On the conservation of optical angular momentum at acoustogyration diffraction of light

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Abstract. A phenomenon of acoustogyration diffraction of light is analysed from the viewpoint of conservation of orbital angular momentum. It is shown that the availability of optical angular momentum in the diffracted optical beam can be necessarily inferred from the requirements of angular-momentum conservation law. As follows from our analysis, circularly polarised diffracted wave should bear a doubly charged vortex, while its wave front should reveal a singularity typical of scalar fields. The efficiency of the spin-to-orbit momentum conversion is governed by the efficiency of acoustogyration diffraction of light.

Keywords: acoustogyration diffraction, conservation of optical angular momentum, optical vortex

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In our recent works a new optical effect of acoustogyration diffraction has been predicted and described following from the standpoints of both electrodynamics and wave optics [1–4]. It has been sown that, in some particular cases, this effect manifests itself as collinear interaction between the longitudinal acoustic wave and (left- or right-handed) circularly polarised plane optical wave propagating along the optic axis in optically uniaxial crystals. As a matter of fact, the acoustogyration interaction appears because the acoustic wave creates a phase grating, in which the imaginary part of optical-frequency impermeability tensor is modulated due to a piezogyration (elastogyration) effect (see, e.g., [5, 6]). In other words, we deal here with the spatial grating (or, in somewhat other terms, with spatial modulation) of the gyration tensor. In the particular case considered in the works [1–4], a collinear diffraction geometry for both the incident (the subscript '*in*') and diffracted (the subscript '*d*') light waves takes place, whereas the longitudinal acoustic (the subscript '*ac*') wave propagates along the same direction ($K_{ac} \parallel k_{in} \parallel k_d$). The relations for the electric field of the wave diffracted in a uniaxial, optically active crystal may be written in the following form:

$$\begin{aligned} \Delta E_1^{\omega\pm\Omega} &= (p_{1233}S_{33}^{\Omega} + ie_{123}\Delta g_{33}k_3)D_2^{\omega} \\ \Delta E_2^{\omega\pm\Omega} &= (p_{2133}S_{33}^{\Omega} + ie_{213}\Delta g_{33}k_3)D_1^{\omega} \end{aligned} , \tag{1}$$

where ΔE_1^{ω} and ΔE_2^{ω} denote respectively the *X*- and *Y*-components of the electric field of the diffracted wave, S_{33}^{Ω} the mechanical strain caused by the acoustic wave with the frequency Ω , D_1^{ω} and D_2^{ω} are respectively the electric displacement of the incident wave, p_{1233} and p_{2133} the photoelastic coefficients ($p_{1233} = p_{2133} = 0$ for optically uniaxial crystals), k_3 the light wave vector, Δg_{33} the increment of the gyration tensor acquired due to elastogyration effect (with δ_{3333} being a fourth-rank axial elastogyration tensor), and $e_{123} = -e_{213} = 1$ stand for the components of the unit Levi-Civita axial tensor. Now let us take into account the relationships $p_{1233} = p_{2133} = 0$ and $\Delta g_{33} = \delta_{3333} S_{33}^{\Omega}$ satisfied in our case. As a result, we rewrite Eqs. (1) as

$$\begin{aligned} \Delta E_1^{\omega} &= i e_{123} \delta_{3333} S_{33}^{\Omega} k_3 D_2^{\omega} \\ \Delta E_2^{\omega} &= -i e_{123} \delta_{3333} S_{33}^{\Omega} k_3 D_1^{\omega} \end{aligned} . \tag{2}$$

As one can easily see, the anisotropic diffraction in the mentioned case can be affected only by the acoustogyration interaction, while the acoustooptic interaction should not occur at all. Moreover, the sign ‘minus’ occurring in the second line of Eq. (2) suggests that the diffracted optical radiation would possess a circular polarisation opposite to that of the incident one. Otherwise, if the incident radiation is polarised linearly, the collinear acoustogyration diffraction would manifest itself in energy transfer between the circularly polarised eigenwaves, while the ‘direction’ of that energy transfer (from the left-handed wave to right-handed one, or vice versa) is determined by the sign of optical activity.

