Singularities of Poynting Vector and the Structure of Optical Fields

I. Mokhun, A. Mokhun and Ju. Viktorovskaya

Chernivtsy University, 2 Kotsybinsky St., 58012 Chernivtsy, Ukraine
e-mail: mokhun@itf.cv.ua

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Abstract

Singularities of Poynting vector are considered for scalar and vector fields. Behaviour of the averaged and instantaneous components is analyzed. Relationships between the Poynting singularities and conventional optical ones and connection with the other special sets of electromagnetic fields are established. Elementary topological regularities and reactions are formulated. The results of computer simulations are presented.

Key words: vortex, polarization singularities, Poynting vector, angular momentum.

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1. Introduction

Propagation of coherent radiation through inhomogeneous media with random fluctuations of local optical characteristics results in forming optical waves characterized by random temporal and spatial distributions of their parameters, such as the intensity, phase and, in a general case, the state of polarization. The fields formed in such a way are referred to as speckle-fields [1,2]. Since the parameters of speckle-fields are described by complex functions of general form, one can expect that diverse peculiarities (singularities and stationary points) are inherent to them, both point-like and extended ones. Under paraxial approximation, optical singularities are divided into the following groups:

1. The optical (or phase) vortices in the scalar fields (the points with a zero intensity) [3-7]. The field phase is indeterminate in these points.

2. The polarization singularities: C-points (field points with a circular polarization) and s-contours (closed lines where the field is linearly polarized) in the vector fields [3,8-13]. The polarization azimuth α and the vibration phase Φ_V (which defines the field vector position with respect to the major axis of the polarization ellipse) are singular in the C-points. The direction of the field vector rotation is indeterminate along the s-contours.

Both the vortices of scalar fields and the C-points of vector fields are characterized by the two topological parameters:

1. The topological charge [14]

$$S = \frac{1}{2\pi}\oint df , \qquad (1)$$

where f means the field phase Φ for the vortex and the vibration phase Φ_V for the C-point. The integration is performed along a small circle that surrounds the singular point, in the counter-clockwise direction. The stable field structures are characterized by the vortex charge $S = \pm 1$ [14] and the C-point charge $S_C = \pm 1/2$ [12].

2. The topological index (the so-called Poincare index) [14], which is calculated in the following way. Under circumference (clockwise or counter-clockwise) of singular point, one determines the direction of rotation of the lines

associated with the quantity under interest (to say, equi-phase lines in the scalar fields). If the rotation direction for these lines coincides with the circumference direction, then one prescribes the sign "+" to the index. Contrary, if the direction of the line rotation is opposite to the circumference direction, then the sign "–" is prescribed to the index. The magnitude (modulus) of the Poincare index is equal to the number of full rotations of the lines calculated for the closed loop.

The positive and negative vortices are characterized by the same Poincare index $N=+1$, while we have $N=-1$ for the saddle point. Both the phase extrema and the vortices have the Poincare index $N=+1$ [14]. The Poincare index I_C for the C - point (further on – simply the C - point index) is equal to $\pm 1/2$ [3]. It is calculated from the rotation direction of the polarization ellipse axes surrounding the C - point. We shall call the C -points with positive (or negative) indices as positive (or negative) C -points. As shown in [12],

$$I_C = hS_C , \qquad (2)$$

where $h=\pm 1$ is the handedness factor that defines the direction of rotation of the field vector (i.e., the right or left polarization) in the analyzed area.

The singularities and stationary points are interconnected in the form of peculiar nets [14-19]. These nets, like field skeletons, constitute the field structure, and the information on the characteristics of these sets enables predicting behaviour of the field at any point, at least qualitatively. At the same time, the field within the area of optical singularity is *absolutely smooth* and includes no discontinuities, strictly obeying the Maxwell equations, etc. For example, indeterminacy of the phase at the centre of phase vortex is, generally speaking, meaningless, while the corresponding amplitude is zero.

Similar considerations are valid for the polarization singularities, viz. the s-contours and the C - points [3]. Indeed, whereas the rotation of axes of the polarization ellipses at some distance from the C - point characterizes a difference in polarization characteristics of the field, the ellipses in the nearest vicinity of the singularity (C - point) differ negligibly from a circle (see Fig. 1) and the very notions of the azimuth and vibration phase, as well as a notion of the phase at the vortex centre, are illegible. In other words, the parameters such as the vibration phase and the azimuth are "unnecessary" for description of the field at the C - point, and the rotation direction of the field vector turns out to be "superfluous" characteristic for the points of the s-contour. Besides, temporal behaviour of the field vector at the C -point (or the s-contour) and close to these elements of the field is almost the same. Conventional optical measurements would not provide discrimination between the points belonging to a singular set and the points lying in the nearest vicinity of this set. These areas are schematically depicted in Fig. 1 respectively as regions A and B. Moreover, optical singularities could only be detected by using indirect interferometric techniques or analyzing the fields resulted from superposition of a singular structure under interest with any reference field [6,20-22].

