Dark and singular cubic–quartic optical solitons with Lakshmanan–Porsezian–Daniel equation by the improved Adomian decomposition scheme

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Abstract. We employ an enhanced version of a decomposition technique, an improved Adomian decomposition method, and confirm computationally its high accuracy for a number of dark and singular cubic–quartic optical solitons which arise from the Lakshmanan–Porsezian–Daniel equation. The overall recurrent scheme applied for the governing model is elucidated and further scrutinized with regard to the optical solitons mentioned above. The computational results are promising and reveal a remarkably high level of precision. Tables of the absolute errors and illustrating plots are provided to augment the main findings of our comparative study.

Keywords: computational methods, Lakshmanan–Porsezian–Daniel model, improved Adomian decomposition method, optical solitons, Kerr-law nonlinearity.

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1. Introduction

Phenomena that appear in quantum optics are modelled by a wide variety of nonlinear evolution equations. Solitons are known as a special class of the solutions of nonlinear evolution equations [1]. A soliton represents a self-reinforcing wave beam that propagates without changing its shape and speed and maintains its stability during mutual collisions [2]. Optical solitons are important, in particular, for optical switching devices and pulse modulation of laser-source light. They form a basic ‘fabric’ for long-range communications industry [3].

The optical solitons arise owing to a very delicate balance among the nonlinear effects, the self-phase modulation and the chromatic dispersion. The presence of this balance leads to transmission of solitons over optical fibres over transcontinental and transoceanic distances [2, 4]. However, this balance can be lost and the soliton transmission can fail when the chromatic dispersion is too low or negligible [5]. Many concepts have been suggested recently to solve this problem. They are given, e.g., by cubic–quartic (CQ) solitons, purely cubic solitons, highly dispersive solitons and Bragg gratings. These technological innovations can salvage the propagation of solitons in optical fibres [6]. Introduction of higher-order dispersions maintains securely the balance between the dispersive effects and the self-phase modulation and so preserves the solitons. The notion of the CQ solitons has appeared in 2017, following the emergence of purely quartic solitons in 2016. Note that the purely quartic solitons have been examined only numerically [3, 4].

The CQ case is applicable when the usual chromatic dispersion is too small and can be neglected. Then it is 'substituted' by a mixture of third- and fourth-order dispersions. The CQ solitons have been dealt with both analytically and numerically. They have given an immense impetus to the field of quantum optics [2–7]. Recently, several mathematical models have appeared for description of the pulse propagation in optical fibres and, in particular, the CQ solitons. They describe substitution for the chromatic dispersion and generalize a well-known nonlinear Schrödinger evolution equations which represent the most fundamental model for the transmission of solitons through optical fibres [2, 4]. Among those models, one can mention a Fokas–Lenells equation [6], a complex dispersive Ginzburg–Landau equation [8], a Lakshmanan–Porsezian–Daniel (LPD) equation [9] and some others.

A perturbed LPD equation models transmission of the solitons through different types of waveguides, using Hamiltonian-type perturbation terms. This LPD model has originally emerged in 1988 during the studies of Heisenberg spins. After that, it has been comprehensively examined and acquired a high reputation [9]. The model has suggested an autonomous approach for studying the dynamics of soliton propagation through metamaterials, photonic-crystal fibres, optical fibres, different types of waveguides, etc. [7]. This model has also been studied with the aid of a wide variety of mathematical techniques, including a method of undetermined coefficients [4], a Lie-symmetry analysis [5], a Riccati-equation expansion approach [7], a sine-Gordon equation approach [9] and a semi-inverse variational principle [10]. Given such substantial analytical approaches at our disposal, it would be imperative to study the above model from a numerical perspective.

Below we will address some analytically derived dark and singular optical solitons found for a CQ-LPD model, using an improved Adomian decomposition method (IADM) [11–14]. Hence, the present work serves as a sequel to the studies previously reported for the same model, which have addressed the bright optical solitons [11]. Numerous works have already confirmed a high efficiency of this improved method when it deals with the Schrödinger equations of different types (see, e.g., Refs. [12–15] and references therein). We will study the model mentioned above with a Kerr-law nonlinearity [16]. The results of our approach will be validated by comparing them with the analytical data available in the recent literature [3].

The present work is organized as follows. Section 2 presents the model under our study, Section 3 outlines the methodology, and Section 4 gives a benchmark of the analytical solutions. Then Section 5 describes the numerical results and Section 6 concludes our study.

2. Governing equation

The CQ-LPD model has been offered in 2021 [5]. Using its dimensionless form and taking perturbation components into account, one can represent this model as [3]

$$\begin{aligned} & iq_t + aiq_{xxx} + bq_{xxxx} + c|q|^2 q = \alpha q_x^2 q^* + \beta |q_x|^2 q + \gamma |q|^2 q_{xx} + \lambda q^2 q_{xx}^* + \delta |q|^4 q \\ & + i\left[\xi \left(|q|^{2m} q \right)_x + \mu \left(|q|^{2m} \right)_x q + \rho |q|^{2m} q_x \right], \qquad i = \sqrt{1}, \end{aligned} \tag{1}$$

where $q = q(x,t)$ is a complex-valued wave field and x and t stand for the spatial and temporal variables, respectively. The first term on the left-hand side of Eq. (1) describes the temporal evolution, which is linear, a and b denote respectively the coefficients of the third- and fourth-order dispersion components, while c represents the coefficient of the nonlinear refractive index governed by the Kerr law. On right-hand side of Eq. (1), the coefficients α,β,γ and λ emanate from the nonlinear dispersion as a result of perturbation, while the two-photon absorption is denoted by δ . Finally, ξ presides over the self-steepening effect, μ and ρ are associated with the nonlinear dispersion effects, and the parameter m stands for the peak acceptable light intensity.

3. Computational approach

Below we present in brief the computational approach used to solve the governing CQ-LPD model, i.e. the IADM [11–15]. We note that the IADM represents an enhanced version of the traditional Adomian decomposition method (ADM), which is an efficient semi-analytical and, at the same time, numerical method for tackling the functional equations.

