Optical solitons for the concatenation model with Kudryashov's approaches

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Abstract. This paper implements two of Kudryashov's approaches to extract optical soliton solutions to the concatenation model that is a conjunction of the nonlinear Schrödinger's equation, Lakshmanan–Porsezian–Daniel model, and the Sasa–Satsuma equation. A full spectrum of soliton solutions emerged along with the parameter constraints that are all comprehensively presented.

Key words: solitons, concatenation model, Kudryashov approach

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1. Introduction

The concatenation model is one of the models that emerged about a decade ago, in 2014, to address soliton communications through optical fibers across transcontinental distances [1, 2]. This is a conjunction of three well-known equations studied in nonlinear fiber optics. They are the nonlinear Schrödinger's equation (NLSE), Lakshmanan–Porsezian–Daniel (LPD) model, and the Sasa–Satsuma equation (SSE). Subsequently, the model gained popularity, and subsequently, an explosion of a variety of results started emerging [3–8]. These range of results are from the Painleve analysis of the model, quiescent solitons for nonlinear chromatic dispersion (CD), conservation laws, rogue waves, application of the method of undetermined coefficients, and many other features [3–8]. For the first time, the current work will implement Kudryashov's approaches to recover soliton solutions to the concatenation model having the Kerr form of self–phase modulation (SPM). A full spectrum of soliton solutions, including the straddled solitons, would be revealed using Kudryashov's approaches. The corresponding parameter constraints would also be enumerated for the existence of such solitons. The derivation details are exhibited in the rest of the paper after a quick intro to the model.

2. Mathematical analysis

The concatenation model [1–3] is given by

$$\begin{aligned} & iq_t + aq_{xx} + b|q|^2 q \\ & +c_1\left[\delta_1 q_{xxxx} + \delta_2 (q_x)^2 q^* + \delta_3 |q_x|^2 q + \delta_4 |q|^2 q_{xx} + \delta_5 q^2 q_{xx}^* + \delta_6 |q|^4 q\right] \\ & +ic_2\left[\delta_7 q_{xxx} + \delta_8 |q|^2 q_x + \delta_9 q^2 q_x^*\right] = 0. \end{aligned} \tag{1}$$

Eq. (1) is the concatenation model that is going to be the focus of study in this work. Here $q = q(x,t)$ is the wave profile and is a complex–valued function with the independent variables being x and t them being the spatial and temporal coordinates, respectively. The first term is the linear temporal evolution, while the coefficients of a and b are CD and SPM, respectively. The first three terms together represents the NLSE component. The coefficients of c_1 and c_2 are the portions from LPD and SSE, respectively, δ_i – represent the coefficients of nonlinearity, q_t gives the temporal dispersion, q_x is the spatial dispersion, q_{xx}, q_{xxx} and q_{xxxx} correspond to the higher-order dispersions. Thus, the conjunction model is truly a concatenation of three familiar models that are visible in optics and optoelectronics. This model will be studied using the two forms of Kudryashov's integration structure. The details are exhibited in the subsequent sections.

The traveling wave hypotheses can be applied as:

$$q(x,t) = Q(\varepsilon)e^{i\phi(x,t)}, \quad \varepsilon = h(x - vt), \quad \phi(x,t) = -kx + \omega t + \theta, \tag{2}$$

where $v, k, \omega, \theta, \phi(x,t)$, $Q(\varepsilon)$, and h represent the soliton's velocity, wave number, frequency, phase constant, phase component, amplitude component, and inverse width, respectively. Using Eq. (2) in Eq. (1), we obtain the real part

$$\begin{aligned} & c_1\delta_1 h^4 Q'''' + \left(c_1\delta_4 h^2 + c_1\delta_5 h^2\right) Q^2 Q'' + \left(c_1\delta_2 h^2 + c_1\delta_3 h^2\right) Q (Q')^2 \\ & + \left(ah^2 - 6c_1\delta_1 h^2 k^2 + 3c_2\delta_7 kh^2\right) Q'' + c_1\delta_6 Q^5 \\ & + \left(b - c_1\delta_2 k^2 + c_1\delta_3 k^2 - c_1\delta_4 k^2 - c_1\delta_5 k^2 + c_2\delta_8 k - c_2\delta_9 k\right) Q^3 \\ & + \left(-\omega - ak^2 + c_1\delta_1 k^4 - c_2\delta_7 k^3\right) Q = 0, \end{aligned} \tag{3}$$

