Highly dispersive optical soliton perturbation with Kudryashov's sextic-power law of nonlinear refractive index

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Abstract. We retrieve for the first time highly dispersive optical solitons with the Kudryashov's nonlinear form. The Hamiltonian perturbation terms with the maximum-intensity parameter are addressed. Twin-singular, dark, bright and singular soliton solutions are derived by means of two different integration schemes. These soliton solutions arise under certain restrictions on the parameters involved, which are analyzed in detail.

Keywords: solitons, Kudryashov's law, dispersive media, perturbations.

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1. Introduction

There is a wide range of forms describing a self-phase modulation, which have been studied in the field of nonlinear fibre optics. Recently, a number of new forms of this modulation have been suggested and successfully studied. One of them has been proposed by Nikolai Kudryashov in 2020. This nonlinear form involves a power-law nonlinearity parameter [1–4]. Up to date, there have been a few results reported for this nonlinear form and the corresponding results have been concerned with chromatic dispersion only.

In the present study, we consider a model combining the Kudryashov's law with six different dispersion effects: chromatic dispersion, inter-modal dispersion, fourth-order dispersion, third-order dispersion, sixth-order dispersion, and fifth-order dispersion. This case corresponds to highly dispersive optical solitons [5–8]. The governing model is a nonlinear Schrödinger's equation that contains six dispersive effects and six forms of nonlinearity. In addition, our model considers the perturbation terms associated with self-steepening effects and nonlinear dispersion, and involves the maximum intensity parameter. We will demonstrate that the F-expansion scheme and the approach of Riccati equation retrieve twin-singular, dark, bright and singular solitons. The restrictions imposed on the domains of model parameters for such solitons to exist (i.e., the constraint conditions) will also be identified.

2. Governing model

We suggest a novel structure of the governing nonlinear Schrödinger's equation with the perturbation terms:

$$\begin{aligned} & iq_t + ia_1 q_x + a_2 q_{xx} + ia_3 q_{xxx} + a_4 q_{xxxx} + ia_5 q_{xxxxx} + a_6 q_{xxxxxx} \\ & + \left(b_1 |q|^n + b_2 |q|^{2n} + b_3 |q|^{3n} + b_4 |q|^{4n} + b_5 |q|^{5n} + b_6 |q|^{6n} \right) q \\ & = i \left[\lambda \left(|q|^{2m} q \right)_x + \theta \left(|q|^{2m} \right)_x q + \mu |q|^{2m} q_x \right], \end{aligned} \tag{1}$$

where $q(x,t)$ is the complex-valued soliton profile and $i = \sqrt{-1}$. In Eq. (1), x and t are the non-dimensional distance and time variables, respectively. The first term in the Eq. (1) corresponds to linear temporal evolution, a_l $(1 \le l \le 6)$ are the coefficients of the inter-modal and chromatic dispersions, as well as the third-, fourth-, fifth- and sixth-order dispersions, respectively. Finally, b_l represent the coefficients of nonlinearity, λ, μ and θ imply respectively the coefficients of self-steepening, nonlinear dispersion and higher-order dispersion, and m and n denote the power-law exponent and the maximum-intensity parameter, respectively.

3. Mathematical analysis

To retrieve highly dispersive optical solitons, we set the wave transformation in the form

$$q(x,t) = Q^{\frac{1}{n}}(\xi) e^{i\varphi(x,t)}, \quad \xi = x - vt, \quad \varphi(x,t) = -\kappa x + \omega t + \theta_0, \tag{2}$$

where ξ is the wave variable, κ is the wave number, ω is the frequency, θ_0 is the phase constant, v is the velocity, $Q(\xi)$ is the amplitude component and $\varphi(x,t)$ is the phase component. Inserting Eq. (2) into Eq. (1) results in the ordinary differential equation

