Cubic-quartic optical solitons for Lakshmanan-Porsezian-Daniel equation by the improved Adomian decomposition scheme

A. A. Al Qarni [1], A. M. Bodaqah [2], A. S. H. F. Mohammed [2], A. A. Alshaery [2], H. O. Bakodah [2] and Anjan Biswas [3, 4, 5, 6]

[1] Department of Mathematics, College of Science, University of Bisha, P.O. Box 551, Bisha 61922, Saudi Arabia, aqarny@ub.edu.sa.
[2] Department of Mathematics, Faculty of Science, University of Jeddah, P.O. Box 80327, Jeddah, Saudi Arabia.
[3] Department of Mathematics and Physics, Grambling State University, Grambling, LA–71245, USA.
[4] Mathematical Modeling and Applied Computation (MMAC) Research Group, Department of Mathematics, King Abdulaziz University, Jeddah–21589, Saudi Arabia.
[5] Department of Applied Sciences, Cross–Border Faculty, Dunarea de Jos University of Galati, 111 Domneasca Street, Galati–800201, Romania.
[6] Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa – 0204, South Africa.

Received: 01.07.2022

Abstract. We study a class of Lakshmanan–Porsezian–Daniel equations endowed with a cubic–quartic nonlinearity. A highly efficient improved Adomian decomposition approach is employed when deriving a generalized numerical scheme. Our numerical results reveal perfect agreement with the analytical optical solutions known from the literature. In other words, our method provides an astonishing level of accuracy and reliability.

Keywords: optical solitons, Lakshmanan–Porsezian–Daniel model, Kerr-law nonlinearity, improved Adomian decomposition method.

UDC: 535.32

1. Introduction

Nonlinear Schrödinger equations are important complex-valued evolution equations. They are utilized far and wide in many technological and scientific applications to model diverse nonlinear states and phenomena. As an example, these equations are highly relevant in the theories of metamaterials and optical fibres [1–3]. Exact analytical solutions of this class of nonlinear equations are widely referred to as solitary wave solutions or solitons, which date back to the 1800s after their first detection by J. S. Russell [4]. Here we remind that "a soliton is defined as a self-reinforcing solitary wave in the form of a wave packet or pulse that always maintains its shape while it travels at steady speed" [5]. Different forms of nonlinearities in optics, which are associated with the complex-valued evolution equations, characterize different interesting aspects of soliton dynamics. In particular, the latest development in mathematical photonics is the introduction of cubic–quartic nonlinearity [6–9], which becomes important when the group-velocity dispersion runs low and thus insignificant. In fact, this development has been pioneered more recently as a follow-up to quartic solitons [10]. Still, the quartic solitons should be thoroughly studied numerically and the corresponding results have to be compared with those obtained analytically.

To improve the situation, below we will consider one of the famous complex-valued evolution equations, a Lakshmanan–Porsezian–Daniel equation [11]. It has emerged primarily in the framework of Heisenberg spin chains. More specifically, we will analyze the Lakshmanan–Porsezian–Daniel equation endowed with the cubic–quartic nonlinearity [12–14]. This specific model of our interest plays a vital role in various optical scenarios and admits diverse applications. In view of this, we remind the readers about various rigorous integration approaches utilized in the literature when tackling different types of the evolution equations. In particular, they include the approaches of Jacobi elliptic function [15], exponential-function expansion [16], modified extended tanh [17], Kudryashov method [18], rational G'/G-expansion [19], sub-equations [20], extended exponential rational method [21], extended auxiliary equation [22] and ansatze method [23] (see Refs. [24–30] for more samples of interrelated analytical approaches).

A number of numerical approaches have also been developed to solve the evolution equations, e.g. an Adomian decomposition method [31, 32], a homotopy Laplace perturbation approach [33] and a homotopy analysis method [34]. Since the Adomian decomposition method [31, 32] has been rightly regarded as a rapidly convergent technique [35, 36], a lot of its reliable modifications have been devised in the literature, including a famous improved Adomian decomposition scheme [37, 38]. This improved scheme has been reported to represent an efficient numerical technique in treating different classes of the evolution equations (see the recent findings mentioned in Refs. [39–41] and references therein).

In fact, the present study makes use of a high efficiency of the improved Adomian decomposition scheme [37, 38] to tackle numerically a cubic–quartic Lakshmanan–Porsezian–Daniel (CQ-LPD) equation [12–14]. This is achieved by deriving a generalized recurrent scheme and subsequently using the available exact solutions as yardsticks for a numerical comparison. The article is organized as follows: Section 2 gives a governing model, Section 3 describes our numerical results, and Section 4 formulates concluding remarks.

2. Governing equation

The dimensionless structure of the CQ-LPD equation in the presence of perturbation terms is given by the relation [24]

$$iq_t + aiq_{xxx} + bq_{xxxx} + c|q|^2 q = \alpha q_x^2 q^* + \beta |q_x|^2 q + \gamma |q|^2 q_{xx} + \lambda q^2 q_{xx}^* + \delta |q|^4 q \\ + i\left[\xi\left(|q|^{2m} q\right)_x + \mu\left(|q|^{2m}\right)_x q + \rho |q|^{2m} q_x \right]. \tag{1}$$

Here t and x denote respectively the temporal and spatial variables, $q(x, t)$ is the wave profile, q_t gives the linear temporal evolution, q_x the linear spatial dispersions, q_{xxx} and q_{xxxx} correspond to the higher-order dispersions, and $i = \sqrt{-1}$. In other words, the first term in the l. h. s. of Eq. (1) corresponds to the linear temporal evolution, while a and b are the coefficients associated with the third- and fourth-order dispersion terms, respectively. The coefficient c describes the Kerr law for the nonlinear refractive index. On the r. h. s. of Eq. (1), the coefficients α, β, γ and λ originate from the perturbation terms coming from the nonlinear dispersion, and δ corresponds to the two-photon absorption. Here ξ governs the self-steepening effect, whereas μ and ρ are due to the nonlinear dispersion effects. Finally, the parameter m corresponds to the peak allowable light intensity.