We begin with decomposing the complex-valued wave field q involved in Eq. (1) into its real and imaginary components:

$$q(x,t) = v_1 + iv_2\,, \tag{2}$$

with v_1 and v_2 standing for real-valued functions. Upon substituting Eq. (2) into Eq. (1), one obtains

$$\begin{aligned}
&i(v_1+iv_2)+ai(v_1+iv_2)_{xxx}+b(v_1+iv_2)_{xxxx}+c|v_1+iv_2|^2(v_1+iv_2)\\
&=\alpha(v_1+iv_2)_x^2(v_1-iv_2)+\beta\left|(v_1+iv_2)_x\right|^2(v_1+iv_2)+\gamma|v_1+iv_2|^2(v_1+iv_2)_{xx}\\
&+\lambda(v_1+iv_2)^2(v_1-iv_2)_{xx}+\delta\left|(v_1+iv_2)\right|^4(v_1+iv_2)\\
&+i\left\{\xi\left[|v_1+iv_2|^{2m}(v_1+iv_2)\right]_x+\mu\left[|v_1+iv_2|^{2m}\right]_x(v_1+iv_2)+\rho|v_1+iv_2|^{2m}(v_1+iv_2)_x\right\}.
\end{aligned} \tag{3}$$

When the variables are separated, the latter relation yields the following coupled equations:

$$v_{1t} = -av_{1xxx} - bv_{2xxxx} + I_1, \tag{4}$$

$$v_{2t} = -av_{2xxx} + bv_{1xxxx} - I_2, \tag{5}$$

where I_1 and I_2 are expressed as

$$\begin{aligned}
I_1 &= -c\left(v_1^2+v_2^2\right)v_2+\alpha\left[\left(v_{2x}^2-v_{1x}^2\right)v_2+2v_{1x}v_{2x}v_1\right]+\beta\left(v_{1x}^2+v_{2x}^2\right)v_2+\gamma\left(v_1^2+v_2^2\right)v_{2xx}\\
&+\lambda\left(v_2^2-v_1^2\right)v_{2xx}+2v_1v_2v_{1xx}+\delta\left(v_1^2+v_2^2\right)^2v_2\\
&+\xi\left\{\left(v_1^2+v_2^2\right)^m v_{1x}+\left[\left(v_1^2+v_2^2\right)^m\right]_x v_1\right\}+\mu\left[\left(v_1^2+v_2^2\right)^m\right]_x v_1+\rho\left(v_1^2+v_2^2\right)^m v_{1x},
\end{aligned} \tag{6}$$

and

$$\begin{aligned}
I_2 &= -c\left(v_1^2+v_2^2\right)v_1+\alpha\left[\left(v_{1x}^2-v_{2x}^2\right)v_1+2v_{1x}v_{2x}v_2\right]+\beta\left(v_{1x}^2+v_{2x}^2\right)v_1+\gamma\left(v_1^2+v_2^2\right)v_{1xx}\\
&+\lambda\left[\left(v_1^2-v_2^2\right)v_{1xx}+2v_1v_2v_{2xx}\right]+\delta\left(v_1^2+v_2^2\right)^2v_1\\
&-\xi\left\{\left(v_1^2+v_2^2\right)^m v_{2x}+\left[\left(v_1^2+v_2^2\right)^m\right]_x v_2\right\}-\mu\left[\left(v_1^2+v_2^2\right)^m\right]_x v_2-\rho\left(v_1^2+v_2^2\right)^m v_{2x}.
\end{aligned} \tag{7}$$

Then the initial conditions become as follows:

$$v_1(x,0)=\left[q(x,0)\right]_R,\ \ v_2(x,0)=\left[q(x,0)\right]_I,$$

where R and I are associated with $\left[q(x,0)\right]$ and denote the real and imaginary parts, respectively.

The traditional ADM decomposes the solution set $\{v_1(x,t),v_2(x,t)\}$ into an infinite series of components as follows:

$$v_1(x,t)=\sum_{n=0}^{\infty}v_{1n}(x,t), \tag{8}$$

$$v_2(x,t)=\sum_{n=0}^{\infty}v_{2n}(x,t), \tag{9}$$

where the components v_{1n} and v_{2n} at $n\ge 0$ are computed recurrently. Re-expressing Eqs. (4) and (5) in an operator style, one obtains

$$L_t v_1=-av_{1xxx}+bv_{2xxxx}-I_1, \tag{10}$$

$$L_t v_2=-av_{2xxx}+bv_{1xxxx}-I_2, \tag{11}$$

where $L_t=\dfrac{\partial}{\partial t}$. After applying the inverse linear operator L_t^{-1} to both sides of Eqs. (10–11), we get

$$v_1(x,t)=v_1(x,0)-aL_t^{-1}v_{1xxx}-bL_t^{-1}v_{2xxxx}+L_t^{-1}I_1, \tag{12}$$

$$v_2(x,t)=v_2(x,0)-aL_t^{-1}v_{2xxx}+bL_t^{-1}v_{1xxxx}-L_t^{-1}I_2. \tag{13}$$

The nonlinear components I_1 and I_2 involved in Eqs. (12) and (13) (see also Eqs. (6) and (7)) are replaced respectively by A_{1n} and A_{2n}, which are referred to as Adomian polynomials. Here we recall that, basing on the precise algorithm devised by Adomian [11–15], one can compute the Adomian polynomials for all the sorts of nonlinearities.

