Peculiarities of acousto-optic diffraction at circularly polarized acoustic waves. Determination of elasto-optic coefficients coupled with shear waves

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Abstract. We develop a new approach for determining some of elasto-optic coefficients (p_{ij} with $i = 1\ldots6$ and $j = 4, 5$) basing on Dixon–Cohen method and acousto-optic diffraction at circularly polarized acoustic waves. Particular cases of crystals that belong to trigonal system and some symmetry groups of tetragonal and hexagonal systems are analyzed. We find that the effective elasto-optic coefficients are different for the alternative cases of diffractions at either right- or left-handed circularly polarized acoustic waves that propagate along Z axis in crystals. One can determine in this way the coefficients p_{44} and p_{45} at the anisotropic diffraction in the crystals belonging to point symmetry groups 4, 4/m, $\bar{4}$, 6, 6/m and $\bar{6}$. For the crystals belonging to symmetry groups 32, 3m and $\bar{3}\mathrm{m}$, it is possible to determine the coefficients p_{14} and p_{44} respectively at the isotropic and anisotropic diffractions. Finally, for the crystals described by the groups 3 and $\bar{3}$, one can determine separately the four coefficients p_{44}, p_{45}, p_{25} and p_{14} following from the anisotropic-diffraction data and the two coefficients p_{14} and p_{25} following from the isotropic-diffraction data.

Keywords: circularly polarized acoustic waves, acousto-optic diffraction, effective elasto-optic coefficients

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1. Introduction

Acousto-optic (AO) diffraction is a well known phenomenon, which consists in interaction of optical wave with the phase grating of refractive index caused by acoustic wave (AW) via elasto-optic effect [1]. The efficiency of AO Bragg diffraction is given by the relation

$$\eta = \sin^2\left(\frac{\pi}{\lambda_0 \cos\Theta_B}\sqrt{\frac{P_{ac}L}{2H}M_2}\right), \tag{1}$$

or, under the condition $\eta \ll 1$, by its simplified version,

$$\eta = \frac{\pi^2 L}{2\lambda_0^2 H \cos\Theta_B} M_2 P_{ac} . \tag{2}$$

Here λ_0 is the wavelength of optical radiation in vacuum, Θ_B the Bragg angle, P_{ac} the power of the AW, L the length of AO interaction, H the width of piezoelectric transducer, and M_2 the AO figure of merit. In fact, the diffraction efficiency is proportional to the AW power, where the proportionality coefficient is just the AO figure of merit. The latter is defined by a set of constitutive coefficients:

$$M_2 = \frac{n_i^3 n_d^3 p_{eff}^2}{\rho v^3}, \quad (3)$$

where n_i and n_d denote the refractive indices of respectively incident and diffracted optical waves, p_{eff} implies the effective elasto-optic coefficient (EEC), v the AW velocity, and ρ the material density. The other parameters entering Eqs. (1) and (2) are geometric or they depend on the acoustic and optical wavelengths, together with phase-matching conditions [2].

Hence, the efficiency of AO diffraction depends mainly on the material properties. Basing on Eqs. (2) and (3), Dixon R. W. and Cohen M. G. have developed an AO method for determining elasto-optic coefficients [3]. In general, it involves the studies of efficiency of AO diffraction in a given material with respect to the efficiency referred to some standard material. For example, one can determine the elasto-optic coefficients p_{ij} (with i, j = 1, 2, 3) using the diffraction at longitudinal AWs, while the coefficients p_{ii} (i = 4, 5, 6) can be measured basing on the diffraction at transverse AWs. The other coefficients are usually combined with each other so that they can be evaluated when utilizing a number of interaction geometries and different crystal cuts. As an example, the coefficients p_{14}, p_{25}, p_{44} and p_{45} are usually superimposed with the other coefficients even for such high-symmetry optically uniaxial crystals as those belonging to the point symmetry groups 3 and $\bar{3}$.

