10\delta-\lambda_1\right)}{\delta\left(\lambda_2+\lambda_3\right)-\lambda_5}}\operatorname{csch}\left[\sqrt{-\delta}\left(z-ct\right)\right]e^{i\left(-\kappa z+\Omega t+\varsigma_0\right)}, \tag{69}$$

provided $\left(10\delta-\lambda_1\right)\left[\delta\left(\lambda_2+\lambda_3\right)-\lambda_5\right]<0,$

while if we set $\rho_1\neq 0,\ \rho_2=0$ in Eq. (66) and (67), we have the bright soliton solutions:

$$u(z,t)=\sqrt{-\frac{2\delta\left(10\delta-\lambda_1\right)}{\delta\left(\lambda_2+\lambda_3\right)-\lambda_5}}\operatorname{sech}\left[\sqrt{-\delta}\left(z-ct\right)\right]e^{i\left(-\kappa z+\Omega t+\varsigma_0\right)}, \tag{70}$$

and

$$v(z,t)=A\sqrt{-\frac{2\delta\left(10\delta-\lambda_1\right)}{\delta\left(\lambda_2+\lambda_3\right)-\lambda_5}}\operatorname{sech}\left[\sqrt{-\delta}\left(z-ct\right)\right]e^{i\left(-\kappa z+\Omega t+\varsigma_0\right)}, \tag{71}$$

provided $\left(10\delta-\lambda_1\right)\left[\delta\left(\lambda_2+\lambda_3\right)-\lambda_5\right]>0.$

Finally, there are several other Type – II and Type – III solutions to Eqs. (2) and (3) that are excluded here for convenience.

## 6. Conclusions

The paper recovered optical soliton solutions for the concatenation model with Kerr's law of nonlinear refractive index. Two integration schemes, namely the simplest equation method and the extended version of the simplest equation method, gave way to the soliton solutions. A full spectrum of solitons, including the bright–singular straddled solitons are recovered. The results are thus interesting and complete the chapter on solitons retrieval for magneto–optic waveguides. The parameter restrictions, that are also known as constraint conditions, are also enumerated in the paper and these conditions guarantee the existence of such enlisted solitons.

The results pave the way for a number of avenues to venture in the future. One immediate thought would be to recover the conservation laws for the model in magneto–optic waveguides. The usage of the multiplier approach would lead to success since the vector-coupled equations are truly non–trivial. Later, the model would be addressed with different forms of SPM which would give an extended or perhaps a generalized perspective of the current results in this paper. Those upcoming results would be then aligned would the pre–existing results [12–15].

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***Анотація.*** *У цій статті приділяється увага відновленню солітонів у магніто-оптичних хвилеводах для моделі конкатенації з нелінійним показником заломлення за законом Керра. Підхід з найпростішим рівнянням, а також розширений метод найпростіших рівнянь спільно розкривають повний спектр розв'язків солітонів для цієї моделі. Обмеження параметрів гарантують існування таких солітонів.*

***Ключові слова:*** *солітони, конкатенаційна модель, магнітооптичний хвилевід, метод найпростіших рівнянь*