In our calculations, complex Eq. (1) is converted into a real system by writing

$$q(x,t) = u_1 + iu_2 \quad , \tag{2}$$

where $\left(u_k, k=1,2\right)$ are real functions. By substituting Eq. (2) into Eq. (1), we have

$$\begin{aligned}
&i\left(u_1+iu_2\right)+ai\left(u_1+iu_2\right)_{xxx}+b\left(u_1+iu_2\right)_{xxxx}+c\left|u_1+iu_2\right|^2\left(u_1+iu_2\right)\\
&=\alpha\left(u_1+iu_2\right)_x^2\left(u_1-iu_2\right)+\beta\left|\left(u_1+iu_2\right)_x\right|^2\left(u_1+iu_2\right)\\
&+\gamma\left|u_1+iu_2\right|^2\left(u_1+iu_2\right)_{xx}+\lambda\left(u_1+iu_2\right)^2\left(u_1-iu_2\right)_{xx}+\delta\left|\left(u_1+iu_2\right)\right|^4\left(u_1+iu_2\right)+\\
&i\left\{\xi\left[\left|u_1+iu_2\right|^{2m}\left(u_1+iu_2\right)\right]_x+\mu\left[\left|u_1+iu_2\right|^{2m}\right]_x\left(u_1+iu_2\right)+\rho\left|u_1+iu_2\right|^{2m}\left(u_1+iu_2\right)_x\right\}.
\end{aligned} \tag{3}$$

Then the following equation can be obtained:

$$u_{1t}=-au_{1xxx}+bu_{2xxxx}-A_1\ , \tag{4}$$

$$u_{2t}=-au_{2xxx}+bu_{1xxxx}-A_2\ . \tag{5}$$

Here we have

$$\begin{aligned}
A_1=&-c\left(u_1^2+u_2^2\right)u_2+\alpha\left[\left(u_{2x}^2-u_{1x}^2\right)u_2+2u_{1x}u_{2x}u_1\right]+\beta\left(u_{1x}^2+u_{2x}^2\right)u_2\\
&+\gamma\left(u_1^2+u_2^2\right)u_{2xx}+\lambda\left(u_2^2-u_1^2\right)u_{2xx}+2u_1u_2u_{1xx}+\delta\left(u_1^2+u_2^2\right)^2u_2\\
&+\xi\left\{\left(u_1^2+u_2^2\right)^m u_{1x}+\left[\left(u_1^2+u_2^2\right)^m\right]_x u_1\right\}\\
&+\mu\left[\left(u_1^2+u_2^2\right)^m\right]_x u_1+\rho\left(u_1^2+u_2^2\right)^m u_{1x}\quad ,
\end{aligned} \tag{6}$$

$$\begin{aligned}
A_2=&-c\left(u_1^2+u_2^2\right)u_1+\alpha\left[\left(u_{1x}^2-u_{2x}^2\right)u_1+2u_{1x}u_{2x}u_2\right]+\beta\left(u_{1x}^2+u_{2x}^2\right)u_1\\
&+\gamma\left(u_1^2+u_2^2\right)u_{1xx}+\lambda\left[\left(u_1^2-u_2^2\right)u_{1xx}+2u_1u_2u_{2xx}\right]+\delta\left(u_1^2+u_2^2\right)^2u_1\\
&-\xi\left\{\left(u_1^2+u_2^2\right)^m u_{2x}+\left[\left(u_1^2+u_2^2\right)^m\right]_x u_2\right\}\\
&-\mu\left[\left(u_1^2+u_2^2\right)^m\right]_x u_2-\rho\left(u_1^2+u_2^2\right)^m u_{2x}\ .
\end{aligned} \tag{7}$$

Note that the initial conditions $u_1\left(x,0\right)=\left[q\left(x,0\right)\right]_R$, $u_2\left(x,0\right)=\left[q\left(x,0\right)\right]_I$ hold true, with R and I representing the real and imaginary parts, respectively.

In the frame of the Adomian decomposition method, one decomposes a solution into infinite sums of components defined as

$$u_1\left(x,t\right)=\sum_{n=0}^{\infty}u_{1n}\left(x,t\right)\ , \tag{8}$$

$$u_2\left(x,t\right)=\sum_{n=0}^{\infty}u_{2n}\left(x,t\right)\ . \tag{9}$$

Here the components $u_{1n}, u_{2n}, n\geq 0$ are to be determined recursively. In the operator form, Eqs. (4) and (5) become as follows:

$$L_t\left(u_1\right)=-au_{1xxx}-bu_{2xxxx}+A_1\ , \tag{10}$$

$$L_t\left(u_2\right)=-au_{2xxx}+bu_{1xxxx}-A_2, \tag{11}$$

where $L_t = \frac{\partial}{\partial t}$.

Applying the inverse operator L_t^{-1} to both sides of Eqs. (10) and (11) gives

$$u_1(x,t) = u_1(x,0) - aL_t^{-1}u_{1xxx} - bL_t^{-1}u_{2xxxx} + L_t^{-1}A_1 , \tag{12}$$

$$u_2(x,t) = u_2(x,0) - aL_t^{-1}u_{2xxx} + bL_t^{-1}u_{1xxxx} - L_t^{-1}A_2. \tag{13}$$

It is assumed that the nonlinear terms in Eqs. (12) and (13) are represented by Eqs. (6) and (7). Then A_{1n} and A_{2n} are the so-called Adomian polynomials which can be constructed for all forms of nonlinearity according to specific algorithms suggested by Adomian.

Substituting the nonlinear terms in Eqs. (6) and (7) and the solution form given by Eqs. (8) and (9) into Eqs. (12) and (13) results in

$$\sum_{n=0}^{\infty}u_{1n}(x,t) = u_1(x,0) - aL_t^{-1}\sum_{n=0}^{\infty}(u_1(x,t))_{xxx} - b_1L_t^{-1}(u_2(x,t))_{xxxx} + L_t^{-1}\sum_{n=0}^{\infty}A_{1n}, \tag{14}$$

$$\sum_{n=0}^{\infty}u_{2n}(x,t) = u_2(x,0) - aL_t^{-1}\sum_{n=0}^{\infty}(u_2(x,t))_{xxx} + bL_t^{-1}(u_1(x,t))_{xxxxx} - L_t^{-1}\sum_{n=0}^{\infty}A_{2n}. \tag{15}$$

Following the decomposition analysis, the following recursive relations can be introduced:

$$u_{1,0}(x,t) = u_1(x,0), \tag{16}$$

$$u_{2,0}(x,t) = u_2(x,0), \tag{17}$$

$$u_{1,k+1}(x,t) = -aL_t^{-1}(u_{1,k}(x,t))_{xxx} - bL_t^{-1}(u_{2,k}(x,t))_{xxxx} + L_t^{-1}A_{1,m}, \tag{18}$$

$$u_{2,k+1}(x,t) = -aL_t^{-1}(u_{2,k}(x,t))_{xxx} + bL_t^{-1}(u_{1,k}(x,t))_{xxxx} - L_t^{-1}A_{1,m} . \tag{19}$$

Let us substitute the terms from Eqs. (18) and (19) into Eqs. (4) and (5). As a result, the approximate solution in the form of Eq. (2) can be found:

$$q(x,t) = u_{1,0} + u_{1,1} + u_{1,2} + \ldots + i(u_{2,0} + u_{2,1} + u_{2,2} + \ldots). \tag{20}$$