Let us substitute the nonlinear components given by Eqs. (6) and (7) and the solution defined by Eqs. (8) and (9) into Eqs. (12) and (13). Then one obtains

$$\sum_{n=0}^{\infty}v_{1n}(x,t)=v_1(x,0)-aL_t^{-1}\sum_{n=0}^{\infty}(v_1(x,t))_{xxx}-b_1L_t^{-1}(v_2(x,t))_{xxxx}+L_t^{-1}\sum_{n=0}^{\infty}A_{1n}, \tag{14}$$

$$\sum_{n=0}^{\infty}v_{2n}(x,t)=v_2(x,0)-aL_t^{-1}\sum_{n=0}^{\infty}(v_2(x,t))_{xxx}+bL_t^{-1}(v_1(x,t)_{xxxxx}-L_t^{-1}\sum_{n=0}^{\infty}A_{2n}. \tag{15}$$

Without much delay, the IADM procedure gives the following overall recurrent scheme for the CQ-LPD model:

$$v_{1,0}(x,t)=v_1(x,0), \tag{16}$$

$$v_{2,0}(x,t)=v_2(x,0), \tag{17}$$

$$v_{1,k+1}(x,t)=-aL_t^{-1}(v_{1,k}(x,t))_{xxx}-bL_t^{-1}(v_{2,k}(x,t))_{xxxx}+L_t^{-1}A_{1,k}, \tag{18}$$

$$v_{2,k+1}(x,t)=-aL_t^{-1}(v_{2,k}(x,t))_{xxx}+bL_t^{-1}(v_{1,k}(x,t))_{xxxx}-L_t^{-1}A_{2,k}, \tag{19}$$

Here the recurrent solution is obtained from Eqs. (18) and (19) upon substituting Eqs. (8) and (9) into Eq. (2) as follows:

$$q(x,t)=\sum_{n=0}^{\infty}v_{1n}(x,t)+i\left(\sum_{n=0}^{\infty}v_{2n}(x,t)\right). \tag{20}$$

4. Dark and singular CQ optical solitons

Now we consider some dark and singular CQ optical solitons, which have been reported analytically in the literature, as benchmark solutions. This consideration becomes imperative for validating the approximate IADM solution associated with the governing CQ-LDP model.

4.1. Dark soliton

Recently, Vega-Guzman et al. [3] have employed the method of undetermined coefficients to reveal some interesting CQ optical solitons for the CQ-LDP model, including the dark optical solitons of the form

$$q(x,t) = A\tanh\left[B(x-vt)\right]e^{i(-kx+\omega t+\theta)}, \tag{21}$$

where k and ω are respectively the wave number and the frequency of the soliton, θ is the centre of the soliton phase, and v denotes the velocity of the soliton which gives its mean position. Here the soliton amplitude and width are represented respectively by A and B. The latter are given by [3]

$$A = \sqrt{\frac{4b\left(10B^2-3k^2\right)B^2}{Z_2+2\left(Z_4+Z_5\right)B^2}}, \tag{22}$$

$$B = \sqrt{\frac{(7Z_4+2Z_5)Z_2+3k^2\left\{40b\delta+\left(Z_4+Z_5\right)\left(Z_4+2Z_5\right)\right\}\pm\Psi_D}{4\left\{100b\delta-\left(Z_4-4Z_5\right)\left(Z_4+Z_5\right)\right\}}}, \tag{23}$$

with

$$\Psi_D = \left\{5Z_2+3k^2\left(Z_4+Z_5\right)\right\}\sqrt{96b\delta+\left(Z_4+2Z_5\right)^2}.$$

They are subject to the following constraint conditions:

$$\left[100b\delta-\left(Z_4-4Z_5\right)\left(Z_4+Z_5\right)\right]$$
$$\times\left[7Z_4+2Z_5)Z_2+3k^2\left\{40b\delta+\left(Z_4+Z_5\right)\left(Z_4+2Z_5\right)\right\}\pm\Psi_D)\right]>0,$$

$$96b\delta+\left(Z_4+2Z_5\right)^2\geq 0,$$

where we have

$$Z_1=\omega+3b^4,$$
$$Z_2=c+\left(2\lambda-\beta\right)k^2,$$
$$Z_3=k\left(\xi+\rho\right)=0,$$
$$Z_4=\alpha+\beta,\quad Z_5=\gamma+\lambda,$$

of which parameters are coupled by the relations

$$v=-8bk^3,$$
$$\xi+2m\left(\xi+\mu\right)+\rho=0,\quad \alpha+\gamma=\lambda.$$

Moreover, the soliton frequency ω can be determined explicitly:

$$\omega=4\left(4B^2-3k^2\right)bB^2-3bk^4-Z_4A^2B^2.$$

Considering all of the parameters mentioned above, we analyze further on the two cases of the dark-soliton solutions by changing the soliton width B.

Case I. The soliton width B takes the form

$$B=\sqrt{\frac{(7Z_4+2Z_5)Z_2+3k^2\left\{40b\delta+\left(Z_4+Z_5\right)\left(Z_4+2Z_5\right)\right\}+\Psi_D}{4\left\{100b\delta-\left(Z_4-4Z_5\right)\left(Z_4+Z_5\right)\right\}}}.$$

Case II. The soliton width B takes the form

$$B=\sqrt{\frac{(7Z_4+2Z_5)Z_2+3k^2\left\{40b\delta+\left(Z_4+Z_5\right)\left(Z_4+2Z_5\right)\right\}-\Psi_D}{4\left\{100b\delta-\left(Z_4-4Z_5\right)\left(Z_4+Z_5\right)\right\}}}.$$

4.2 Singular soliton

Now we analyze the singular optical soliton reported by Vega-Guzman et al. [3] for the CQ-LDP model and consider Eq. (21). Below we do not present too much details since many of them have been given with regard to the dark optical solitons. With no delay, we consider again the two cases of the singular soliton solutions by changing the soliton width B.

Case I. This case involves the following types.

Type I. The soliton width B takes the form

$$B=\sqrt{-\frac{(7Z_4+2Z_5-5\Psi_B)Z_2+3\left\{40b\delta+\left(Z_4+Z_5\right)\left(Z_4+2Z_5-\Psi_B\right)k^2\right\}}{2\left\{100b\delta-\left(Z_4-4Z_5\right)\left(Z_4+Z_5\right)\right\}}}.$$

Type II. The soliton width B takes the form

$$B=\sqrt{-\frac{(7Z_4+2Z_5+5\Psi_B)Z_2+3\left\{40b\delta+\left(Z_4+Z_5\right)\left(Z_4+2Z_5+\Psi_B\right)k^2\right\}}{2\left\{100b\delta-\left(Z_4-4Z_5\right)\left(Z_4+Z_5\right)\right\}}}.$$

Case II. This case involves the following types.