Notice that excitation of circular AWs produces shear mechanical deformations. For instance, the deformation components e_4 and e_5 arise when the AW propagates along Z axis. These components can lead to AO coupling that depend upon the elasto-optic coefficients p_{ij} (i = 1...6 and j = 4, 5). Recently, we have reported that the case of diffraction of circularly polarized optical waves at AWs in optically active crystals represents a separate area of acousto-optics [4–6]. In particular, when the circular optical waves represent eigenwaves in a material medium, one can consider the isotropic and anisotropic types of AO coupling between two left-handed (LH) (or right-handed (RH)) optical waves or, alternatively, between the RH and LH optical waves. This situation can happen, e.g., in the crystals that belong to gyrotropic cubic point symmetry groups or at the optical wavelength that corresponds to a so-called isotropic point in anisotropic crystals [7]. If the isotropic approximation [6] holds true, the same situation can also be realized in the vicinity of optic axes.

From this point of view, the situation described above is very similar to the AO interaction between linearly polarized optical eigenwaves, i.e. the interaction between the two ordinary (or extraordinary) waves or between the ordinary and extraordinary waves, under condition that the AWs are circularly polarized. As far as we know, the diffraction at the circular AWs has not yet been analyzed in the literature. In the present work, we will show that, in crystals of some particular symmetries, the diffractions occurring at the RH and LH AWs can differ from each other. Moreover, we will demonstrate that it is possible to determine some of the elasto-optic coefficients with the Dixon–Cohen method, when using the circular AWs.

2. Results and discussion

Let us consider propagation of a circularly polarized AW along the acoustic axis in a crystal. For example, this can be a direction of four-fold symmetry axis in paratellurite. The transverse AWs propagating along this direction (i.e., along the Z axis) have the same velocities that do not depend on the orientation of displacement vector in the XY plane. Hence, these waves suffer no linear acoustic birefringence. The circular acoustic birefringence, which is caused by the acoustic activity, can be ignored since it cannot manifest itself in our case. When the incident optical wave propagates along the *X* (or *Y*) axis and diffracts at a thick acoustic grating in tangential Bragg regime, one can consider the two alternative interaction planes, *XZ* and *YZ* (see Fig. 1).

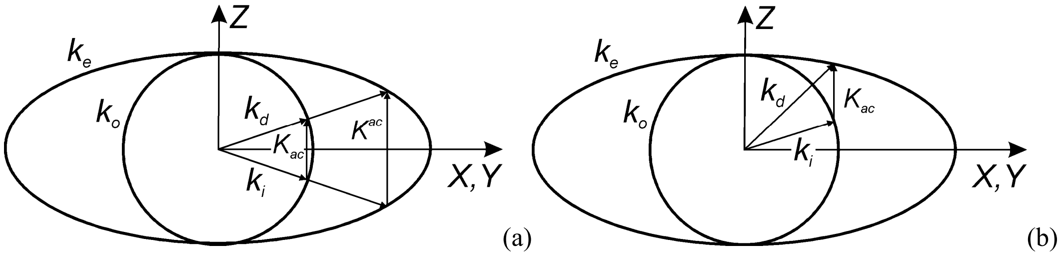

Fig. 1. Schematic view of AO interactions in optically positive crystals, which arise in the cases of isotropic (a) and anisotropic (b) diffractions.

The components of deformation tensor for the RH and LH AWs can be represented by the systems of equations

$$\begin{cases} e_5^{RH} = -e_0 \cos\delta \\ e_4^{RH} = e_0 \sin\delta \end{cases} \text{and} \begin{cases} e_5^{LH} = e_0 \cos\delta \\ e_4^{LH} = e_0 \sin\delta \end{cases}, \tag{4}$$

where e_0 is the unit amplitude of deformation and δ its phase. In the case of isotropic diffraction inside the *XZ* and *YZ* interaction planes, the EEC is determined by the elasto-optic coefficients p_{14}, p_{15}, p_{24}, p_{25}, p_{34} and p_{35}, which are equal to zero for the symmetry group 422. Thus, the isotropic AO interaction (see Fig. 1a) with the circular AW in this experimental geometry cannot be implemented in the paratellurite crystals.