3. Numerical results

In this section we analyze three distinct scenarios for the CQ-LPD problem given in Eq. (1) and demonstrate how the improved Adomian decomposition method described in Section 2 might be applied. We analyze a bright-soliton solution of the model, which has been derived recently by J. Vega-Guzman et al. [42]:

$$q(x,t) = A\,\mathrm{sech}[B(x - vt)]e^{i(-kx+\omega t+\theta)}, \tag{21}$$

where *k* is the wave number of the solution, ω the soliton frequency, θ the centre of phase of the soliton, v the velocity of the soliton which gives its mean position, and *A* and *B* are respectively the soliton amplitude and its width. The latter parameters are given by [42]

$$A = \sqrt{\frac{4(3k^2 + 5B^2)bB^2}{Z_2 - (Z_4 + Z_5)B^2}} , \tag{22}$$

$$B = \sqrt{-\frac{(7Z_4 + 2Z_5 \pm 5\Psi_B)Z_2 + 3\{40b\delta + (Z_4 + Z_5)(Z_4 + 2Z_5 \pm \Psi_B)k^2\}}{2\{100b\delta - (Z_4 - 4Z_5)(Z_4 + Z_5)\}}} . \tag{23}$$

Here we have

$$\Psi_B = \sqrt{96b\delta + (Z_4 + 2Z_5)^2},$$

provided that

$$\left[100b\delta - (Z_4 - 4Z_5)(Z_4 + Z_5)\right]\times$$
$$\left[(7Z_4 + 2Z_5 \pm 5\Psi_B)Z_2 + 3\left\{40b\delta + (Z_4 + Z_5)(Z_4 + 2Z_5 \pm \Psi_B)k^2\right\}\right] < 0$$

and

$$96b\delta + (Z_4 + 2Z_5)^2 \geq 0, \qquad Z_1 = \omega + 3b^4$$

(implying the conditions for existing Z_1 and Z_3 in Eqs. (22) and (23) [42]),

$$Z_2 = c + (2\lambda - \beta)k^2,$$
$$Z_3 = k(\xi + \rho) = 0,$$
$$Z_4 = \alpha + \beta,$$
$$Z_5 = \gamma + \lambda,$$
$$v = -8bk^3,$$
$$\xi + 2m(\xi + \mu) + \rho = 0,$$
$$\alpha + \gamma = \lambda.$$

Let us consider the CQ-LPD problem given by Eq. (1) with $a = 4bk$, $\rho = \xi - 2\mu = -3$, $\delta = \dfrac{3\left(4\gamma k^2 - c - 2k\mu - 2k\xi\right)^2}{2b\left(3k^2 + 5\right)^2}$, $m = 1$, $\mu = 1$, $k = 0.1$ and $\xi = -1$, [42]. The initial condition at $t = 0$ follows from Eq. (21). Now the following particular cases can be distinguished.

I. A first bright soliton is given by

$$q(x,0) = A\,\mathrm{sech}\left[B(x)\right]e^{i(-kx+\theta)}, \tag{24}$$

with

$$B = \sqrt{-\frac{(7Z_4 + 2Z_5 - 5\Psi_B)Z_2 + 3\left\{40b\delta + (Z_4 + Z_5)(Z_4 + 2Z_5 - \Psi_B)k^2\right\}}{2\left\{100b\delta - (Z_4 - 4Z_5)(Z_4 + Z_5)\right\}}}. \tag{25}$$

In our numerical simulations, we consider the three following subcases for the fixed model parameters.

Case 1: $b = 0.1$, $c = 0.1$, $\gamma = 0.1$, $\alpha = 0.1$, $\beta = 0.1$.

Case 2: $b = 1$, $c = 1$, $\gamma = 0.1$, $\alpha = 1.0\times10^{-7}$, $\beta = 1.0\times10^{-7}$.

Case 3: $b = 0.5$, $c = 0.5$, $\gamma = 0.1$, $\alpha = 0.001$, $\beta = 0.001$.

II. A second bright soliton is given by Eq. (24) with

$$B = \sqrt{-\frac{(7Z_4 + 2Z_5 + 5\Psi_B)Z_2 + 3\left\{40b\delta + (Z_4 + Z_5)(Z_4 + 2Z_5 + \Psi_B)k^2\right\}}{2\left\{100b\delta - (Z_4 - 4Z_5)(Z_4 + Z_5)\right\}}}. \tag{26}$$

In our numerical simulations, we consider the following three subcases of the fixed model parameters.

Case 1: $b = 0.4$, $c = 0.1$, $\gamma = -1.5$, $\alpha = -0.1$, $\beta = 1.5$.

Case 2: $b = 0.7$, $c = 0.1$, $\gamma = -1.7$, $\alpha = 0.001$, $\beta = 2$.

Case 3: $b = 1$, $c = 0.1$, $\gamma = -2$, $\alpha = 1.0\times 10^{-7}$, $\beta = 2.5$.

Below we report the absolute error differences between the exact solutions and the approximate solutions derived with the improved Adomian decomposition method for the above three cases (see Tables 1–6). Besides, we portray the appropriate solutions for different t values, which correspond to the regions $-60 \le x \le 60$ (see Figs. 1–2) and $-20 \le x \le 20$ (see Figs. 3–6). With no loss of generality, the data presented in Figs. 1–6 can be considered as self-explanatory in illustrating an excellent performance of our numerical method. For convenience, the absolute disagreement values are noted in Figs. 1–6 by the figures situated above the bell-shaped curves. As a matter of fact, only a small discrepancy can be seen for the peaks of the curves. Moreover, this discrepancy can be made still less whenever the model parameters are suitably chosen and more iterations/approximants are considered in the series summation.

Table 1. Absolute errors for the Case 1 of the first bright soliton ($\boldsymbol{x}=60$).

t	Error
0.0	0.000000
0.1	0.009999984856
0.2	0.01999998484
0.3	0.02999998484
0.4	0.03999998483
0.5	0.04999998483

Table 2. Absolute errors for the Case 2 of the first bright soliton ($\boldsymbol{x}=60$).

t	Error $x = 60$
0.0	0.000000
0.1	9.999817309×10^{-9}
0.2	1.999981083×10^{-8}
0.3	2.999980429×10^{-8}
0.4	3.999979763×10^{-8}
0.5	4.999979085×10^{-8}

Table 3. Absolute errors for the Case 3 of the first bright soliton ($\boldsymbol{x}=60$).

t	Error
0.0	0.000000
0.1	0.00009999999071
0.2	0.0001999999906
0.3	0.0002999999906
0.4	0.0003999999905
0.5	0.0004999999903

Table 4. Absolute errors for the Case 1 of the second bright soliton ($\boldsymbol{x}=60$).

t	Error
0.0	0.000000
0.1	0.009999999993
0.2	0.02000000000
0.3	0.02999999999
0.4	0.03999999999
0.5	0.04999999998

Table 5. Absolute errors for the Case 2 of the second bright soliton ($\boldsymbol{x}=60$).

t	Error
0.0	0.000000
0.1	9.907824491×10^{-6}
0.2	0.0000199078459
0.3	0.0000299078702
0.4	0.0000399078952
0.5	0.0000499079206

Table 6. Absolute errors for the Case 3 of the second bright soliton ($\boldsymbol{x}=60$).

t	Error
0.0	0.000000
0.1	9.598051×10^{-9}
0.2	1.9188718×10^{-8}
0.3	2.8771896×10^{-8}
0.4	3.8347413×10^{-8}
0.5	4.7915154×10^{-8}