Type I. The soliton width B takes the form

$$B=\sqrt{\frac{(7Z_4+2Z_5)Z_2+3k^2\left\{40b\delta+\left(Z_4+Z_5\right)\left(Z_4+2Z_5\right)\right\}+\Psi_D}{4\left\{100b\delta-\left(Z_4-4Z_5\right)\left(Z_4+Z_5\right)\right\}}}.$$

Type II. The soliton width B takes the form

$$B=\sqrt{\frac{(7Z_4+2Z_5)Z_2+3k^2\left\{40b\delta+\left(Z_4+Z_5\right)\left(Z_4+2Z_5\right)\right\}-\Psi_D}{4\left\{100b\delta-\left(Z_4-4Z_5\right)\left(Z_4+Z_5\right)\right\}}}.$$

5. Numerical results

Now we demonstrate the main results acquired via our numerical simulations. In particular, we report the absolute errors and provide graphical illustrations of the dark and singular soliton solutions which have been described in Section 4. To be specific, we take the following parameter values for the governing CQ-LPD model [3]:

$$a=4bk,\ \rho=-3\xi-2\mu,\ \delta=0.1,\ m=1,\ \mu=1,$$
$$k=0.1\ ,\ \xi=1,\ \alpha=\beta=1\times10^{-6},\ \gamma=0.1.$$

In addition, we prescribe the initial conditions at $t=0$ given by Eq. (21) to the dark soliton solutions and, accordingly, to the corresponding singular soliton solutions.

The absolute errors, i.e. the differences between the benchmark analytical solutions [3] and our numerical solutions derived by the IADM are gathered in Tables 1–6. Figs. 1–6 show the plots of the analytical and numerical solutions taken at different values of the temporal variable $t=\{0,0.1,0.2,0.3,0.4,0,5\}$, where the spatial variable x ranges in the interval $-20\leq x\leq 20$. In fact, the error differences are remarkably low. Moreover, a particular case of ideal correspondence between the exact and computational results can be seen in Figs. 1–6 at $t = 0$. Finally, one can see that the level of accuracy of the numerical model decreases slightly with increasing t value.

Table 1. Absolute errors for the Case-I dark optical soliton.

t	Error at $x = 60$
0.0	0.000000
0.1	9.60×10^{-8}
0.2	1.919×10^{-7}
0.3	2.881×10^{-7}
0.4	3.839×10^{-7}
0.5	4.798×10^{-7}

Table 2. Absolute errors for the Case-II dark optical soliton.

t	Error at $x = 60$
0.0	0.000000
0.1	8.438×10^{-7}
0.2	0.0000016876
0.3	0.0000025314
0.4	0.0000033752
0.5	0.0000042189

Table 3. Absolute errors for the Case-I Type-I singular optical soliton.

t	Error at $x = 60$
0.0	0.000000
0.1	9.999999991×10^{-9}
0.2	1.999999999×10^{-8}
0.3	2.99999999×10^{-8}
0.4	3.999999999×10^{-8}
0.5	4.999999999×10^{-8}

Table 4. Absolute errors for the Case-I Type-II singular optical soliton.

t	Error at $x = 60$
0.0	0.000000
0.1	1.000000000×10^{-8}
0.2	2.000000000×10^{-8}
0.3	3.00000000×10^{-8}
0.4	4.000000000×10^{-8}
0.5	5.000000000×10^{-8}

Table 5. Absolute errors for the Case-II Type-I singular optical soliton.

t	Error at $x = 60$
0.0	0.000000
0.1	9.6×10^{-9}
0.2	1.92×10^{-8}
0.3	2.89×10^{-8}
0.4	3.85×10^{-8}
0.5	4.81×10^{-8}

Table 6. Absolute errors for the Case-II Type-II singular optical soliton.

t	Error at $x = 60$
0.0	0.000000
0.1	8.4×10^{-9}
0.2	1.69×10^{-8}
0.3	2.53×10^{-8}
0.4	3.37×10^{-8}
0.5	4.22×10^{-8}