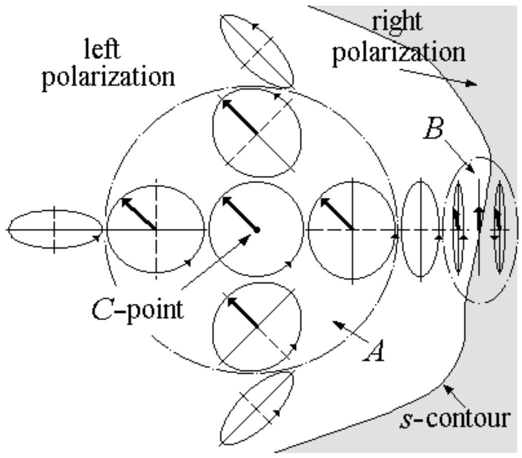


Fig. 1. Behaviour of polarization characteristics in immediate proximity to polarization singularities.

It seems that the above considerations should lead to obvious conclusion that the only reason for studies of optical singularities would be related to their role in the formation of structure of light fields, both scalar and vector ones. On the other hand, the presence of singularity in any field parameter unavoidably gives rise to some physical peculiarities of that field in its vicinity. Then a question arises: what is a physical manifestation of optical singularities or, in other words, which is a specific behaviour of physical system under the influence of electromagnetic wave?

It is known that the specific behaviour of the physical system in the case of scalar fields is associated with the orbital angular momentum of electromagnetic field existing in the vicinity of optical vortex [23]. Such the momentum arises due to a specific (helicoidal) phase surface in the vicinity of the vortex and, generally speaking, due to peculiar temporal behaviour of the field. In other words, physical manifestation of the scalar singularity is reflected in peculiar temporal behaviour of the electromagnetic field components. In the end, polarization singularities must also be considered as temporal peculiarities of the field: the polarization azimuth, the vibration phase and the sense of rotation of the field vector determine both the spatial and temporal behaviour of the field. It is clear that physical manifestations of the vector singularities should be linked to a temporal behaviour of the field, which, after all, is reduced to a specific magnitude of the angular momentum of electromagnetic field in the vicinity of those singularities, as well as to some behaviour of this characteristic in the mentioned area, which is different from that inherent to the other areas of the field.

It is common knowledge (see, e.g., [23, 24]) that a ponderomotive effect of electromagnetic wave on a physical system is associated with the Poynting vector. In any case, spatial distribution of the parameters of this vector, such as its magnitude and orientation, is one of the main factors determining mechanical influence of the wave on that system. Besides, the characteristics of the Poynting vector are directly related to the angular momentum (see [24]). Naturally, the characteristics of the Poynting vector components for the fields of general form (including the modulus and orientation of its transversal component) can be considered as some spatially distributed parameters of that field. In general, these distributions would possess singularities. Similarly to the common optical singularities, the Poynting vector singularities can be obviously connected into nets, which must determine, at least in a qualitative manner, behaviour of the Poynting vector anywhere. In other words, they must form a skeleton of the field and determine the regularities governing spatial distributions of the field parameters.

Similarly to the distributions of the phase and intensity or the polarization and intensity considered above (see [17,18,25,26]), the characteristics of such the singular sets and the behaviour of the Poynting vector are expected to be associated with the characteristics of nets of the common singularities.

Taking into account the above reasoning, one can conclude that the analysis of the Poynting vector singularities and establishing the corresponding topological regularities are highly relevant problems. In addition, the applied aspects of this consideration are strictly linked to research and development of so-called optical tweezers representing one of urgent topics in the modern optics (see, e.g., [27]).

2. General assumptions. Components of the Poynting vector

Let us perform our analysis in the paraxial approximation. In contrast to the traditional approach [23,28], we will consider not only the time-averaged Poynting vector, but also the instantaneous one. The following reasons justify such the consideration:

1. Temporal averaging has a sense for the optical waves alone, because of too rapid temporal changes of fields. Concerning the electromagnetic waves of radio frequency

domain, the oscillation period is often comparable with the relaxation time of physical systems. In this case, the influence of the wave on the system is determined by the behaviour of *non-averaged Poynting vector* or, at least, by the behaviour of this vector averaged over much smaller time interval δt. Similarly, the concept of the Nye disclination is a *fundamental notion* for radio waves, which *loses its fruitfulness* in optics [3,11].

2. Let us note also that the consideration of temporal behaviour of the Poynting vector components provides additional information necessary for *deeper understanding* of the processes leading to formation of the averaged Poynting vector.

Let z axis be coinciding with the prevailing direction of the wave energy propagation. The orientation of x and y axes is not relevant and may be specified arbitrary. It can be shown [29,30] that the following relations for the Poynting vector components are valid under the paraxial approximation:

$$\begin{cases} P_x \approx -\dfrac{c}{4\pi k}\{E_x T_2 - E_y T_1\} \\ P_y \approx -\dfrac{c}{4\pi k}\{E_y T_2 + E_x T_1\} \\ P_z \approx \dfrac{c}{4\pi}\{E_x^2 + E_y^2\} \end{cases}, \tag{3}$$

where

$$\begin{cases} T_1 = E_x \dfrac{\partial \Phi_x}{\partial y} - E_y \dfrac{\partial \Phi_y}{\partial x} + \dfrac{1}{A_x}\dfrac{\partial A_x}{\partial y} E_{x,\frac{\pi}{2}} - \dfrac{1}{A_y}\dfrac{\partial A_y}{\partial x} E_{y,\frac{\pi}{2}} \\ T_2 = E_x \dfrac{\partial \Phi_x}{\partial x} + E_y \dfrac{\partial \Phi_y}{\partial y} + \dfrac{1}{A_x}\dfrac{\partial A_x}{\partial x} E_{x,\frac{\pi}{2}} + \dfrac{1}{A_y}\dfrac{\partial A_y}{\partial y} E_{y,\frac{\pi}{2}} \end{cases} \tag{4}$$

and

$$\begin{cases} E_i = A_i \cos(\omega t + \Phi_i - kz) \\ E_{i,\frac{\pi}{2}} = A_i \sin(\omega t + \Phi_i - kz) \end{cases}. \tag{5}$$

