Indicative Surfaces for Crystal Optical Effects

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Abstract

This paper has mainly a pedagogical meaning. Our aim is to demonstrate a correct general approach for constructing indicative surfaces of higher-rank tensors. We reconstruct the surfaces of piezooptic tensor for β-BaB_2O_4 and $LiNbO_3$ crystals, which have been incorrectly presented in our recent papers.

Keywords: indicative surface, piezooptic tensor

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Introduction

Many works have been devoted to analysis of anisotropy of piezooptic (PO) effect in crystals on the basis of construction of so-called indicative surfaces (see [1-5] and our previous studies [6,7]). Unfortunately, the indicative surfaces in those studies have been constructed, using the inconsistent formula

$$\pi'_{ijkl} = \alpha_{im}\alpha_{jn}\alpha_{kp}\alpha_{lt}\pi_{mnpt}\,, \qquad (1)$$

where π'_{ijkl} means the PO tensor written in the current (or 'new') Cartesian coordinate system, π_{mnpt} the PO tensor written in the 'old' Cartesian coordinate system and $\alpha_{im}, \alpha_{jn}, \alpha_{kp}, \alpha_{lt}$ the matrix components of directional cosines between the 'old' and 'new' systems. It is necessary to note that Eq. (1) is often used for rewriting tensors in different Cartesian systems or, in some special cases, for constructing the so-called representation surfaces (see, e.g., [8]), though not the indicative ones. In this paper, we present the relation for the indicative surfaces of higher-order crystal optical effects, specifically for the PO one, and reconstruct in this way the surfaces presented in our previous works.

Results and Discussion

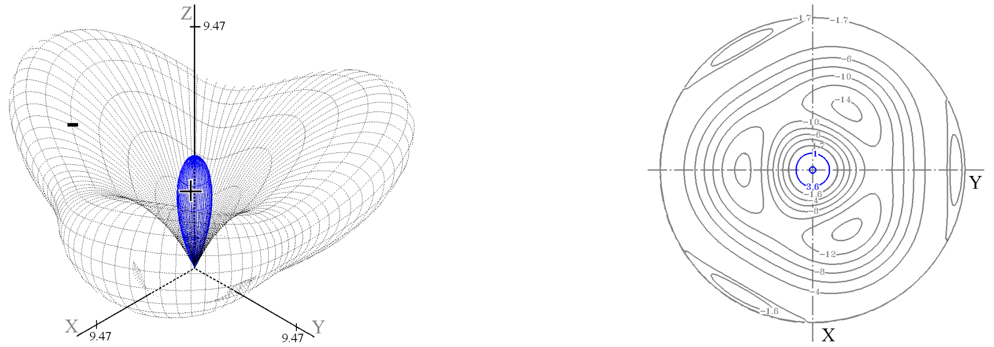
Using a spheric coordinate system (R,Θ,φ), one can represent the relation for the indicative surfaces of symmetric tensors of higher ranks as follows [9]:

$$R(\Theta,\varphi) = T_{i_1\ldots i_p} n_{i_1}\ldots n_{i_p}\,, \qquad (2)$$

where R is the module of the spheric coordinate system, $T_{i_1\ldots i_p}$ the tensor of a rank p and $n_{i_1}\ldots n_{i_p}$ the transformation relation between Cartesian and spheric coordinates ($n_1 = \sin\Theta\cos\varphi, n_2 = \sin\Theta\sin\varphi, n_3 = \cos\Theta$). Applying Eq. (2) to the PO effect, we find easily the relation for the surface of the PO tensor:

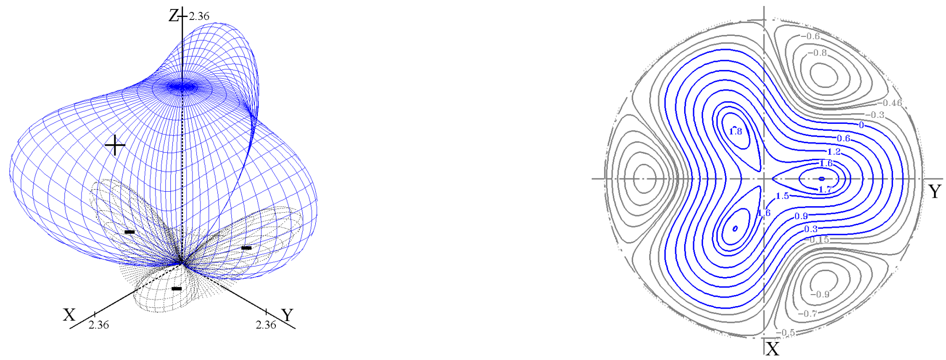
$$R(\Theta,\varphi) = \pi_{ijkl} n_i n_j n_k n_l\,. \qquad (3)$$

Eq. (2) has been successfully used while constructing the indicative surfaces of second-rank axial gyration tensor, elastic module tensor, etc. (see, e.g., [9,10]). Since for the case of PO effect $R \sim \pi_{ijkl} \sim \Delta n/\sigma_{kl}$, the R value would reflect in some cases anisotropy of the induced