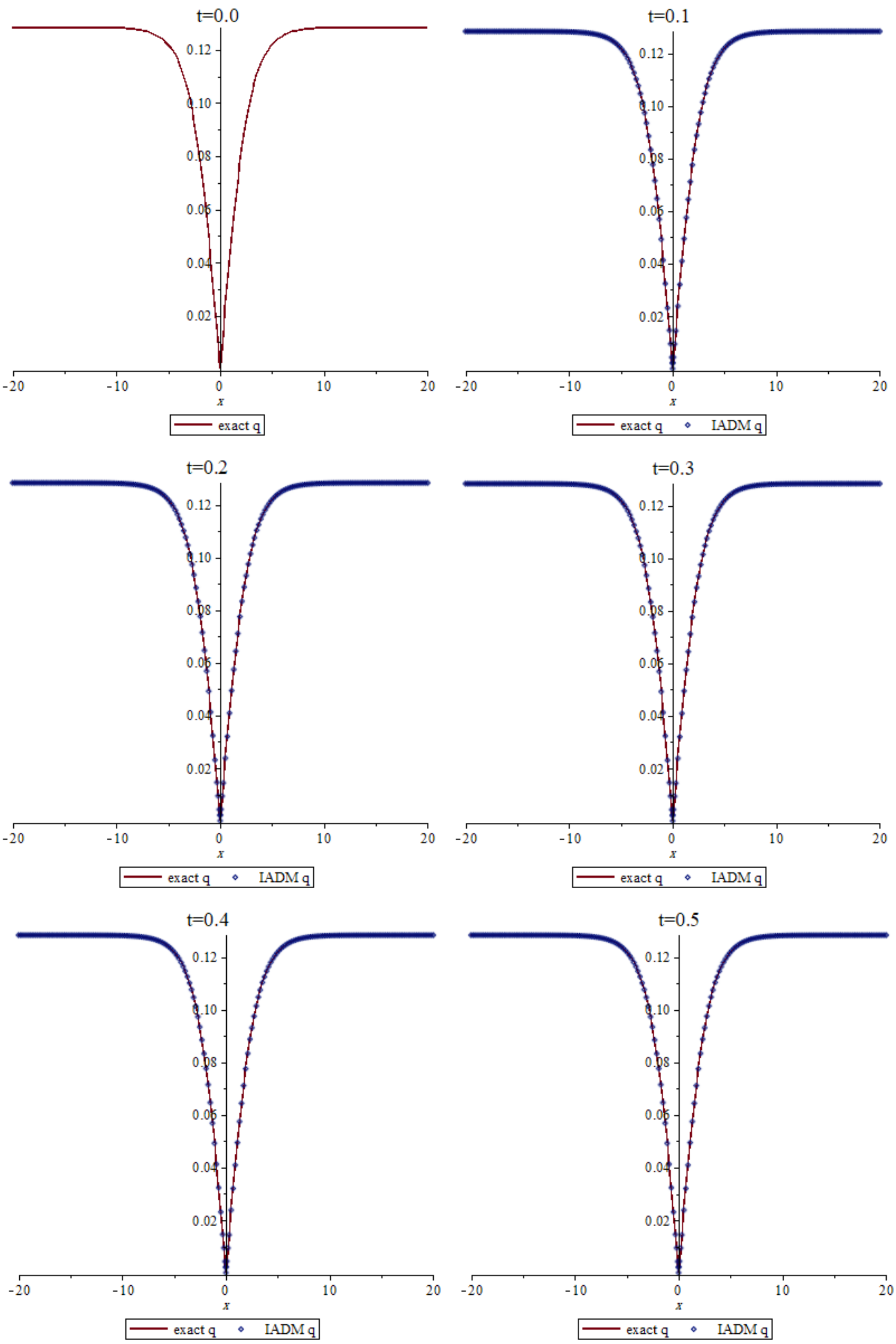

Fig. 1. Comparison of the exact and IADM-simulated solutions for the Case-I dark optical soliton.

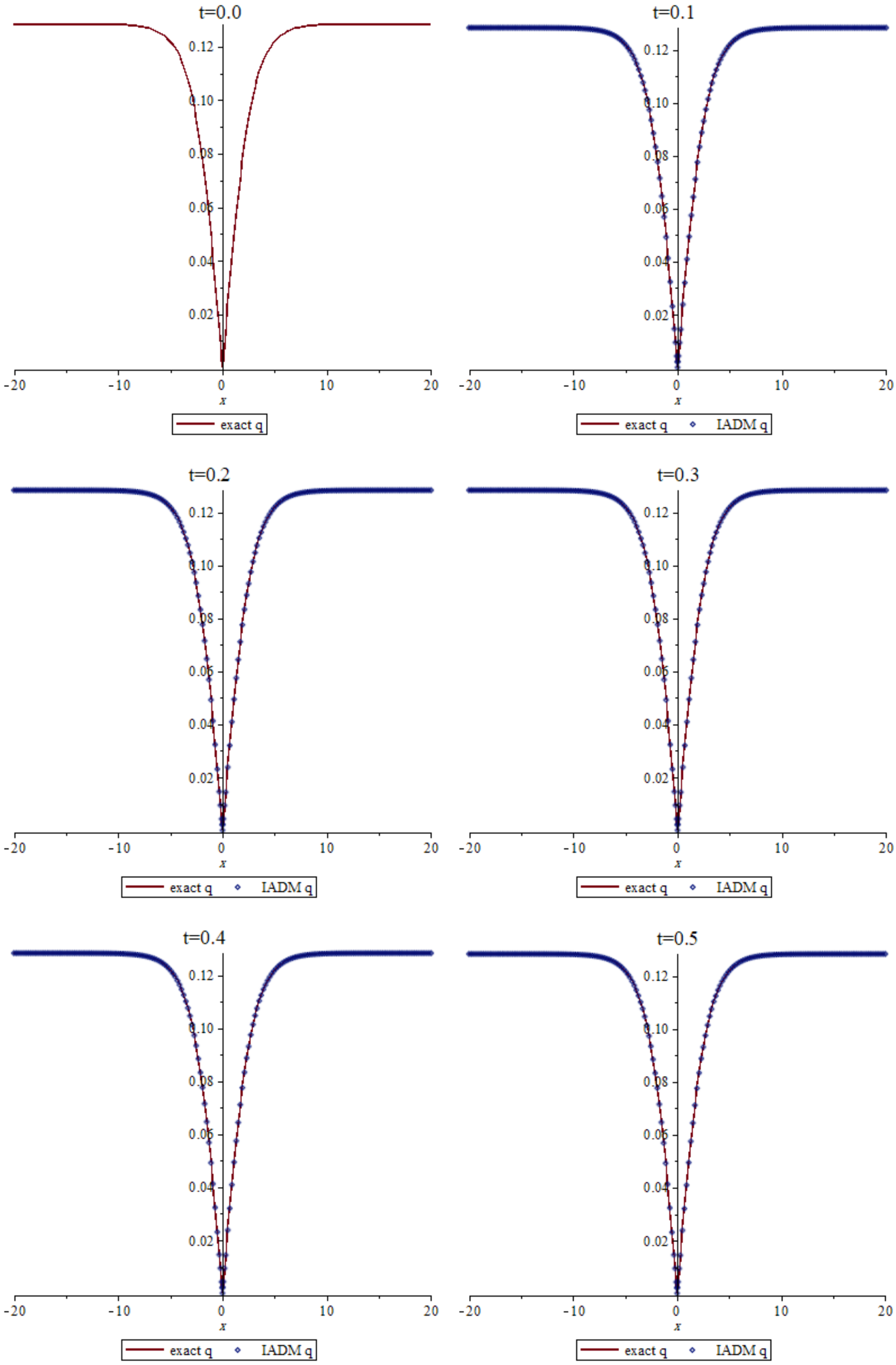

Fig. 2. Comparison of the exact and IADM-simulated solutions for the Case-II dark optical soliton.