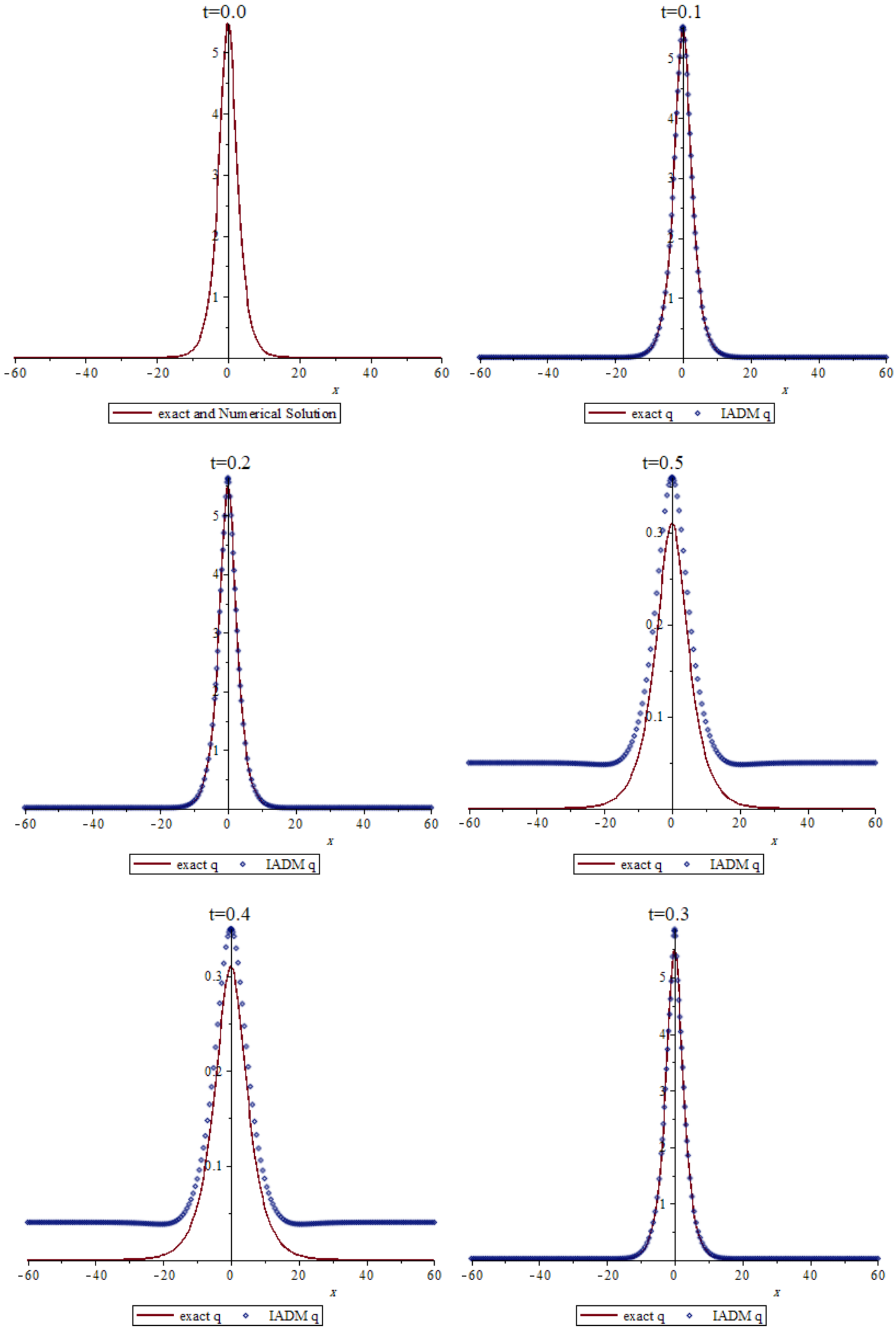

Fig. 1. Comparison of the exact solution and that of the improved Adomian decomposition method for the Case 1 of the first bright soliton ($-60 \le x \le 60$). In all figures, the abbreviation 'IADM' implies 'improved Adomian decomposition method'.