Consider now the anisotropic diffraction (see Fig. 1b) of the ordinary incident optical wave at the RH AW inside the *XZ* interaction plane. Then the system of equations for the electric-field components of the diffracted wave can be written as

$$\begin{cases} E_1 = \Delta B_{12} D_2 = (p_{64} e_4^{RH} + p_{65} e_5^{RH}) D_2 = 0 \\ E_3 = \Delta B_{32} D_2 = (p_{44} e_4^{RH} + p_{45} e_5^{RH}) D_2 = p_{44} \sin\delta D_2 \end{cases}, \tag{5}$$

where ΔB_{ij} denotes the increment of optical impermeability tensor and D_j is the unit electrical induction of the incident optical wave. After averaging the phase over period ($\overline{\delta} = \pi / 4$), one gets the EEC equal to $p_{eff} = \frac{\sqrt{2}}{2} p_{44}$, since the relations p_{45}, p_{64}, $p_{65} = 0$ hold true. Under the same diffraction conditions, we still have the same EEC for the case of LH AW.

At the anisotropic diffraction of ordinary incident optical wave at the RH AW inside the interaction plane *YZ*, the electric field of the diffracted optical wave reads as

$$\begin{cases} E_2 = \Delta B_{21} D_1 = (p_{64} e_4^{RH} + p_{65} e_5^{RH}) D_1 = 0 \\ E_3 = \Delta B_{31} D_1 = (p_{54} e_4^{RH} + p_{55} e_5^{RH}) D_1 = -p_{55} \cos\delta D_1 \end{cases}. \tag{6}$$

Since we have $p_{55} = p_{44}$, the EEC is equal to $p_{eff} = -\frac{\sqrt{2}}{2} p_{44}$. The EEC obtained for the alternative case of diffraction at the LH AW is equal to $p_{eff} = \frac{\sqrt{2}}{2} p_{44}$, i.e. the module of the EEC remains the same. Hence, the EEC modules are the same for the *XZ* and *YZ* interaction planes in

TeO_2. In other words, these parameters are invariant under changing sign of the circular AW.

Nonetheless, the AO figures of merit can differ for the cases of diffractions at the LH and RH AWs, since these waves differ by their propagation velocity due to acoustic activity. Then it is not necessary to excite the circular AW for determining the p_{44} coefficient or, in some other terms, it is enough to excite one of the transverse waves. The same is true for the symmetry groups 4/mmm, 4mm, $\bar{4}2\text{m}$, 622, 6mm, 6/mmm and $\bar{6}\text{m}2$ that belong to either tetragonal or hexagonal systems.

Let us now consider the crystals belonging to the symmetry groups 4, 4/m, $\bar{4}$, 6, 6/m and $\bar{6}$, and the case of anisotropic diffraction in the geometry illustrated in Fig. 1b. The electric field of the diffracted wave is given by

$$E_3 = \Delta B_{32} D_2 = (p_{44} e_4^{RH} + p_{45} e_5^{RH}) D_2 = \frac{\sqrt{2}}{2} (p_{44} - p_{45}) D_2 \quad (7)$$

when the optical wave diffracts at the RH AW inside the XZ plane. The diffraction at the LH AW under the same conditions yields in the relation

$$E_3 = \Delta B_{32} D_2 = (p_{44} e_4^{RH} + p_{45} e_5^{RH}) D_2 = \frac{\sqrt{2}}{2} (p_{44} + p_{45}) D_2 . \quad (8)$$

Then the EEC is equal to $p_{eff} = \frac{\sqrt{2}}{2}(p_{44} - p_{45})$ in the first case, while we have $p_{eff} = \frac{\sqrt{2}}{2}(p_{44} + p_{45})$ in the second case. Hence, the two above measurements can enable determining separately the coefficients p_{44} and p_{45}. Moreover, here the diffraction efficiencies differ for the cases of diffractions at the RH and LH AWs, since they are determined by different EECs.