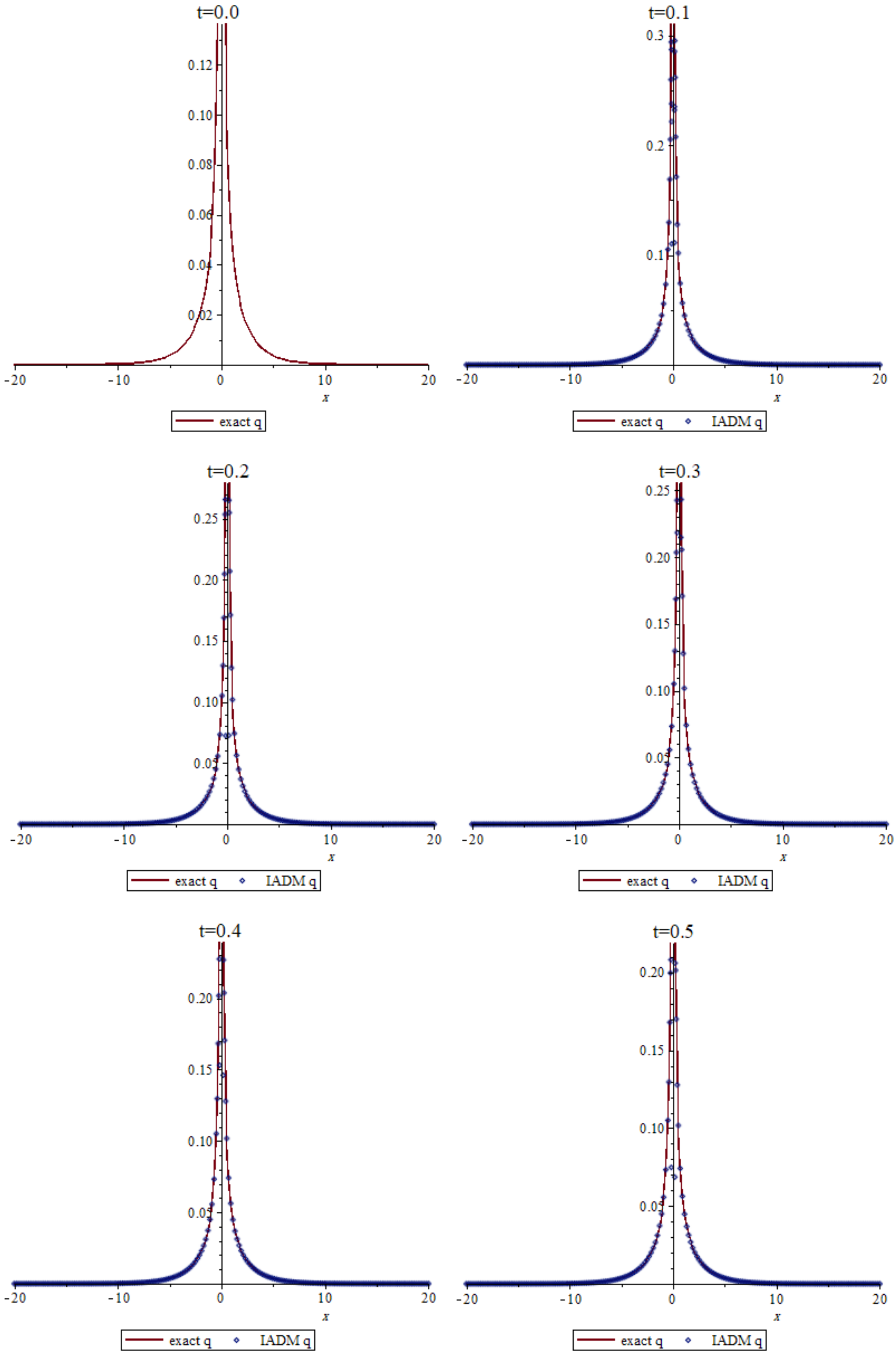

Fig. 3. Comparison of the exact and IADM-simulated solutions for the Case-I Type-I singular optical soliton.