Here A_i and Φ_i are respectively the amplitudes and the phases of the electric field components, c the velocity of light, $k = \dfrac{2\pi}{\lambda}$ the wave number, ω the circular frequency of oscillation and $i,l = x,y$. It follows from Eqs. (3)–(5) that the Poynting vector components in the paraxial approximation can be represented as functions determined by the x and y components alone. Just these equations and their versions will be basic in our further analysis.

3. Singularities of Poynting vector in scalar fields

Let us specify at first the notion of a scalar field. As a rule, one considers any uniformly polarized field as a scalar one, irrespective of the type of its polarization [3,6]. Hereinafter, we reduce the notion of the scalar field to linearly polarized one, whereas behaviour of the Poynting vector for elliptically polarized fields can be very sophisticated. In part, elliptically polarized waves possess a so-called spin angular momentum [24].

3.1. Instantaneous singularities of scalar fields

The basic scalar equations for the wave polarized along the y axis (here a choice of the axis is not relevant) have the following form:

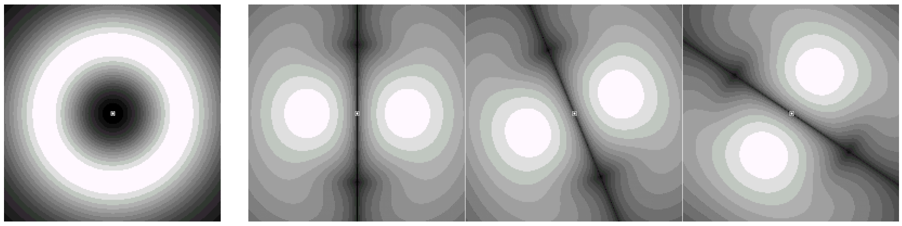
$$\begin{cases} P_x \approx -\dfrac{c}{4\pi k} E T_2 \\ P_y \approx -\dfrac{c}{4\pi k} E T_1 \\ P_z \approx \dfrac{c}{4\pi} E^2 \end{cases}, \tag{6}$$

$$\begin{cases} T_1 = E \dfrac{\partial \Phi}{\partial y} + \dfrac{1}{A}\dfrac{\partial A}{\partial y} E_{\frac{\pi}{2}} \\ T_2 = E \dfrac{\partial \Phi}{\partial x} + \dfrac{1}{A}\dfrac{\partial A}{\partial x} E_{\frac{\pi}{2}} \end{cases}, \tag{7}$$

$$\begin{cases} E = A\cos(\omega t + \Phi - kz) \\ E_{\frac{\pi}{2}} = A\sin(\omega t + \Phi - kz) \end{cases}. \tag{8}$$

It follows from Eqs. (6)–(8) that the singularities of the Poynting vector could arise in the two cases:

(*i*) all of the three components vanish simultaneously; this case corresponds to appearance of disclination;



a *b* *c* *d*

Fig. 2. Rotation of edge disclination in the vicinity of isotropic vortex: *a* – intensity distribution for an isotropic vortex and *b-d* – instantaneous distributions of the modulus of transversal Poynting vector component for different moments, which determines the position of disclination. The temporal step separating figures *b-c* is 1/12 of the oscillation period.

(*ii*) only the transversal component vanishes; this corresponds to simultaneous vanishing of T_1 and T_2. Really, orientation of the transversal component of the Poynting vector (its azimuth being $\theta = \arctan\left(P_y / P_x\right)$) is then indeterminate.

Thus, the appearance of singularity of the Poynting vector for the case when the three components vanish simultaneously requires more precise definition of the disclination of scalar fields. In contrast to vector fields, where disclinations are the lines moving within 3D space (or the corresponding points in any cross-section of the field), disclinations in the scalar field degenerate into zero surfaces (or the corresponding closed lines in any cross-section of the field). So, contrary to the vector field, where the disclinations are “point-like” singularities, they represent moving “edge” singularities in the scalar field. Moreover, the point disclinations do not exist in the scalar field, following from the field continuity and the fact that the amplitude of linearly polarized wave vanishes at each point of the field twice per period of its oscillations.

Such behaviour of the field is illustrated in Fig. 2 by the behaviour of transversal component of the Poynting vector in the vicinity of isotropic vortex [15].