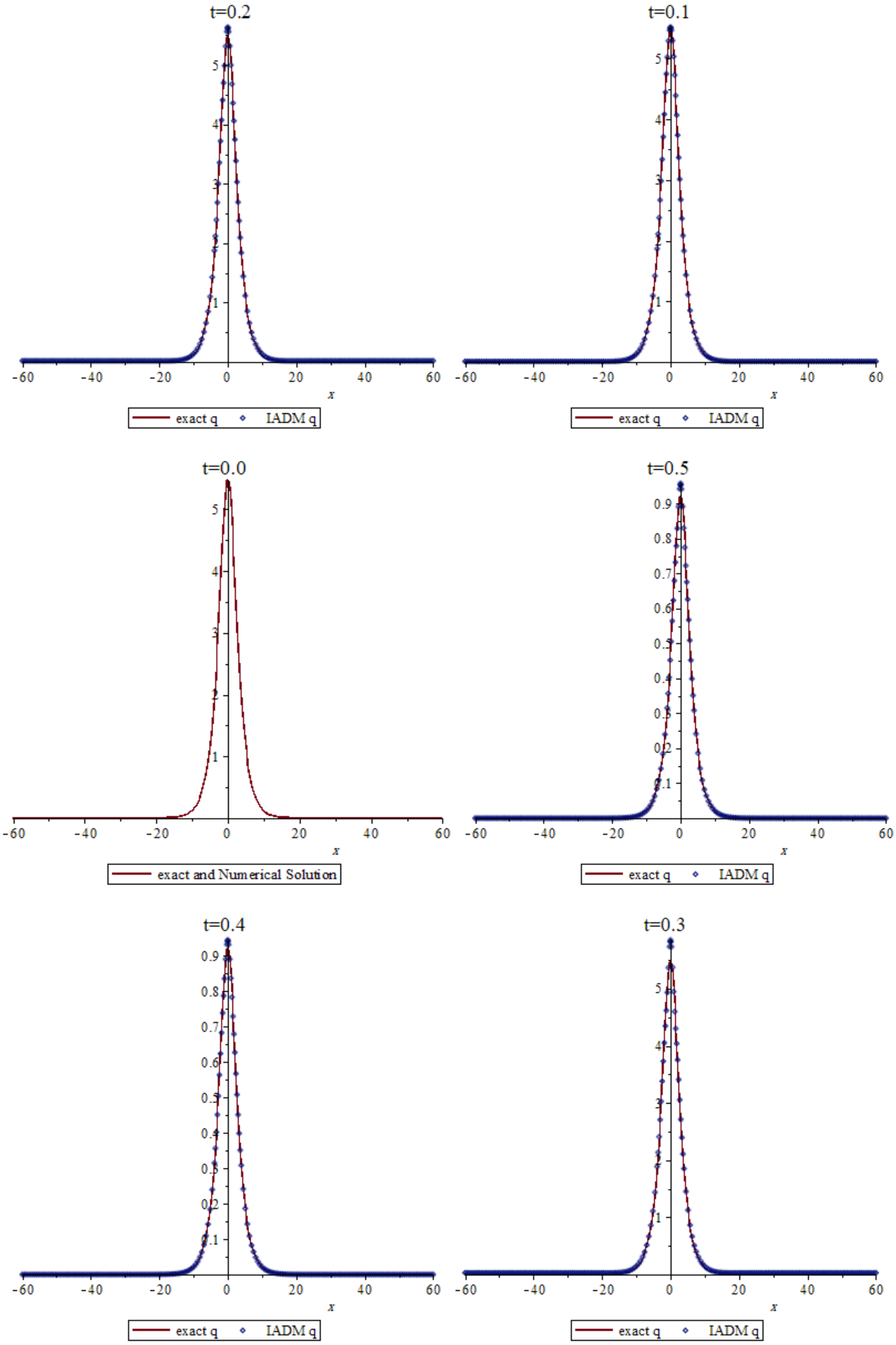

Fig. 2. Comparison of the exact solution and that of the improved Adomian decomposition method for the Case 2 of the first bright soliton ($-60 \leq x \leq 60$).

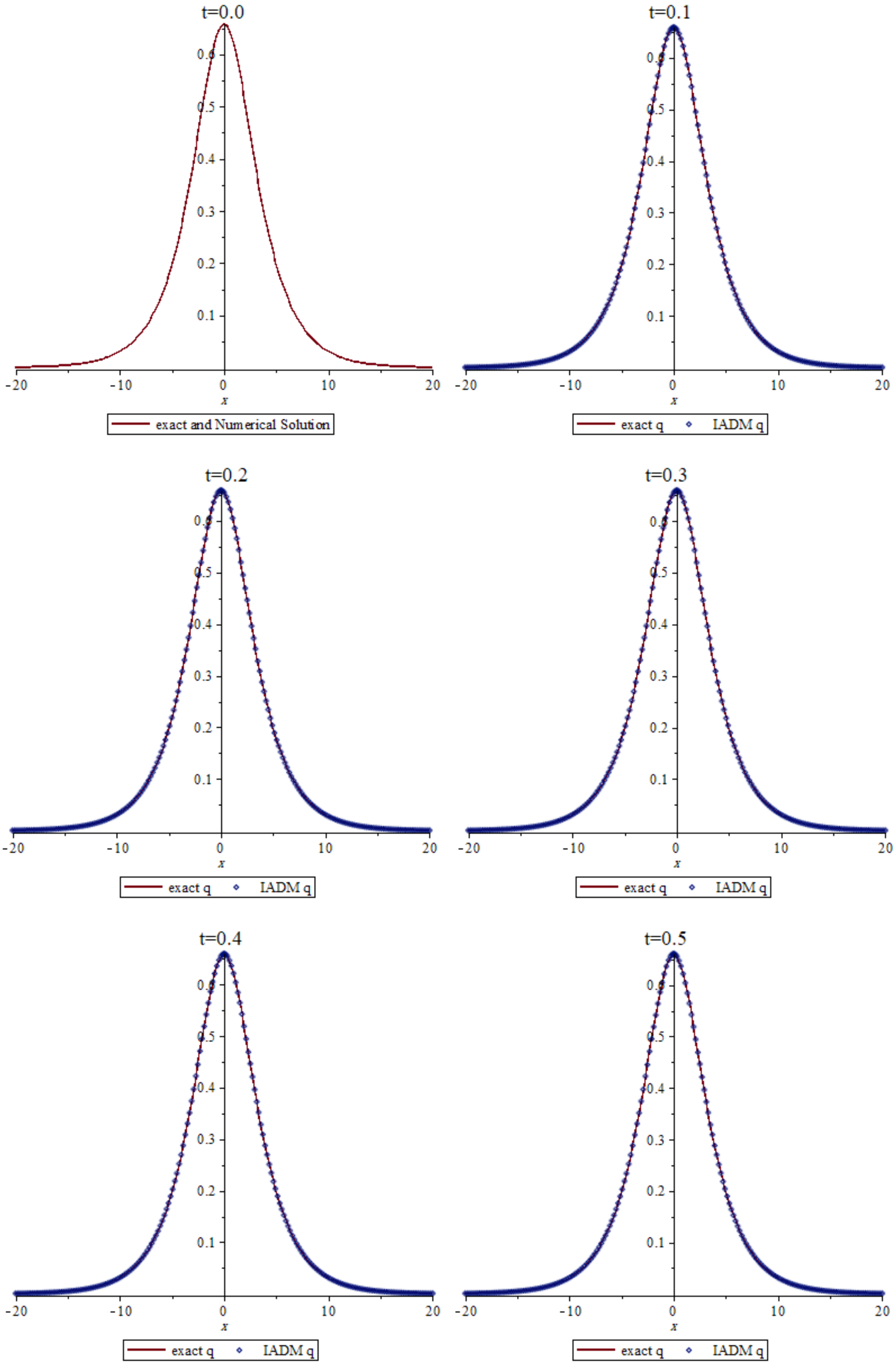

Fig. 3. Comparison of the exact solution and that of the improved Adomian decomposition method for the Case 3 of the first bright soliton ($-20 \le x \le 20$).