$$\begin{aligned} & a_6 (1-n)(1-3n)(1-2n)(1-5n)(1-4n)(Q')^6 - \kappa(\mu+\lambda) n^6 U^{\frac{2m}{n}+6} \\ & +15n(1-2n)(1-n)(1-4n)(1-3n) Q (Q')^4 Q'' \\ & +45n^2 (1-n)(1-3n)(1-2n) Q^2 (Q')^2 (Q'')^2 \\ & +20n^2 (1-n)(1-3n)(1-2n) Q^2 (Q')^3 Q''' + 10n^4 (1-n) Q^4 (Q''')^2 \\ & +60n^3 (1-n)(1-2n) Q^3 Q' Q'' Q''' + 15n^3 (1-n)(1-2n) Q^3 (Q'')^3 \\ & +15n^3 (1-n)(1-2n) Q^3 (Q')^2 Q^{(iv)} + 15n^4 (1-n) Q^4 Q'' Q^{(iv)} \\ & +6n^4 (1-n) Q^4 Q' Q^{(v)} + n^5 Q^5 Q^{(vi)} \\ & + \left(\left(-5\kappa^6 a_6 + \kappa a_1 - \omega \right) Q^6 + b_1 Q^7 + b_2 Q^8 + b_3 Q^9 + b_4 Q^{10} + b_5 Q^{11} + b_6 Q^{12} \right) n^6 = 0, \end{aligned} \tag{3}$$

along with the soliton velocity

$$v = a_1 - 6a_6 \kappa^5, \tag{4}$$

and the constraint conditions

$$\begin{aligned} & a_2 = 15 a_6 \kappa^4, a_3 = -20 a_6 \kappa^3, a_4 = -15 a_6 \kappa^2, \\ & a_5 = 6 a_6 \kappa, \lambda + \mu + 2m(\theta + \lambda) = 0, \end{aligned} \tag{5}$$

where $' = \frac{d}{d\xi}$, $'' = \frac{d^2}{d\xi^2}$, $''' = \frac{d^3}{d\xi^3}$, ${}^{(iv)} = \frac{d^4}{d\xi^4}$, ${}^{(v)} = \frac{d^5}{d\xi^5}$ and ${}^{(vi)} = \frac{d^6}{d\xi^6}$.

3.1. Method of Riccati equation

Using the homogeneous balance, one can admit the following formal solution of Eq. (3):

$$Q(\xi)=A_0+A_1V(\xi),\ \ A_1\neq 0, \tag{6}$$

where A_0 and A_1 are constants, and the newly introduced function $V(\xi)$ satisfies the Riccati equation

$$V'(\xi)=S_2V^2(\xi)+S_1V(\xi)+S_0,\qquad S_2\neq 0, \tag{7}$$

with S_0, S_1 and S_2 being constants. Substituting Eqs. (6) and (7) into Eq. (3) yields

$$m=3n, S_1=\frac{A_0{}^2S_2+A_1{}^2S_0}{A_0A_1}, b_1=\frac{a_6S_2\left(A_0{}^2S_2-A_1{}^2S_0\right)^5(n+2)\left(n^2+n+1\right)\left(n^2+3n+3\right)}{n^6A_1{}^6A_0{}^5}, \tag{8}$$

$$b_2=-\frac{S_2{}^2a_6\left(A_0{}^2S_2-A_1{}^2S_0\right)^4(n+1)\left(31n^4+90n^3+105n^2+60n+15\right)}{n^6A_1{}^6A_0{}^4}, \omega=\frac{\beta}{n^6A_1{}^6A_0{}^6},$$
$$b_3=\frac{10S_2{}^3a_6\left(A_0{}^2S_2-A_1{}^2S_0\right)^3(2n+1)(3n+2)\left(3n^2+3n+1\right)(n+1)}{n^6A_1{}^6A_0{}^3}, \tag{9}$$

$$b_4=-\frac{5S_2{}^4a_6\left(A_0{}^2S_2-A_1{}^2S_0\right)^2(3n+1)(2n+1)\left(13n^2+12n+3\right)(n+1)}{n^6A_1{}^6A_0{}^2},$$
$$b_5=\frac{3a_6S_2{}^5(2n+1)(5n+2)(4n+1)(3n+1)\left(A_0{}^2S_2-A_1{}^2S_0\right)(n+1)}{n^6A_0A_1{}^6}, \tag{10}$$