and the imaginary part

$$\begin{aligned} & \left(-4c_1\delta_1 kh^3 + c_2\delta_7 h^3\right) Q''' + \left(-hv - 2ahk + 4c_1\delta_1 k^3 h - 3c_2\delta_7 hk^2\right) Q' \\ & + \left(-2c_1\delta_2 hk - 2c_1\delta_4 hk + 2c_1\delta_5 kh + c_2\delta_8 h + c_2\delta_9 h\right) Q^2 Q' = 0, \end{aligned} \tag{4}$$

where $Q' = \dfrac{dQ(\varepsilon)}{d\varepsilon}$, $Q'' = \dfrac{d^2Q(\varepsilon)}{d\varepsilon^2}$, $Q''' = \dfrac{d^3Q(\varepsilon)}{d\varepsilon^3}$, and $Q'''' = \dfrac{d^4Q(\varepsilon)}{d\varepsilon^4}$. Employing the following constraints in Eq. (4)

$$\begin{aligned} & -4c_1\delta_1 kh^3 + c_2\delta_7 h^3 = 0, \\ & -2c_1\delta_2 hk - 2c_1\delta_4 hk + 2c_1\delta_5 kh + c_2\delta_8 h + c_2\delta_9 h = 0. \end{aligned} \tag{5}$$

Thus, one can obtain the velocity of the soliton as

$$v = -2ak + 4c_1\delta_1 k^3 - 3c_2\delta_7 k^2. \tag{6}$$

Now, Eq. (3) can be rewritten as

$$A_1Q''''+A_2Q^2Q''+A_3Q\left(Q'\right)^2+A_4Q''+A_5Q^5+A_6Q^3+A_7Q=0, \tag{7}$$

where

$$\begin{aligned}
&A_1=c_1\delta_1h^4,\\
&A_2=c_1\delta_4h^2+c_1\delta_5h^2,\\
&A_3=c_1\delta_2h^2+c_1\delta_3h^2,\\
&A_4=ah^2-6c_1\delta_1h^2k^2+3c_2\delta_7kh^2,\\
&A_5=c_1\delta_6,\\
&A_6=b-c_1\delta_2k^2+c_1\delta_3k^2-c_1\delta_4k^2-c_1\delta_5k^2+c_2\delta_8k-c_2\delta_9k,\\
&A_7=-\omega-ak^2+c_1\delta_1k^4-c_2\delta_7k^3.
\end{aligned} \tag{8}$$

3. Kudryashov's method

In this section, the Kudryashov method [10] is applied for obtaining the soliton solutions of Eq. (7). Let us assume that Eq. (7) has the solution of the form

$$Q(\varepsilon)=B_0+B_1R(\varepsilon), \tag{9}$$

where B_0 and B_1 are arbitrary constants with $B_1\neq 0$, and $R(\varepsilon)$ satisfies the ordinary differential equation (ODE)

$$R'(\varepsilon)=R^2(\varepsilon)-R(\varepsilon), \tag{10}$$

and has of the form

$$R(\varepsilon)=\frac{1}{1+\eta e^{\varepsilon}}, \tag{11}$$

where η is a constant. Substituting the values from Eq. (9) and Eq. (10) into Eq. (7) and collecting the coefficients of different powers of $R(\varepsilon)$ equal to zero, we have

$$\begin{aligned}
&0=A_5B_1^5+2A_2B_1^3+A_3B_1^3+24A_1B_1,\\
&0=5A_5B_0B_1^4+4A_2B_0B_1^2-3A_2B_1^3+A_3B_0B_1^2-2A_3B_1^3-60A_1B_1,\\
&0=10A_5B_0^2B_1^3+2A_2B_0^2B_1-6A_2B_0B_1^2+A_2B_1^3-2A_3B_0B_1^2\\
&\quad+A_3B_1^3+A_6B_1^3+50A_1B_1+2A_4B_1,\\
&0=10A_5B_0^3B_1^2-3A_2B_0^2B_1+2A_2B_0B_1^2+A_3B_0B_1^2+3A_6B_0B_1^2-15A_1B_1-3A_4B_1,\\
&0=5A_5B_0^4B_1+A_2B_0^2B_1+3A_6B_0^2B_1+A_1B_1+A_4B_1+A_7B_1,\\
&0=A_5B_0^5+A_6B_0^3+A_7B_0.
\end{aligned} \tag{12}$$