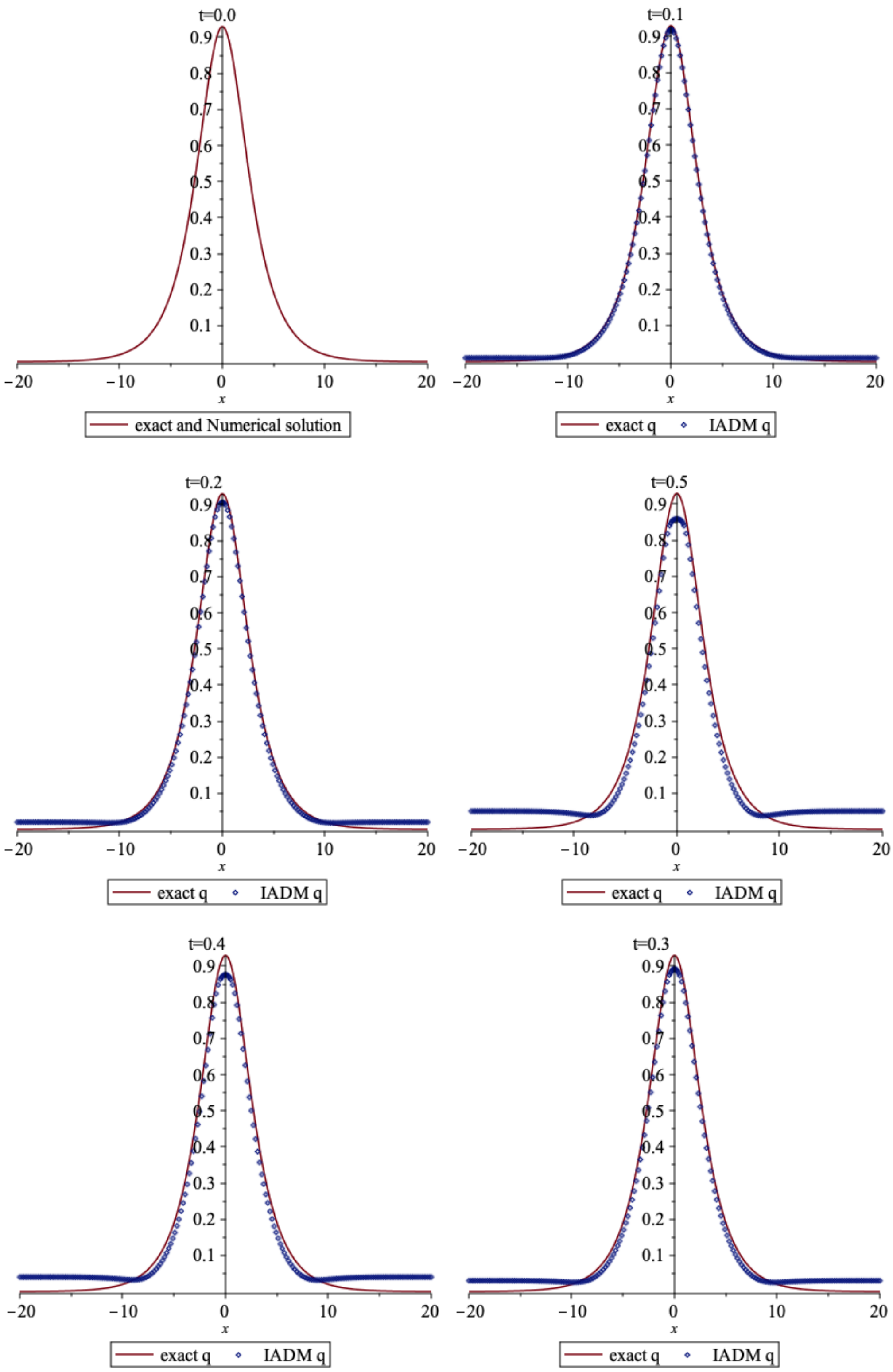

Fig. 4. Comparison of the exact solution and that of the improved Adomian decomposition method for the Case 1 of the second bright soliton ($-20 \le x \le 20$).

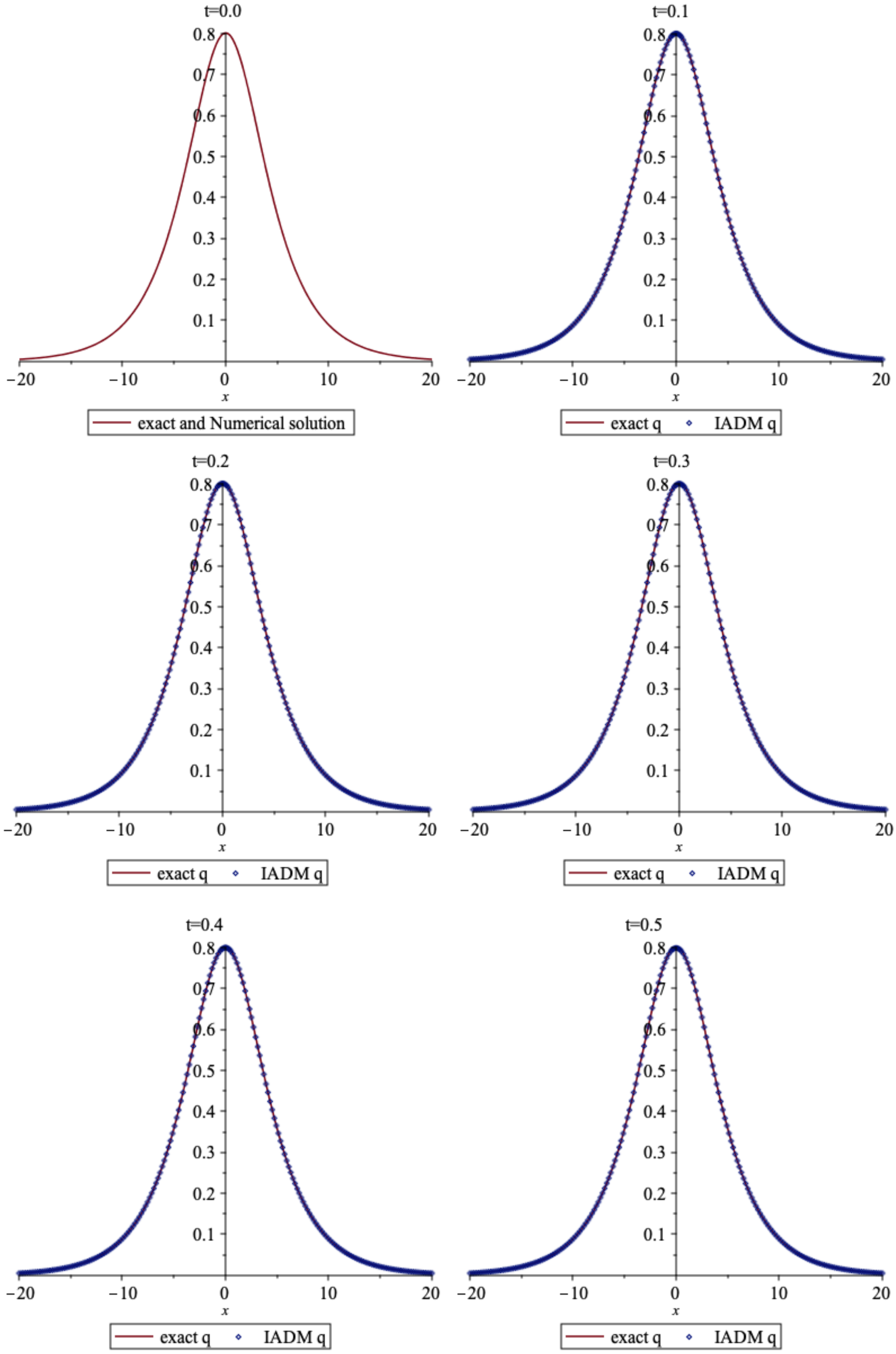

Fig. 5. Comparison of the exact solution and that of the improved Adomian decomposition method for the Case 2 of the second bright soliton ($-20 \le x \le 20$).