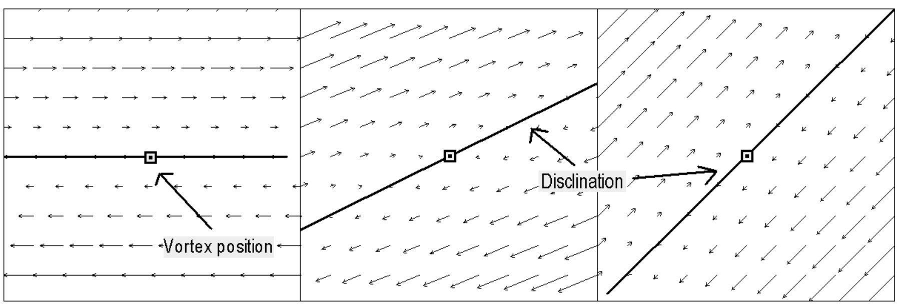
It can be seen that the transverse component rotates around the vortex centre with the frequency of oscillations, and the rotation direction is determined by the sign of topological charge of the vortex.

Instantaneous orientation of the Poynting vector component for different moments is presented in Fig. 3. It is described by the relation

$$\gamma = S\frac{\pi}{2} - S(\omega t - kz)\,, \qquad (9)$$

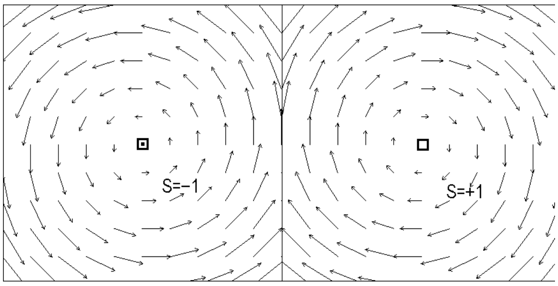
where S is the topological charge of the vortex.

It follows from Eq. (9) and Fig. 3 that the azimuth of the Poynting vector for some instant does not depend on x and y. It suffers a jump-like change while crossing the disclination. Thus, the well-known circulation of the averaged Poynting vector (see Fig. 4) occurring in the vicinity of the vortex centre (cf. [24]) results from averaging of “similarly oriented” vectors.



a *b* *c*

Fig. 3. Instantaneous orientation of transverse component of the Poynting vector for different moments. The temporal step between the figures is 1/16 of the oscillation period.



a *b*

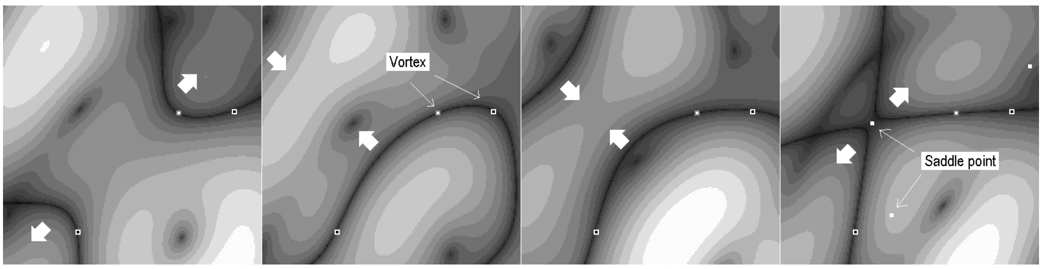
Fig. 4. Circulation of averaged Poynting vector in the vicinity of vortex centre.

It is seen from this figure that the azimuth of the averaged component of the Poynting vector has a singularity at the vortex centre, which is a kind of the "centre" [31]. Both cases depicted in Fig. 4*a* and 4*b* are associated with the positive Poincare index $N = +1$. That is why one should introduce an additional parameter, e.g. a chirality V, in order to characterize comprehensively the above singularity of the Poynting vector. Let us assume that the field propagates towards the observer. Let the positive chirality $V = +1$ (see Fig. 4*b*) correspond to the clockwise vector circulation, whereas the negative chirality $V = -1$ (see Fig. 4*a*) to the counter-clockwise circulation. Hereafter we will refer to such the azimuth singularities of the Poynting vector (as well as the singularities similar to them) as the "*vortex singularities*".

The situation is much more complicated for the scalar field of general form, but the behaviour of the Poynting vector is the same as in the case of isotropic vortex. Temporal behaviour of the transversal component for the area of random scalar field is illustrated in Fig. 5. Disclinations rotate according to the signs of their topological charges. Disclinations rotating in the opposite directions and corresponding to the adjacent vortices converge at the saddle points of phase (cf. Fig. 5*b* and 5*c*). Then they diverge again in the direction orthogonal to that of the convergence (see Fig. 5*d* and 5*a*). The direction of motion of the disclinations is indicated by white arrows in Fig. 5.

The second kind of instantaneous singularities arising in scalar fields is constituted by the singularities of transversal component of the Poynting vector that correspond to its zero magnitude and a non-zero z-component magnitude. These singularities are point-like. Possible realizations of the point-like singularities can be reduced to the structures shown in Fig. 6. Contrary to the vortex singularities, here the angular momentum of the field averaged over spatial coordinates and short temporal interval δt vanishes in the nearest vicinity of the singularity. Hereafter we will refer to the singularities of this kind as "*passive singularities*".