$$b_6=\frac{\kappa\lambda n^6A_1{}^6+\kappa\mu n^6A_1{}^6-120n^5S_2{}^6a_6-274n^4S_2{}^6a_6}{n^6A_1{}^6}$$
$$+\frac{-225n^3S_2{}^6a_6-85n^2S_2{}^6a_6-15nS_2{}^6a_6-S_2{}^6a_6}{n^6A_1{}^6}, \tag{11}$$

$$\beta=a_6\Big(A_0{}^{12}S_2{}^6-6A_0{}^{10}A_1{}^2S_0S_2{}^5+15A_0{}^8A_1{}^4S_0{}^2S_2{}^4-20A_0{}^6A_1{}^6S_0{}^3S_2{}^3+$$
$$15A_0{}^4A_1{}^8S_0{}^4S_2{}^2-6A_0{}^2A_1{}^{10}S_0{}^5S_2+A_1{}^{12}S_0{}^6-5\kappa^6n^6A_1{}^6A_0{}^6+\kappa n^6A_1{}^6\frac{a_1}{a_6}A_0{}^6\Big). \tag{12}$$

Using Eqs. (8)–(12) and well-known solutions of Eq. (7), one can retrieve the following results:

$$q(x,t)=\sqrt[n]{\frac{A_0{}^2S_2-A_1{}^2S_0}{2A_0S_2}}\left(1-\tanh\left[\frac{\left(A_0{}^2S_2-A_1{}^2S_0\right)\left(x-\left(a_1-6a_6\kappa^5\right)t\right)}{2A_0A_1}\right]\right)^{\frac{1}{n}}e^{i(-\kappa x+\omega t+\theta_0)}, \tag{13}$$

$$q(x,t)=\sqrt[n]{\frac{A_0{}^2S_2-A_1{}^2S_0}{2A_0S_2}}\left(1-\coth\left[\frac{\left(A_0{}^2S_2-A_1{}^2S_0\right)\left(x-\left(a_1-6a_6\kappa^5\right)t\right)}{2A_0A_1}\right]\right)^{\frac{1}{n}}e^{i(-\kappa x+\omega t+\theta_0)}, \tag{14}$$

where $\left(A_0{}^2S_2-A_1{}^2S_0\right)A_0S_2>0.$ Eqs. (13) and (14) represent dark and singular soliton solutions, respectively.

3.2. F-expansion approach

According to the homogeneous balance, Eq. (3) satisfies a formal solution

$$Q(\xi) = A_0 + A_1 F(\xi), \ \ A_1 \neq 0, \tag{15}$$

where A_0 and A_1 are constants, and the new function $F(\xi)$ satisfies the ordinary differential equation

$$F'(\xi) = \sqrt{PF^4(\xi) + QF^2(\xi) + R}, \tag{16}$$

with P, Q and R being constants. Substituting Eqs. (15) and (16) into Eq. (3) yields the following results.

Case 1.

$$m = 3n, P = \frac{Q^2}{4R}, \omega = -\frac{5\kappa^6 n^6 a_6 - \kappa n^6 a_1 + 8Q^3 a_6}{n^6}, A_0 = \pm\sqrt{\frac{2Q^3 a_6 (n+1)\alpha}{n^6 b_2}}, \tag{17}$$

$$A_1 = \pm\sqrt{-\frac{a_6 (n+1) Q^4 \alpha}{n^6 R b_2}}, b_1 = \pm\frac{2\sqrt{2b_2} Q^2 a_6 (n+2)\left(n^2+3n+3\right)\left(n^2+n+1\right)}{n^3 \sqrt{Q a_6 \alpha (n+1)}}, \tag{18}$$