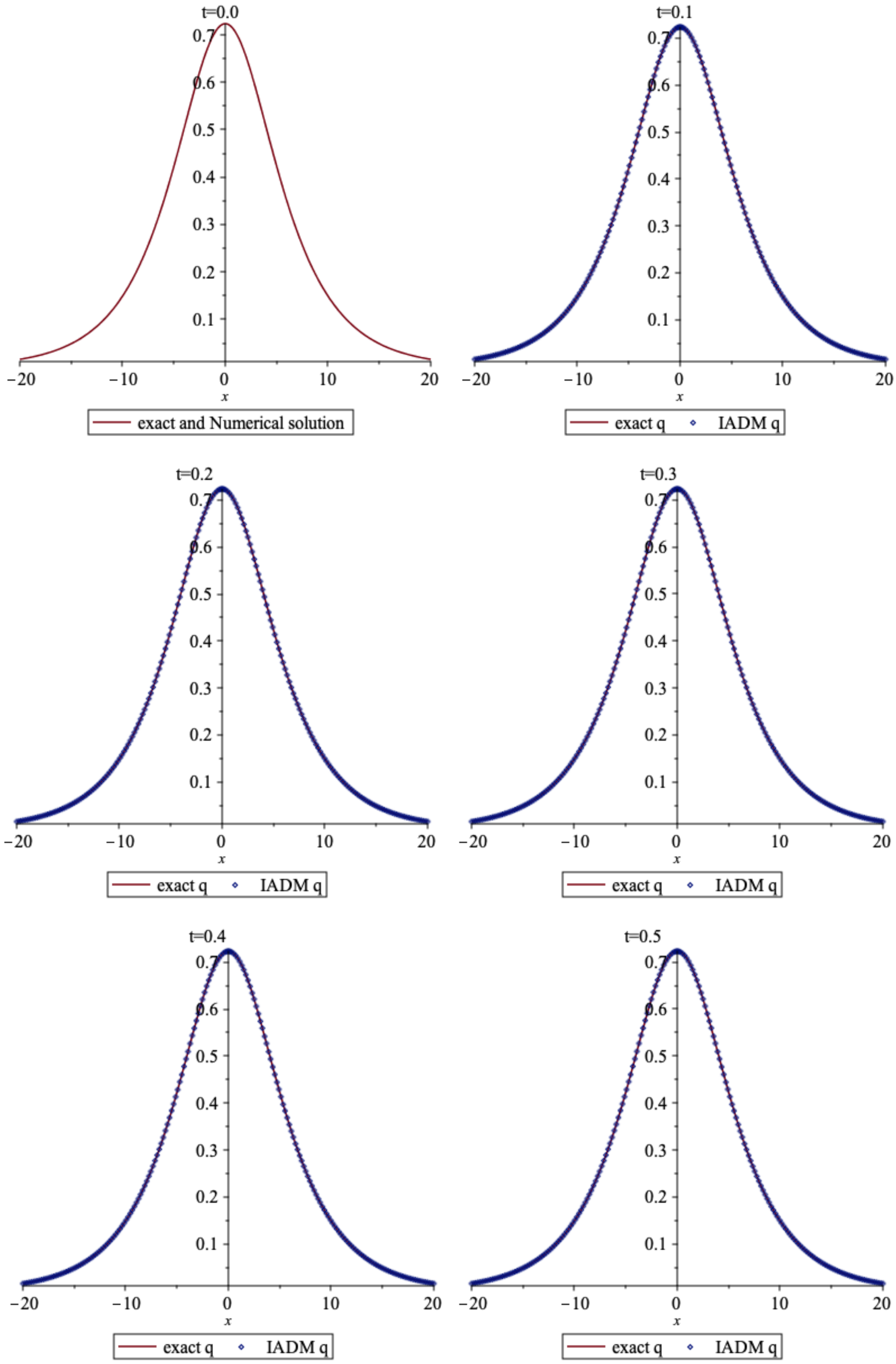

Fig. 6. Comparison of the exact solution and that of the improved Adomian decomposition method for the Case 3 of the second bright soliton ($-20 \leq x \leq 20$).

4. Conclusion

Summing up, in the present work we have studied a class of the complex-valued evolution equations, which is commonly called the Lakshmanan–Porsezian–Daniel equation. This is done for the important case of the cubic–quartic nonlinearity. A highly efficient improved Adomian decomposition method has been employed to derive a generalized numerical scheme and subsequently obtain simulated numerical results. Moreover, the known analytical optical solutions have been sampled from the literature and compared with our numerical data. It is the astonishing fact that the numerical results reported by us are in perfect agreement with the exact solutions. This testifies the advanced accuracy of our numerical scheme.

References

1. Seadawy A R and Lu D, 2017. Bright and dark solitary wave soliton solutions for the generalized higher order nonlinear Schrodinger equation and its stability. Res. Phys. **7**: 43–48.
2. Justin M, Hubert M B, Betchewe G, Doka S Y and Crepin K T, 2018. Chirped solitons in derivative nonlinear Schrödinger equation. Chaos, Solitons & Fractals. **107**: 49–54.
3. Bansal A, Biswas A, Zhou Q and Babatin M M, 2018. Lie symmetry analysis for cubic–quartic nonlinear Schrödinger’s equation. Optik. **169**: 12–15.
4. Russell J S. Report on waves. Made to the Meetings of the British Association in 1842–43. Print Book, English, 1845.
5. Asghar Ali, Aly R Seadawy and Dianchen Lu, 2018. New solitary wave solution of some nonlinear models and their applications. Adv. Diff. Equations. **232**: 1687–7.
6. Biswas A, Triki H, Zhou Q, Moshokoa S P, Ullah M Z and Belic M, 2017. Cubic–quartic optical solitons in Kerr and power law media. Optik. **144**: 357–362.
7. Biswas A, Ullah M Z, Zhou Q, Moshokoa S P and Triki H, Belic M, 2017. Resonant optical solitons with quadratic–cubic nonlinearity by semi-inverse variational principle. Optik. **145**: 18–21.
8. Bansal A, Biswas A, Zhou Q and Babatin M M, 2018. Lie symmetry analysis for cubic–quartic nonlinear Schrödinger's equation. Optik. **169**: 12–15.
9. Biswas A, Ekici M, Sonmezoglu A and Belic M R, 2019. Highly dispersive optical solitons with quadratic–cubic law by exp-function. Optik. **186**: 431–435.
10. Blanco-Redondo A, De Sterke C M, Sipe J E, Krauss T F, Eggleton B J and Husko C, 2016. Pure-quartic solitons. Nature Commun. **7**: 1–9.
11. Lakshmanan M, Porsezian K and Daniel M, 1988. Effect of discreteness on the continuum limit of the Heisenberg spin chain. Phys. Lett. A. **133**: 483–488.
12. Kumar S, Biswas A, Zhou Q, Yıldırım Y, Alshehri H M and Belic M R, 2021. Straddled optical solitons for cubic–quartic Lakshmanan–Porsezian–Daniel model by Lie symmetry. Phys. Lett. A. **417**: 127706.
13. Biswas A, Dakova A, Khan S, Ekici M, Moraru L and Belic M R, 2021. Cubic–quartic optical soliton perturbation with Fokas–Lenells equation by semi–inverse variation. Semicond. Phys. Quant. Electron. Optoelectron. **24**: 431–435.
14. Zayed E M, Nofal T A, Gepreel K A, Shohib R and Alngar M E, 2021. Cubic–quartic optical soliton perturbation with Lakshmanan–Porsezian–Daniel model. Optik. **233**: 166385.
15. Gepreel K A, Nofal T A and Althobaiti A A, 2012. The modified rational Jacobi elliptic functions method for nonlinear differential difference equations. J. Appl. Math. **2012**: 427479.
16. Islam M E, Khan K, Akbar M A and Islam R, 2013. Traveling wave solution of nonlinear