Let us assign the term "positive passive singularities" for those presented in Fig.6*a* (by analogy with the flux behaviour in the vicinity of positive electric charge). Singularities shown in Fig.6*b* and 6*c* will be respectively called as "negative" and "saddle passive singularities". Specific behaviour of the transversal component of the Poynting vector in the areas corresponding to all kinds of point-like singularities, which is derived due to computer simulations, is illustrated in Fig. 7. It is seen from Fig. 6 and 7 that the passive singularities can be characterized by both positive (see Fig. 6*a* and 6*b*, as well



a *b* *c* *d*

Fig. 5. Temporal behaviour of the transverse component modulus for a random scalar field. The direction of movement of the disclication is indicated by white arrows. The temporal step separating figures is ¼ of the oscillation period.

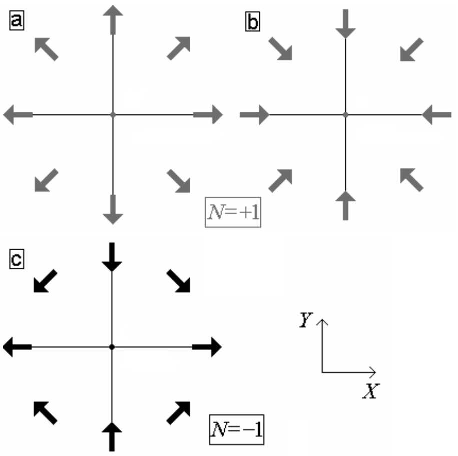
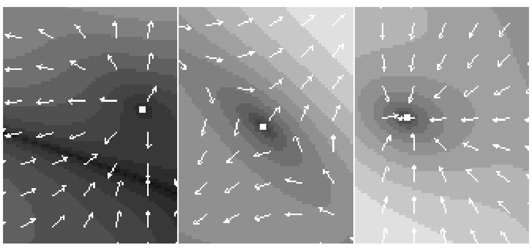


Fig. 6. Instantaneous “passive” singularities: *a* – positive singularities, *b* – negative singularity and *c* – saddle singularity.

as Fig.7*a* and 7*c*) and negative (see Fig. 6*c* and 7*b*) Poincare indices.

The adjacent passive singularities with different signs of the Poincare index are connected into singular nets by the current lines of transversal component of the Poynting vector. For that, a saddle character of the saddle singularity provides a topological connection between the singularities with the positive indices. Therefore these singularities are born and annihilate by pairs (with plus- and minus- indices), without arising of additional singularities. The motion of such the singularities is governed by a number of regularities. In part, the analysis of Eqs. (6)–(8) (i.e., the impossibility for vanishing E and $E_{\frac{\pi}{2}}$ simultaneously) yields the conclusion that the passive point-like singularities unavoidably pass through all stationary points of the phase and intensity.



a *b* *c*

Fig. 7. Behaviour of transversal component of the Poynting vector in the areas corresponding to all kinds of point-like singularities (computer simulations).

3.2. Singularities of the averaged Poynting vector of scalar fields

The averaged Eqs. (6)–(8) take the following form:

$$\begin{cases} \overline{P}_x = -\dfrac{c^2A^2}{8\pi\omega}\dfrac{\partial\Phi}{\partial x} \\ \overline{P}_y = -\dfrac{c^2A^2}{8\pi\omega}\dfrac{\partial\Phi}{\partial y} \\ \overline{P}_z = \dfrac{cA^2}{4\pi} \end{cases}, \qquad (10)$$

where A denotes the amplitude and Φ the phase.

Similarly to the preceding case of the instantaneous Poynting vector, the two kinds of singularities can arise:

1. All the components of the averaged Poynting vector vanish (see Fig. 8*b* and 8*c*). This case corresponds to the averaged vortex singularity localized at the vortex centre. The amplitude A is zero. Conventional circulation of the Poynting vector is observed in the vicinity of the vortex centre. Such a singularity of the azimuth of the Poynting vector is characterized by the positive Poincare index. Singularities with different chirality correspond to the vortices differing in the sign of the topological charge.
2. Only the transversal component vanishes (see Fig.8*a*, 8*d* and 8*e*). Then the averaged passive singularities take place. As follows from Eqs. (8), their coordinates coincide with those of the stationary points of phase. The direction of the energy flow at these points coincides with the z axis.

In other words, these points just determine the “prevailing direction” of the energy flow of scalar waves. A possible behaviour of the Poynting vector in the nearest vicinity of those singularities derived with computer simulations is illustrated in Fig. 9.

The negative (saddle) passive singularities provide topological connection between the vortex singularities with the same chirality, while the adjacent vortices with the opposite directions of circulation of the Poynting vector

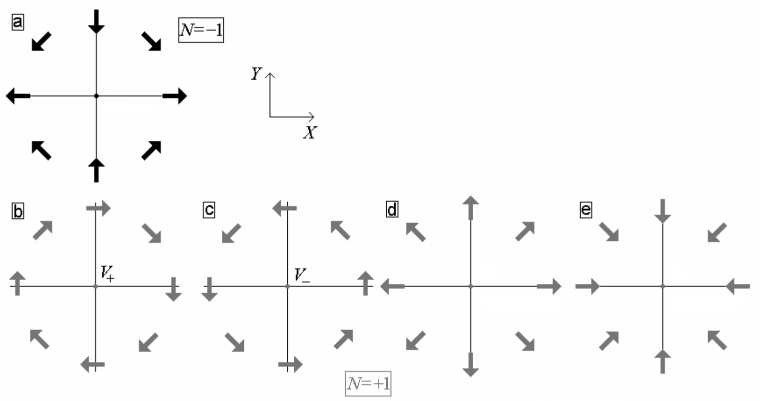


Fig. 8. Possible behaviour of the Poynting vector in the vicinity of averaged singularities of a scalar field: *a, d, e* – passive singularities and *b, c* – vortex ones.

are directly connected by the current lines of transversal component of the Poynting vector.