$$b_3 = \pm\frac{5n^3 b_2 (3n+2)(2n+1)\left(3n^2+3n+1\right)\sqrt{2}}{2\alpha Q\sqrt{\frac{Q a_6 (n+1)\alpha}{b_2}}}, \tag{19}$$

$$b_4 = \frac{5(2n+1)(3n+1)\left(13n^2+12n+3\right)n^6 {b_2}^2}{8Q^3 a_6 (n+1)\alpha^2},$$

$$b_5 = \pm\frac{(15n+6)(2n+1)(3n+1)(4n+1)n^9 {b_2}^2\sqrt{2}}{(32n+32)\alpha^2 Q^4 a_6\sqrt{\frac{Q a_6 (n+1)\alpha}{b_2}}}, \tag{20}$$

$$\alpha = 31n^4 + 90n^3 + 105n^2 + 60n + 15,$$

$$\begin{aligned} b_6 = \frac{1}{64Q^6 {a_6}^2 (n+1)^2 \alpha^3}\Big(&120n^{16}{b_2}^3 + 154n^{15}{b_2}^3 + 71n^{14}{b_2}^3 + 14n^{13}{b_2}^3 + n^{12}{b_2}^3 \\ &+321995520Q^6\kappa\mu n^{11}{a_6}^2 + 703990080Q^6\kappa\mu n^{10}{a_6}^2 + 1137041280Q^6\kappa\mu n^9{a_6}^2 \\ &+1401065280Q^6\kappa\mu n^8{a_6}^2 + 1339891200Q^6\kappa\mu n^7{a_6}^2 + 1000814400Q^6\kappa\mu n^6{a_6}^2 \\ &+321995520Q^6\kappa\lambda n^{11}{a_6}^2 + 703990080Q^6\kappa\lambda n^{10}{a_6}^2 + 1137041280Q^6\kappa\lambda n^9{a_6}^2 \\ &+1401065280Q^6\kappa\lambda n^8{a_6}^2 + 1339891200Q^6\kappa\lambda n^7{a_6}^2 + 1000814400Q^6\kappa\lambda n^6{a_6}^2 \\ &+102703744Q^6\kappa\mu n^{12}{a_6}^2 + 102703744Q^6\kappa\lambda n^{12}{a_6}^2 + 20419328Q^6\kappa\mu n^{13}{a_6}^2 \\ &+20419328Q^6\kappa\lambda n^{13}{a_6}^2 + 1906624Q^6\kappa\mu n^{14}{a_6}^2 + 1906624Q^6\kappa\lambda n^{14}{a_6}^2 \\ &+581904000Q^6\kappa\mu n^5{a_6}^2 + 259675200Q^6\kappa\mu n^4{a_6}^2 + 86400000Q^6\kappa\mu n^3{a_6}^2 \\ &+20304000Q^6\kappa\mu n^2{a_6}^2 + 3024000Q^6\kappa\mu n{a_6}^2 + 581904000Q^6\kappa\lambda n^5{a_6}^2 \\ &+259675200Q^6\kappa\lambda n^4{a_6}^2 + 86400000Q^6\kappa\lambda n^3{a_6}^2 + 20304000Q^6\kappa\lambda n^2{a_6}^2 \\ &+3024000Q^6\kappa\lambda n{a_6}^2 + 216000Q^6\kappa\mu{a_6}^2 + 16000Q^6\kappa\lambda{a_6}^2\Big). \end{aligned} \tag{21}$$

Using Eqs. (17)–(21) and well-known solutions of Eq. (16), one derives

$$q(x,t)=\pm\sqrt[2n]{-\frac{16a_6(n+1)\alpha}{n^6 b_2}}\left(1+\tanh\left(x-\left(a_1-6a_6\kappa^5\right)t\right)\right)^{\frac{1}{n}}e^{i(-\kappa x+\omega t+\theta_0)}, \tag{22}$$

$$q(x,t)=\pm\sqrt[2n]{-\frac{16a_6(n+1)\alpha}{n^6 b_2}}\left(1+\coth\left(x-\left(a_1-6a_6\kappa^5\right)t\right)\right)^{\frac{1}{n}}e^{i(-\kappa x+\omega t+\theta_0)}, \tag{23}$$