Let us consider the diffraction at the circular AW in the crystals that belong to the trigonal groups 32, 3m and $\bar{3}\text{m}$. The electric field of the optical wave diffracted at the RH AW in the case when isotropic interaction between the ordinary optical waves occurs inside the XZ plane is as follows:

$$E_2 = \Delta B_{22} D_2 = (p_{24} e_4^{RH} + p_{25} e_5^{RH}) D_2 = -\frac{\sqrt{2}}{2} p_{14} D_2 ,\; p_{eff} = -\frac{\sqrt{2}}{2} p_{14} . \quad (9)$$

The EEC modules remain the same when the isotropic interaction inside the YZ plane occurs or when the optical wave diffracts at the LH AW. Then one can determine the elasto-optic coefficient p_{14}. Finally, the EEC is equal to zero at the isotropic interaction between the extraordinary optical waves. The reason is the absence of appropriate elasto-optic coefficients in the four-rank tensor.

Let us proceed to the cases of anisotropic diffraction inside the XZ and YZ planes in the crystals belonging to the symmetry groups mentioned above. Let the optical waves diffract at the RH or LH AWs. Then the EEC is defined as $p_{eff} = \frac{\sqrt{2}}{2}(p_{14}^2 \sin^2\varphi + p_{44}^2 \cos^2\varphi)^{1/2}$, where φ is the angle between the X (or Y) axis and the wave vector of incident optical wave. Having determined the p_{14} coefficient value in the isotropic-diffraction study, one can also found the p_{44} coefficient.

Finally, we consider the crystals that belong to the point symmetry groups 3 and $\bar{3}$. Let the isotropic interaction between the ordinary optical waves take place inside the XZ and YZ planes. The EEC modules calculated for the cases of diffractions at the RH and LH AWs read respectively as

$$\left|p_{eff}\right| = \frac{\sqrt{2}}{2}(p_{25} + p_{14}) \text{ and } \left|p_{eff}\right| = \frac{\sqrt{2}}{2}(p_{25} - p_{14}) . \quad (10)$$

This fact enables one to determine the elasto-optic coefficients p_{14} and p_{25} separately. Note that the EECs and, hence, the efficiencies of AO diffractions differ for the cases of interactions with the RH and LH AWs. On the other hand, the EEC is equal to zero when the extraordinary optical waves interact with each other and with circular AWs.

The EEC for the case of anisotropic interaction inside the *XZ* plane and diffraction at the RH AW is as follows:

$$p_{eff} = \frac{\sqrt{2}}{2}\left((p_{44} - p_{45})^2\cos^2\varphi + (p_{25} - p_{14})^2\sin^2\varphi\right). \quad (11)$$

For the *YZ* plane we have

$$p_{eff} = \frac{\sqrt{2}}{2}\left((p_{44} + p_{45})^2\cos^2\varphi + (p_{25} - p_{14})^2\sin^2\varphi\right). \quad (12)$$

Notice that the EECs differ for the *XZ* and *YZ* interaction planes.

The EEC obtained for the cases of diffractions at the LH AW are given by

$$p_{eff} = \frac{\sqrt{2}}{2}\left((p_{44} + p_{45})^2\cos^2\varphi + (p_{25} + p_{14})^2\sin^2\varphi\right) \quad (13)$$

for the *XZ* plane and

$$p_{eff} = \frac{\sqrt{2}}{2}\left((p_{44} - p_{45})^2\cos^2\varphi + (p_{25} + p_{14})^2\sin^2\varphi\right) \quad (14)$$

for the *YZ* plane. As a consequence, one can determine the four coefficients p_{44}, p_{45}, p_{25} and p_{14} separately by determining the combinations of coefficients given by Eqs. (11)–(14). Notice that, again, the EECs differ for the cases of AO interactions with the RH and LH AWs.