4. Singularities of the Poynting vector of vector fields

4.1. Instantaneous singularities of vector fields

Similarly to the case of scalar fields and in accordance with Eqs. (3)–(5), the instantaneous singularities of the Poynting vector arise at the field points, where a disclination or instantaneous zero amplitude of the transversal component of this vector occur. It is known [3,13] that the disclinations are point-like singularities of vector fields. It has been shown [3] that the disclinations move along the s-contours, they are born and annihilate. The number of disclinations at the s-contour can only change by even number, i.e. similarly to all

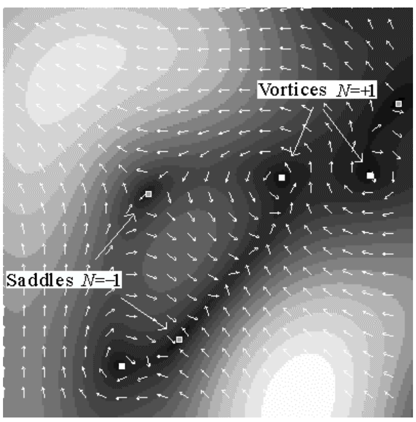


Fig. 9. Possible behaviour of the Poynting vector in the nearest vicinity of the averaged singularities (computer simulations).

the topological defects, disclinations are born and annihilate in pairs [13]. Motion of disclinations, their interconnection and connection with the other structures of field obey topological regularities. This is why the events associated with the singularities of the Poinging vector resulted from discinations must obey similar regularities.

As follows from Eqs. (3)–(5), there are no limitations on the sign of singularity associated with the disclination. Moreover, the positive instantaneous singularities of the Poynting vector can be both the vortex and passive singularities. This circumstance is illustrated by the results of computer simulations presented in Fig. 10. Note that such newly arising singularities can be both vortex ones, i.e. both singularities can be characterized by the same Poincare indices, differing only by chirality. A difference in chirality is sufficient for providing interconnection between the born Poynting vortices, though insufficient for forming topological connection with the other field structures.

Hence, the two following scenarios of the birth and annihilation events are possible for the singularities of the Poynting vector associated with disclinations:

1. Assume that two vortex singularities N_V^+ are born at the s-contour, their chiralities being opposite and their Poincare indices the same (both positive). In agreement with the conservation law for the total topological index

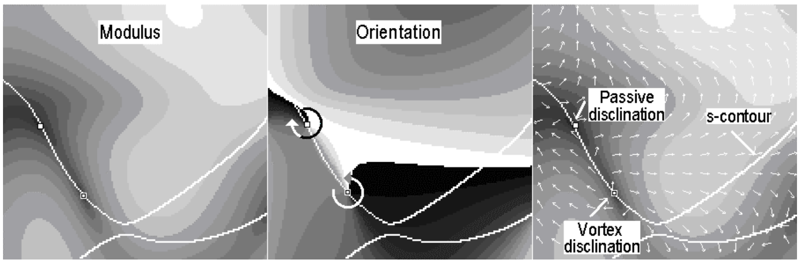


Fig. 10. Singularities of the Poynting vector associated with disclinations for random vector field: *a* – instantaneous distribution of the modulus of transversal component of the Poynting vector, *b* – distribution of instantaneous azimuth of the transversal component (the levels of grey correspond to different orientations of the vector and the direction of arrows indicates the sign of Poincare index of singularities) and *c* – distribution of the modulus and azimuth of the transversal component (orientation of the Poynting vector is indicated by white arrows).

[14], the two singularities N_P^+ with the negative index must be born simultaneously with the birth event of the former singularities. This should occur at the same point, i.e. at the s-contour. Evidently, these are the passive singularities, which leave the s-contour just after they have appeared and "walk" to the region of elliptical polarization. The topological reaction

$$2N_V^+ + 2N_P^- \Leftrightarrow 0 \qquad (11)$$

corresponds to this event, i.e. four singularities of the transversal component of the Poynting vector appear and disappear.

2. One of the singularities $N_{V,P}^+$ associated with the disclination has the positive index (it does not matter whether the vortex or passive singularity is concerned) and another one the negative one. In this case the topological reaction of appearance-disappearance of singularities is transformed to the form

$$N_{V,P}^+ + N_P^- \Leftrightarrow 0 \,. \qquad (12)$$

Only two singularities of the Poynting vector take part in this reaction.