evolution equation via exp(–Φ(η)) – expansion method. Gl. J. Sci. Front. Res. Dec. Sci. **13**: 1–10.
17. Raslan K R, Khalid K A and Shallal M A, 2017. The modified extended tanh method with the Riccati equation for solving the space–time fractional EW and MEW equations. Chaos, Solitons & Fractals. **103**: 404–409.
18. Nuruddeen R I and Nass A M, 2018. Exact solitary wave solution for the fractional and classical GEW–Burgers equations: an application of Kudryashov method. Taibah Uni. J. Sci. **12**: 309–314.
19. Islam M T, Akbar M A and Azad M A K, 2015. A rational (G'/G)-expansion method and its application to modified KdV–Burgers equation and the (2+1)-dimensional Boussineq equation. Nonlin. Stud. **6**: 1–11.
20. Gepreel K A and Althobaiti A A, 2014. Exact solutions of nonlinear partial fractional differential equations using fractional sub-equations method. Indian J. Phys. **88**: 293–300.
21. Althobaiti A, Althobaiti S, El-Rashidy K and Seadawy A R, 2021. Exact solutions for the nonlinear extended KdV equation in a stratified shear flow using modified exponential rational method. Res. Phys. **29**: 104723.
22. Mahak N, Akram G, 2020. The modified auxiliary equation method to investigate solutions of the perturbed nonlinear Schrodinger equation with Kerr law nonlinearity. Optik. **207**: 164467.
23. Nuruddeen R I, 2018. Multiple soliton solutions for the (3+1) conformable space–time fractional modified Korteweg–de-Vries equations, Ocean J. Eng. Sci. **3**: 11–18.
24. Nuruddeen R I, Aboodh K S and Khalid K A, 2018. Analytical investigation of soliton solutions to three quantum Zakharov–Kuznetsov equations. Commun. Theor. Phys. **70**: 405–412.
25. Cattani C, Sulaiman T A, Baskonus H M and Bulut H, 2018. On the soliton solutions to the Nizhnik–Novikov–Veselov and the Drinfel'd–Sokolov systems. Opt. Quant. Electron. **50**: 138.
26. Chen H T and Hong-Qing Z, 2004. New double periodic and multiple soliton solutions of the generalized (2+1)-dimensional Boussinesq equation. Chaos Soliton & Fractals. **20**: 765–769.
27. Seadawy A R and Lu D, 2017. Bright and dark solitary wave soliton solutions for the generalized higher order nonlinear Schrodinger equation and its stability. Res. Phys. **7**: 43–48.
28. Islam M H, Khan K, Akbar M A and Salam M A, 2014. Exact traveling wave solutions of modified KdV–Zakharov–Kuznetsov equation and viscous Burgers equation. Springer Plus. **3**: 105.
29. Liu W, Zhang Y, Wazwaz A M and Zhou Q, 2019. Analytic study on triple-S, triple-triangle structure interactions for solitons in inhomogeneous multi-mode fiber. Appl. Math. Comp. **361**: 325–331.
30. Guan X, Liu W, Zhou Q and Biswas A, 2020. Some lump solutions for a generalized (3+1)-dimensional Kadomtsev–Petviashvili equation. Appl. Math. Comp. **366**: 124757.
31. Adomian G. Nonlinear stochastic operator equations. San Diego: Academic Press, 1986.
32. Adomian G. Solving frontier problems of physics. The decomposition method. Boston: Kluwer Academic Publishers, 1994.
33. Gepreel K A, Nofal T A and Althobaiti A A, 2014. Numerical solutions of the nonlinear partial fractional Zakharov–Kuznetsov equations with time and space fractional, Sci. Res.Ess. **9**: 471–482.
34. Shakhanda R, Goswami P and He J-H and Althobaiti A, 2021. An approximate solution of the time-fractional two-mode coupled Burgers equations. Fractal and Fractional. **5**: 196.
35. Cherruault Y, 1990. Convergence of Adomian's methods. Math. Comp. Model. **14**: 83–86.
36. Cherruault Y and Adomian G, 1993. Decomposition methods: a new proof of convergence, Math. Comp. Model. **18**: 103–106.

37. Al Qarni A A, Banaja M A and Bakodah H O, 2015. Numerical analyses optical solitons in dual core couplers with Kerr law nonlinearity. Appl. Math. **6**: 1957–1967.
38. Banaja MA, AlQarni AA, Bakodah HO, Zhou Qin, Moshokoa Seithuti P., Biswas Anjan, 2017. The investigate of optical solitons in cascaded system by improved adomian decomposition scheme. Optik, **130**: 1107-1114.
39. Mohammed A S H F and Bakodah H O, 2020. Numerical investigation of the Adomian-based methods with w-shaped optical solitons of Chen–Lee–Liu equation. Phys. Scripta. **96**: 035206.
40. Mohammed A S H F and Bakodah H O, 2021. Approximate solutions for dark and singular optical solitons of Chen–Lee–Liu Model by Adomian-based methods. Int. J. Appl. Comp. Math. **7**: 1–12.
41. Al-Qarni A A, Bakodah H O, Alshaery A A, Biswas A, Yildirim Y, Moraru L and Moldovanu S, 2022. Numerical simulation of cubic–quartic optical solitons with Perturbed Fokas–Lenells equation using improved Adomian decomposition algorithm. Mathematics. **10**: 138.
42. Vega-Guzman J, Biswas A, Kara A H, Mahmood M F, Ekici M, Alshehri H M and Belic M R, 2021. Cubic–quartic optical soliton perturbation and conservation laws with Lakshmanan–Porsezian–Daniel model: undetermined coefficients. J. Nonlin. Opt. Phys. Math. **30**: 2150007.

A. A. Al Qarni, A. M. Bodaqah, A. S. H. F. Mohammed, A. A. Alshaery, H. O. Bakodah and Anjan Biswas. 2022. Cubic–quartic optical solitons obtained with the Lakshmanan–Porsezian–Daniel equation by an improved Adomian decomposition scheme. Ukr.J.Phys.Opt. **23**: 228 – 242. doi: 10.3116/16091833/23/4/228/2022

Анотація. *Досліджено клас рівнянь Лакшманана–Порсезіана–Данієля, наділених кубічно-квартичною нелінійністю. В отриманні узагальненої числового підходу використано високоефективну покращену схему розкладання Адоміана. Наші чисельні результати виявляють ідеальну згоду з аналітичними рішеннями для оптики, відомими з літератури. Іншими словами, наш метод забезпечує вражаючий рівень точності та надійності.*

***Ключові слова**: оптичні солітони, модель Лакшманана–Порсезіана–Данієля, керрівська нелінійність, покращений метод розкладання Адоміана*