After solving the above system of algebraic equations, we have the following results

$$\begin{aligned}
&B_0=\sqrt{-\frac{6A_4-20A_7}{4A_2-A_3-8A_6}},\quad B_1=\pm2\sqrt{-\frac{6A_4-20A_7}{4A_2-A_3-8A_6}},\\
&A_1=\frac{2A_2A_4+A_3A_4-4A_4A_6-4A_2A_7-4A_3A_7+8A_6A_7}{4A_2-A_3-8A_6},\\
&A_5=\frac{\left(4A_2-A_3-8A_6\right)\left(-6A_4A_6+4A_2A_7-A_3A_7+12A_6A_7\right)}{4\left(3A_4-10A_7\right)^2}.
\end{aligned} \tag{13}$$

Using Eq. (13), the solution of the Eq. (7) can be written as

$$Q(\varepsilon)=\sqrt{-\frac{2(3A_4-10A_7)}{4A_2-A_3-8A_6}}\left(1\pm\frac{2}{1+\eta e^{\varepsilon}}\right). \tag{14}$$

From Eq. (2) and Eq. (14), and after simplification, the straddled dark–singular soliton solution of Eq. (1) can be written as

$$q(x,t)=\sqrt{-\frac{2(3A_4-10A_7)}{4A_2-A_3-8A_6}}\left\{1\pm\frac{2}{1+\eta\cosh\left[h(x-vt)\right]+\eta\sinh\left[h(x-vt)\right]}\right\} \times e^{i(-kx+\omega t+\theta)}, \tag{15}$$

provided

$$(3A_4-10A_7)(4A_2-A_3-8A_6)<0. \tag{16}$$

In particular, upon choosing $\eta=\pm1$, one recovers dark and singular optical 1–soliton solutions respectively as:

$$q(x,t)=-\sqrt{-\frac{2(3A_4-10A_7)}{4A_2-A_3-8A_6}}\left[\tanh\left(\frac{h(x-vt)}{2}\right)\right]e^{i(-kx+\omega t+\theta)}, \tag{17}$$

and

$$q(x,t)=-\sqrt{-\frac{2(3A_4-10A_7)}{4A_2-A_3-8A_6}}\left[\coth\left(\frac{h(x-vt)}{2}\right)\right]e^{i(-kx+\omega t+\theta)}. \tag{18}$$

4. Modified new Kudryashov's approach

In this section, an effective algorithm namely, the modified new Kudryashov method [10], is applied to obtain more soliton solutions. Let us assume that Eq. (7) has the solution of the form

$$Q(\varepsilon)=B_0+B_1R(\varepsilon), \tag{19}$$

where B_0 and B_1 are arbitrary constants with $B_1\neq0$, and $R(\varepsilon)$ satisfies the ODE

$$\left(R'(\varepsilon)\right)^2=\left[R^2(\varepsilon)\left(1-\chi\left(R(\varepsilon)\right)^2\right)\right]\ln(A)^2,\quad 0<A\neq1, \tag{20}$$

and has the form

$$R(\varepsilon)=\frac{4k}{4k^2A^{\varepsilon}+\chi A^{-\varepsilon}}. \tag{21}$$

Substituting Eq. (19) and Eq. (20) into Eq. (7) and collecting the coefficients of different powers of $R(\varepsilon)$ equal to zero, we get

$$\begin{aligned}
&0=A_5B_1^5-2\chi A_2B_1^3\ln(A)^2-\chi A_3B_1^3\ln(A)^2+24\chi^2A_1B_1\ln(A)^4,\\
&0=5A_5B_0B_1^4-4\chi A_2B_0B_1^2\ln(A)^2-\chi A_3B_0B_1^2\ln(A)^2,\\
&0=10A_5B_0^2B_1^3-2\chi A_2B_0^2B_1\ln(A)^2+A_2B_1^3\ln(A)^2+A_3B_1^3\ln(A)^2\\
&\quad+A_6B_1^3-20\chi A_1B_1\ln(A)^4-2\chi A_4B_1\ln(A)^2,\\
&0=10A_5B_0^3B_1^2+2A_2B_0B_1^2\ln(A)^2+A_3B_0B_1^2\ln(A)^2+3A_6B_0B_1^2,\\
&0=5A_5B_0^4B_1+A_2B_0^2B_1\ln(A)^2+3A_6B_0^2B_1+A_1B_1\ln(A)^4+A_4B_1\ln(A)^2+A_7B_1,\\
&0=A_5B_0^5+A_6B_0^3+A_7B_0.
\end{aligned} \tag{22}$$