To end up our discussion, we would like to remind that the circular AWs can be excited in different ways, e.g., when using acoustic quarter-wave plates [8] or a Fresnel parallelepiped [9]. However, a known effect of conical refraction usually manifests itself in the trigonal crystals whenever the circular AWs propagate along the three-fold axis. The angle of the internal conical refraction depends on relationships among the elastic-stiffness coefficients C_{ij}. It can vary notably for different crystals. For instance, this angle can approach zero (e.g., in $Pb_5Ge_3O_{11}$, since we have $C_{14} \approx 0$ [10]) or reach high enough values (e.g., 17.17 deg in quartz or even 30.75 deg in calcite [11]). In the case of internal conical acoustic refraction, a conically shaped acoustic energy walk-off arises, so that the orientation of linear polarization of the AW reveals azimuthal dependence on the angle π around the ring of refraction. Therefore, special requirements must be met in order to satisfy the conditions necessary for the AO diffraction at the circular AWs in such crystals. Namely, in the case of tangential Bragg AO interactions, the numerical aperture of the incident optical beam should be large enough for covering the aperture of a conical acoustic beam.

3. Conclusion

Issuing from the results of the present work, one can conclude that the AO diffraction at the circular AWs can be considered as an additional approach, which is useful when determining the elasto-optic coefficients via the Dixon–Cohen technique. Using this approach, one can find separately some of the elasto-optic coefficients (p_{ij} with $i = 1\ldots6$ and $j = 4, 5$). In particular, one can measure the coefficients p_{44} and p_{45} under conditions of anisotropic diffraction in the crystals belonging to the symmetry groups 4, 4/m, $\overline{4}$, 6, 6/m and $\overline{6}$. For the crystals that belong to the trigonal symmetries 32, 3m and $\overline{3}$m, it is possible to find the coefficients p_{14} and p_{44} respectively at the isotropic and anisotropic diffractions. For the crystals described by the groups 3 and $\overline{3}$, one can find separately the four coefficients p_{44}, p_{45}, p_{25} and p_{14}, using the studies of anisotropic diffraction. Finally, the coefficients p_{14} and p_{25} for the same crystals can be evaluated following from the isotropic-diffraction data. It is also worth noting that the EECs can be defined by different relationships for the cases of diffraction either at the RH AW or at the LH AW.

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Анотація. *Розроблено новий підхід до визначення деяких із еластооптичних коефіцієнтів (p_{ij} із $i = 1...6$ та $j = 4, 5$) на основі методу Діксона–Коена та акустооптичної дифракції на циркулярно поляризованих акустичних хвилях. Проаналізовано конкретні випадки кристалів, що належать до тригональної системи і деяких груп симетрії тетрагональної та гексагональної систем. Виявлено, що ефективні еластооптичні коефіцієнти різні для альтернативних випадків дифракції на правих або лівих акустичних хвилях, які поширюються вздовж осі Z у кристалах. Так можна визначити коефіцієнти p_{44} і p_{45} при анізотропній дифракції в кристалах, що належать до точкових груп симетрії 4, 4/m, $\bar{4}$, 6, 6/m і $\bar{6}$. Для кристалів, що належать до груп симетрії 32, 3m і $\bar{3}m$, можна знайти коефіцієнти p_{14} і p_{44} відповідно при ізотропній і анізотропній дифракції. Нарешті, для кристалів, що описуються групами 3 і $\bar{3}$, можна окремо визначити чотири коефіцієнти p_{44}, p_{45}, p_{25} і p_{14} на підставі даних для анізотропної дифракції і два коефіцієнти p_{14} і p_{25} на підставі даних для ізотропної дифракції.*