$$q(x,t)=\pm\sqrt[2n]{-\frac{a_6(n+1)\alpha}{4n^6 b_2}}\left(1+coth\left(x-\left(a_1-6a_6\kappa^5\right)t\right)\pm csch\left(x-\left(a_1-6a_6\kappa^5\right)t\right)\right)^{\frac{1}{n}} \times e^{i(-\kappa x+\omega t+\theta_0)}, \tag{24}$$

where $a_6 b_2<0$. Eqs. (22)–(24) stand respectively for dark, singular and twin-singular soliton solutions.

Case 2.

$$Q=\frac{PA_0^4+RA_1^4}{A_0^2A_1^2},\ m=3n,\ n=2,\ b_5=\frac{8505P^3A_0a_6}{8A_1^6},$$
$$b_1\frac{7Pa_6\left(247P^2A_0^8+22R^2A_1^8-217PRA_0^4A_1^4\right)}{2A_0^3A_1^6}, \tag{25}$$

$$b_2=-\frac{21Pa_6\left(7733P^2A_0^8+93R^2A_1^8-3778PRA_0^4A_1^4\right)}{64A_0^4A_1^6},$$
$$b_4=\frac{105P^2a_6\left(1687PA_0^4-107RA_1^4\right)}{64A_0^2A_1^6}, b_3=\frac{75P^2a_6\left(389PA_0^4-85RA_1^4\right)}{8A_0A_1^6}, \tag{26}$$
$$b_6=\frac{64\kappa\lambda A_1^6+64\kappa\mu A_1^6-10395P^3a_6}{64A_1^6},$$

$$\omega=\frac{6845P^3A_0^{12}a_6-9291P^2RA_0^8A_1^4a_6+2511PR^2A_0^4A_1^8a_6}{64A_1^6A_0^6}$$
$$+\frac{-R^3A_1^{12}a_6-320\kappa^6A_1^6a_6A_0^6+64\kappa A_1^6a_1A_0^6}{64A_1^6A_0^6}. \tag{27}$$

Using Eqs. (25)–(27) and well-known solutions of Eq. (16), one can arrive at

$$q(x,t)=\pm\sqrt[4]{\frac{162393a_6}{64b_2}}\left(1+\text{sech}\left(x-\left(a_1-6a_6\kappa^5\right)t\right)\right)^{\frac{1}{2}}e^{i\left(-\kappa x-\left(5\kappa^6a_6-\kappa a_1+\frac{6845a_6}{64}\right)t+\theta_0\right)}, \tag{28}$$

where $a_6b_2>0$. Eq. (28) represents a bright soliton solution.

4. Conclusions

The present study retrieves for the first time the optical soliton solutions to the nonlinear Schrödinger's equation that involves six dispersion terms and six forms of nonlinear effects. The perturbation terms associated with the nonlinear dispersion and the self-steepening nonlinearity are also incorporated. Our theoretical findings have been substantiated using two different integration methodologies.

The optical solitons obtained in our work can be referred to as highly dispersive soliton solutions that exist whenever a specific relationship between the power-law parameter and the maximum-intensity parameter holds true. In particular, the bright soliton solutions become possible only at a specific value of the power-law nonlinearity parameter and, hence, a specific value of the maximum-intensity parameter. Finally, we have clarified the constraints imposed on the model parameters, which are necessary for our soliton solutions to exist.

Disclosure

The authors claim that there is no conflict of their interests.

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Анотація. *Уперше одержано високодисперсні оптичні солітони із нелінійної форми Кудряшова. Розглянуто гамільтонові члени збурення з параметром максимальної інтенсивності. За допомогою двох різних схем інтегрування одержано подвійні сингулярні, темні, яскраві та сингулярні солітонні розв'язки. Ці розв'язки з'являються за певних обмежень на модельні параметри, які детально проаналізовано нами.*