Similar to the disclinations themselves, the singularities of the Poynting vector associated

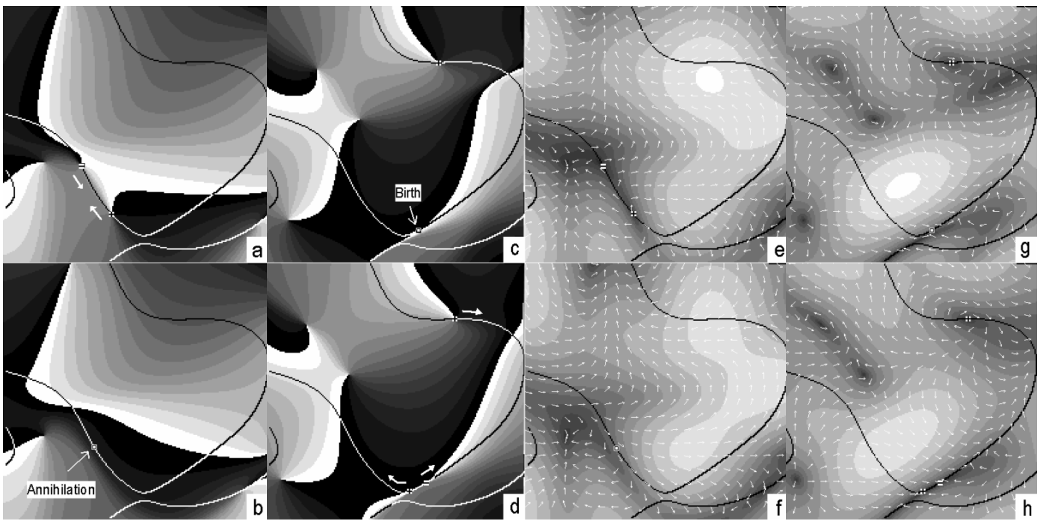
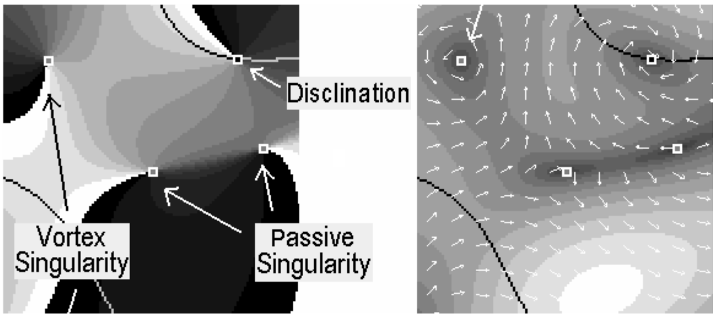


Fig. 11. Behaviour of singularities of the Poynting vector associated with the disclinations for different moments. Temporal step separating the figures is 1/8 of the oscillation period: *a–d* – distribution of the azimuth of transversal component of the Poynting vector (the levels of grey correspond to various orientation of the vector) and *e–h* – distributions of the modulus and azimuth of the transversal component. Orientation of the component is illustrated by thin white arrows. Bold white arrows in fragments *a–d* indicate the direction of motion of the singularities. Symbols ⊞ and ⊟ are the instantaneous singularities of the Poynting vector with the positive and negative indices, respectively. Solid lines are the s-contours.



a *b*

Fig. 12. Instantaneous singularities associated with zero magnitude of the transversal component of the Poynting vector: *a* – distribution of the component azimuth indicated by shades of grey and *b* – the azimuth (indicated by white arrows) and the modulus of the component (indicated by shades of grey).

with disclinations move along the s-contour, disappear and are born again (see Fig. 11). Their positions are repeated twice per the oscillation period. The singularities birth and annihilation events can be accompanied by appearance and disappearance of additional singularities, when only the transversal component of the Poynting vector vanishes.

In general case, these singularities do not belong to the s-contour. Similar singularities can appear independent of the birth events of the disclinations (see Fig. 12). For that, the singularities not connected with the s-contour can be both passive and vortex ones.

Temporal behaviour of the instantaneous vortex singularity, which is born in the area of elliptical polarization, is illustrated in Fig. 13. One can see that the singularity passes through the area of nonuniform polarization.

In conclusion, we are to stress that the "instantaneous" angular momentum averaged over spatial coordinates and short temporal interval δt, being observed in the vicinity of the vortex singularity, has a maximal magnitude that exceeds the momentum magnitude in the other field areas with the same energy per unit area, irrespective of the origin of singularity (e.g., from the disclination or in some other way).

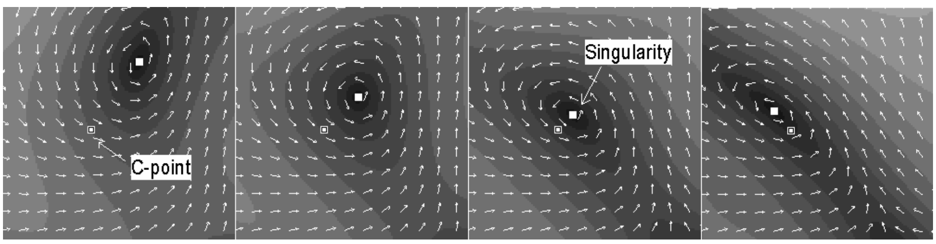
4.2. Averaged Poynting vector of vector fields

As shown in the works [29,30,32,33], the appearance of singularity of the averaged Poynting vector in the area of elementary polarization cell is related to the presence of C-points with specific characteristics. Obviously, this pattern of the phenomenon is directly extended over the case of singularities of the Poynting vector for the fields of general form. Namely, the appearance of the Poynting vector singularity is always associated with the C-point located near this singularity (see Fig. 14).