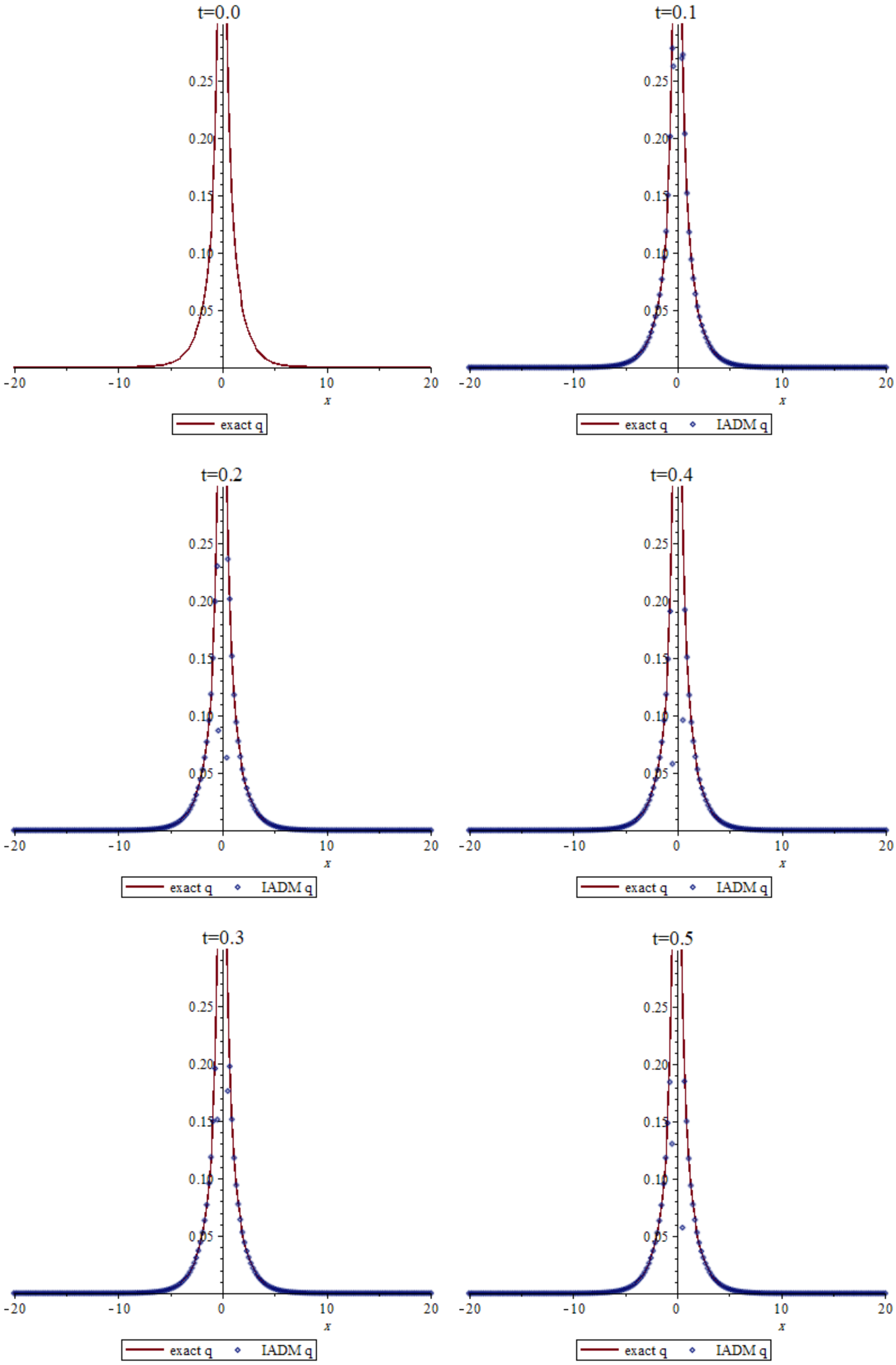

Fig. 4. Comparison of the exact and IADM-simulated solutions for the Case-I Type-II singular optical soliton.

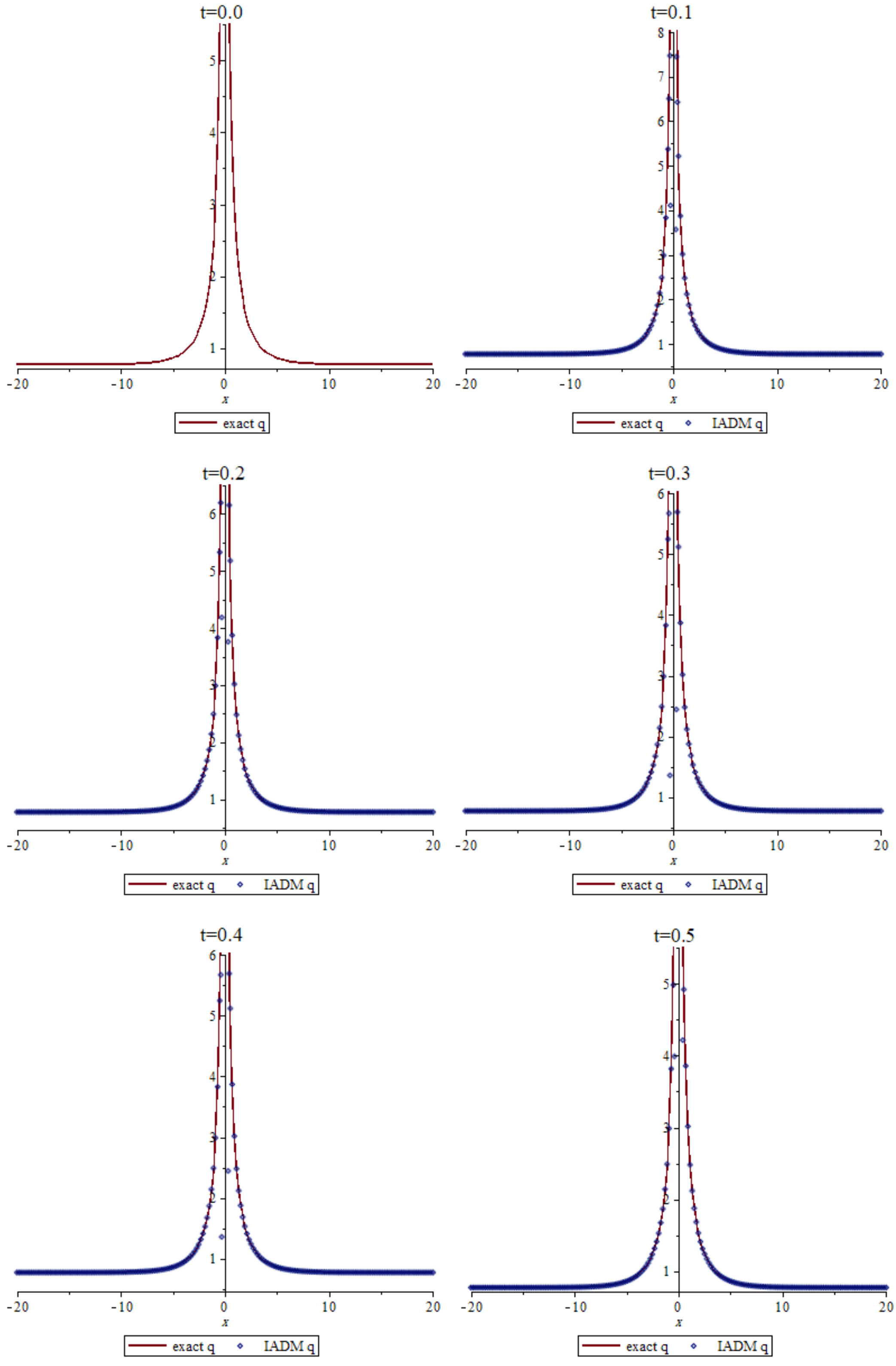

Fig. 5. Comparison of the exact and IADM-simulated solutions for the Case-II Type-I singular optical soliton.