After solving the above system of algebraic equations, we have the following result

$$B_0=0,\quad B_1=\pm\sqrt{-\frac{20\chi A_7+18\chi A_4\ln(A)^2}{A_6+A_2\ln(A)^2+A_3\ln(A)^2}},\quad A_1=-\frac{A_7+A_4\ln(A)^2}{\ln(A)^4},$$

$$A_5=-\frac{\ln(A)^2(A_2+A_3)+A_6}{2\left(9A_4\ln(A)^2+10A_7\right)^2} \tag{23}$$

$$\times\ln(A)^4(6A_2A_4-3A_3A_4)+\ln(A)^2(8A_2A_7-12A_4A_6-2A_3A_7)-12A_6A_7.$$

From Eq. (23) solution of the Eq. (7) can be written as

$$Q(\varepsilon)=\pm\sqrt{-\frac{2\left\{9A_4\ln(A)^2+10A_7\right\}}{A_2\ln(A)^2+A_3\ln(A)^2+A_6}}\left(\frac{4k}{4k^2A^{\varepsilon}+\chi A^{-\varepsilon}}\right). \tag{24}$$

From Eq. (2) and Eq. (24), the straddled bright–singular soliton solution of Eq. (1) can be written as

$$q(x,t)=\pm\sqrt{-\frac{2\left\{9A_4\ln(A)^2+10A_7\right\}}{A_2\ln(A)^2+A_3\ln(A)^2+A_6}}$$
$$\times\left\{\frac{4k}{\left(4k^2+\chi\right)\cosh\left[h\ln A(x-vt)\right]+\left(4k^2-\chi\right)\sinh\left[h\ln A(x-vt)\right]}\right\} \tag{25}$$
$$\times e^{(-kx+\omega t+\theta)},$$

provided

$$\left\{9A_4\ln(A)^2+10A_7\right\}\left\{A_2\ln(A)^2+A_3\ln(A)^2+A_6\right\}<0. \tag{26}$$

In particular, upon choosing $\chi=\pm4k^2$, one arrives at bright and singular 1-soliton solutions given by

$$q(x,t)=\pm\sqrt{-\frac{2\left\{9A_4\ln(A)^2+10A_7\right\}}{A_2\ln(A)^2+A_3\ln(A)^2+A_6}}\left[sech\left(h(x-vt)\right)\right]e^{(-kx+\omega t+\theta)}, \tag{27}$$

and

$$q(x,t)=\pm\sqrt{-\frac{2\left\{9A_4\ln(A)^2+10A_7\right\}}{A_2\ln(A)^2+A_3\ln(A)^2+A_6}}\left[csch\left(h(x-vt)\right)\right]e^{(-kx+\omega t+\theta)}. \tag{28}$$

5. Conclusions

The current paper recovered a variety of optical soliton solutions to the concatenation model by the usage of two of Kudryashov's schemes. This gave way to a full spectrum of optical solitons that have collectively emerged from the two schemes. The parameter constraints, that naturally emerged from the two schemes, guarantee the existence of such solitons. These solitons will now be applied to address further issues with the model, namely to study the model with fractional temporal evolution. The application of the model to additional devices would be taken into consideration. A few such devices would be with Bragg gratings, magneto–optic waveguides, and optical metamaterials, as well as to extend the study with differential group delay and dispersion–

flattened fibers. The results of such research activities are currently awaited, and they would be disseminated with time. The numerical analysis of the models would also be addressed using the Laplace–Adomian decomposition scheme and several others. These results are currently awaited and would be soon visible after getting them aligned with the preexisting works [1–10].

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Анотація. *У цій статті реалізовано два підходи Кудряшова для виділення оптичних солітонних розв'язків моделі конкатенації, яка є кон'юнкцією нелінійного рівняння Шредінгера, моделі Лакшманана–Порсезіана–Даніеля та рівняння Саса–Сацуми. Отримано повний спектр солітонних рішень разом із обмеженнями параметрів, які вичерпно представлені в роботі.*

Ключові слова: *солітони, модель конкатенації, підходи Кудряшова*