The type of singularity (vortex or passive) depends upon a relation of signs associated with the topological charge of the vibration phase of the C-point and with the handedness factor h. The vortex singularity arises when these signs are opposite:

$$S_C = -h/2 \,. \tag{13}$$

The passive singularity is formed when the signs of S_C and h are the same. As follows from Eq. (2) and Eq. (13), the vortex singularity of transversal component of the Poynting vector corresponds to the C-points with the negative index (i.e., the negative C-points), and the



a *b* *c* *d*

Fig. 13. Motion of the instantaneous vortex singularity born in the area of nonuniform polarization.

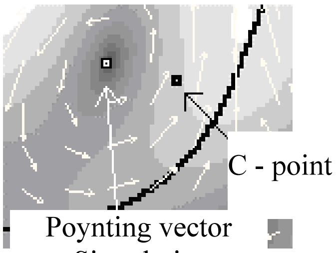


Fig. 14. Displacement of the Poynting vector singularity with respect to the position of *C*-point.

passive singularity of the Poynting vector appears near the positive *C*-points.

The latter circumstance is illustrated by the data of computer simulations performed for a random vector field (see Fig. 15). It is seen that the negative C-points are located near the vortex singularity indicated by the numbers 1,1. The passive singularities are attracted towards the positive C-points (numbers 2,2). Notice that the vortex singularities differ in the chirality ($V=+1$ or $V=-1$), which is determined by the sign of the handedness factor for the area containing the negative C-point. The transversal component of the Poynting vector circulates clockwise around the Poynting vortex in the areas of right polarization ($h=+1$, $V=+1$) and in the opposite direction in the areas of left polarization ($h=-1$, $V=-1$).

In conclusion, we represent Tables 1 and 2 summarizing the main properties of the Poynting vector singularities and their connection with the conventional optical singularities.

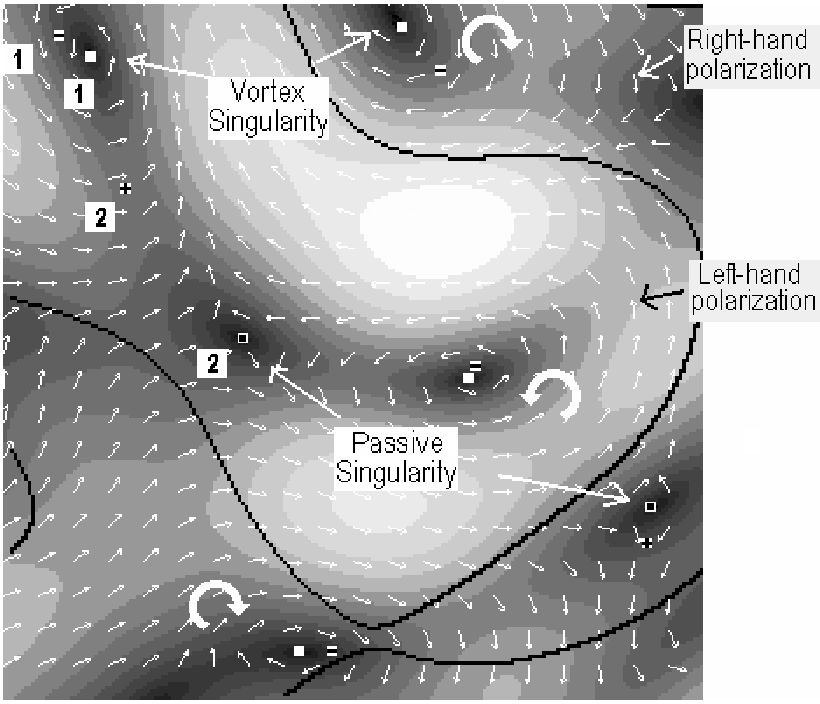


Fig. 15. Interconnection between the singularities of transversal component of the Poynting vector and the C-points. The symbols ⊟ and ⊞ label the negative and positive C-points, respectively, and the symbols □ and ▣ the vortex and passive singularities. Chirality of the vortex singularities is indicated by bold white arrows and black solid lines represent the s-contours. The numbers give the examples of specified pairs of the C-points associated with the singularities

.

Table 1. Instantaneous singularities of the Poynting vector.

Kind of Poynting vector singularity	Edge	Vortex singularity (VS) *	Passive singularity (PS)
Scalar field	Its localization coincides with equi-phase lines	Do not exist	Moving PSs unavoidably pass through the stationary points of phase and intensity
Vector field	Does not exist	As a rule, VSs coincide with disclinatons	PSs can appear independent of disclinations

* VSs have the same topological index and differ in chirality.

Table 2. Averaged singularities of the Poynting vector.

Kind of Poynting vector singularity	Vortex singularity (VS)	Passive singularity (PS)
Scalar field	1. Localization of VSs coincides with the vortices. 2. Chirality of VSs is determined by the topological charge of the vortex phase.	1. Positions of PSs coincide with the stationary points of phase. 2. As a rule, PSs in the far region are located in the saddle points of phase.
Vector field	1. VSs are associated with the negative C-points. 2. Generally, localizations of VSs and C-points are different. 3. Chirality of PSs is determined by the handedness factor within the region where PS occurs.	1. PSs are associated with the positive C-points. 2. Generally, localizations of PSs and C-points are different.

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