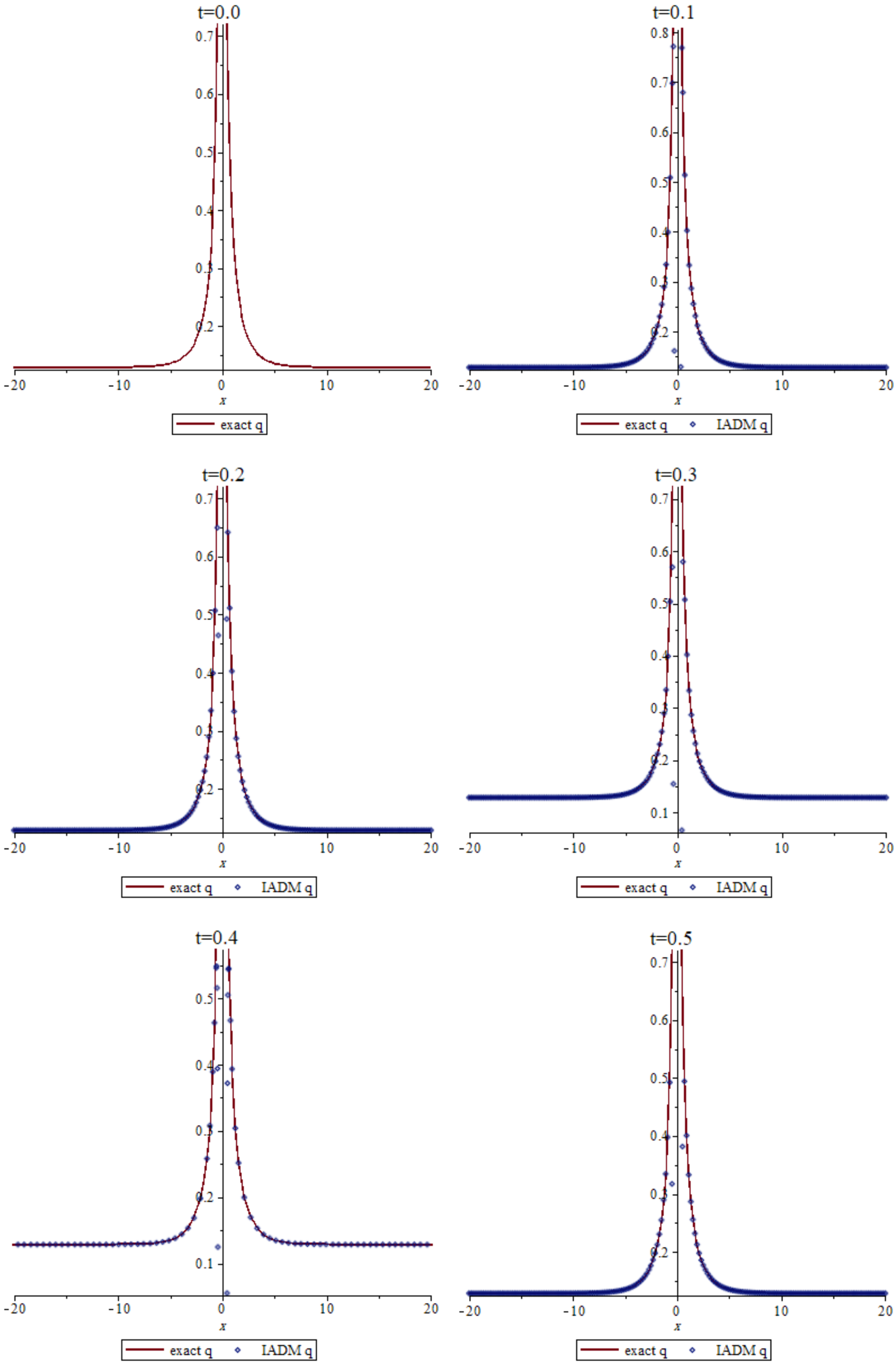

Fig. 6. Comparison of the exact and IADM-simulated solutions for the Case-II Type-II singular optical soliton.

6. Conclusion

In the present study, we have developed the computational scheme based on an enhanced version of the traditional ADM, the IADM, to tackle numerically the CQ-LPD equation. In particular, we have derived the recurrent computational scheme for the governing model. In order to validate further our scheme, we have compared our results with some of the dark and singular optical soliton solutions found analytically in the recent work by Vega-Guzman et al. [3]. Amazingly, our computational results are very promising. The corresponding absolute errors provided in Section 5 testify that our numerical findings are characterized by an extremely high accuracy. The comparative plots in Figs. 1–6 portray an almost perfect match between the contending curves.

It is worth noting that the computational approach developed in the present work can be extended to other classes of differential Schrödinger equations concerned with more complicated nonlinearities. In the future, the above model will be studied using additional approaches and the appropriate computational data will be compared with the recent results [17–23].

Disclosure. The authors claim that there is no conflict of their interests.

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Анотація. *Ми використали вдосконалену версію методу декомпозиції – покращений метод декомпозиції Адоміана – і чисельно підтвердили його високу точність для низки темних і сингулярних кубічно-квартичних оптичних солітонів, які з'являються з рівняння Лакшманана–Порсезіана–Даніеля. Пояснено загальну рекурентну схему, застосовану для керуючої математичної моделі, та додатково досліджено її на прикладі згаданих вище*

оптичних солітонів. Результати наших обчислень багатообіцяючі та демонструють надзвичайно високий рівень точності. Основні результати нашого порівняльного дослідження доповнено таблицями абсолютних похибок та ілюстраційними графіками.

Ключові слова: *обчислювальні методи, модель Лакшманана–Порсезіана–Деніела, удосконалений метод декомпозиції Адоміана, оптичні солітони, нелінійність за законом Керра.*