As shown earlier by the authors [1–4], the diffraction efficiency η in the conditions of acoustogyration diffraction is determined by the relation

$$\eta = \sin^2 \left[\frac{\pi}{2\lambda_0} \sqrt{\frac{2\delta_{3333}^2 P_a L}{\rho v_{33}^3 n^2 H}} \right], \tag{3}$$

where L represents the interaction length of the acoustic and optical waves, n the refractive index, and λ_0 the wavelength of optical radiation. In our case $S_{33} = \sqrt{2P_a / \rho v_{33}^3 LH}$, with P_a being the acoustic wave power, ρ the density of crystal, v_{33} the acoustic wave velocity, and H the width of the acoustic beam. Here the acoustogyration figure of merit is given by

$$M_{ag} = \frac{\delta_{kn33}^2}{\rho v_{33}^3 n^2} . \tag{4}$$

Thus, the acoustogyration diffraction efficiency can finally be presented as

$$\eta = \sin^2 \left[\frac{\pi}{2\lambda_0} \sqrt{2 M_{ag} P_a \frac{L}{H}} \right] \approx \frac{\pi^2}{2\lambda_0^2} M_{ag} P_a \frac{L}{H} . \tag{5}$$

In fact, the acoustogyration diffraction effect manifests itself as a diffraction of, e.g., the right-handed optical photon (the photon with a spin angular momentum (SAM) equal to $S^{in} = \hbar$) at the longitudinal acoustic phonon (the corresponding SAM and orbital angular momentum (OAM) are equal to zero), with the appearance of the left-handed diffracted photon (the photon with the SAM equal to $S^{d} = -\hbar$). Then the spin angular momentum is changed as $\hbar \to -\hbar$. In such a case the total angular momentum changes its value by $2\hbar$.

It is necessary to notice that the spin angular momentum of a circularly polarised wave cannot be transformed into a mechanical momentum of sample, due to the Beth effect [7], whenever the light propagates along optically isotropic directions. However, the principle of conservation of the total angular momentum requires equality of the momentum before the acoustogyration interaction and after the process is completed. Thus, the diffracted photon should possess an additional optical angular momentum equal to $2\hbar$, which should obviously correspond to orbital angular momentum. The latter is acquired as a result of acoustogyration interaction (i.e., $L^d = 2\hbar$). Then the equations describing conservation of the total angular momentum can be written as

$$\begin{aligned} S^{in} &= \hbar \\ S^d + L^d &= -\hbar + 2\hbar = \hbar \end{aligned} . \tag{6}$$

This means that the diffracted (left-handed) circularly polarised wave should bear a doubly charged vortex, while its wave front should possess a singularity which is proper of scalar fields. Therefore the diffracted wave should possess a helical wave front, though the zero-order wave should remain plane one in case if the incident wave is the same. It is evident that the efficiency of such a spin-to-orbit conversion is determined by the same relation as the efficiency of acoustogyration light diffraction, i.e. by Eq. (5). It is interesting to notice that the OAM sign for the diffracted beam is opposite to that of its SAM.

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Анотація. *В роботі проаналізоване явище акустогіраційної дифракції світла з точки зору збереження оптичного кутового моменту. Показано, що наявність орбітального кутового моменту дифрагованого променя обов'язково випливає з умови виконання закону збереження кутового моменту імпульсу. З наведеного аналізу випливає, що циркулярно поляризований дифрагований промінь повинен переносити оптичний вихор з подвійним зарядом тоді як його хвильовий фронт повинен містити сингулярність скалярного поля. При цьому ефективність спін-орбітального перетворення відповідатиме ефективності акустогіраційної дифракції світла.*