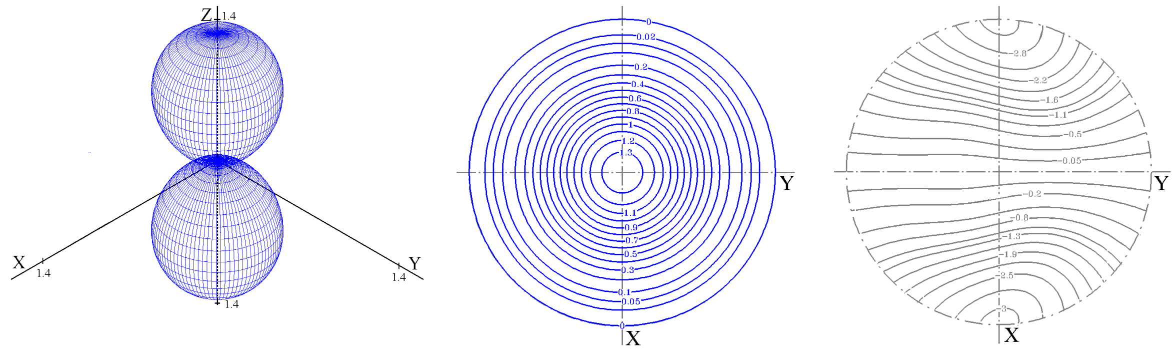
(a) (b)

Fig. 1. Half the indicative surface of the PO tensor (a) and its stereographic projection (b) for β-BaB_2O_4 crystals (in the units of *pm*^{2}/*N*).

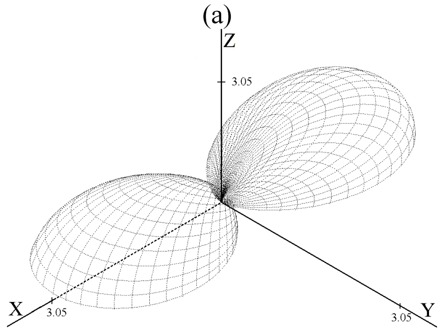


(a) (b)

Fig. 2. Half the indicative surface of the PO tensor (a) and its stereographic projection (b) for $LiNbO_3$ crystals (in the units of *pm*^{2}/*N*).



(a) (b) (d)



c

Fig. 3. Indicative surface of the part of PO tensor corresponding to the stress σ_{33} (a), its stereographic projection (b), half the indicative surface of the part of PO tensor corresponding to the stress σ_{11} (c) and its stereographic projection (d) for β-BaB_2O_4 crystals (in the units of *pm*^{2}/*N*).

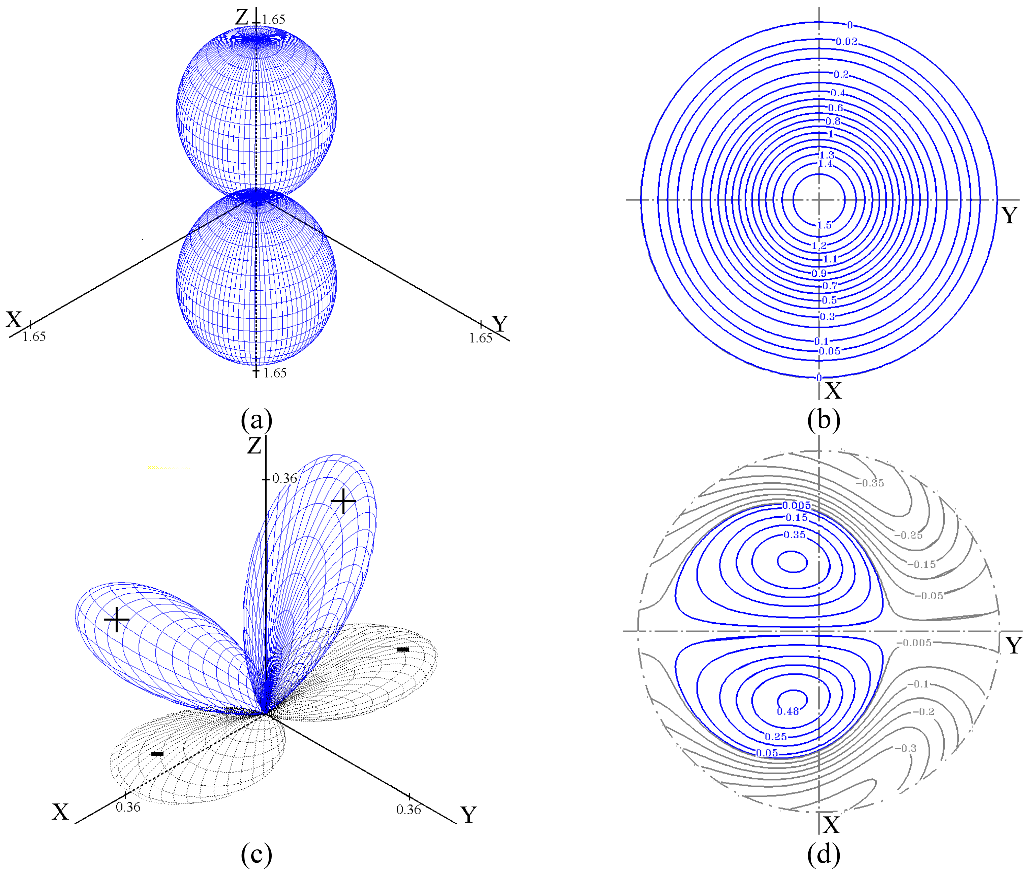


Fig. 4. Indicative surface of the part of PO tensor corresponding to the stress σ_{33} (a), its stereographic projection (b), half the indicative surface of the part of PO tensor corresponding to the stress σ_{11} (c) and its stereographic projection (d) for $LiNbO_3$ crystals (in the units of pm^2/N).

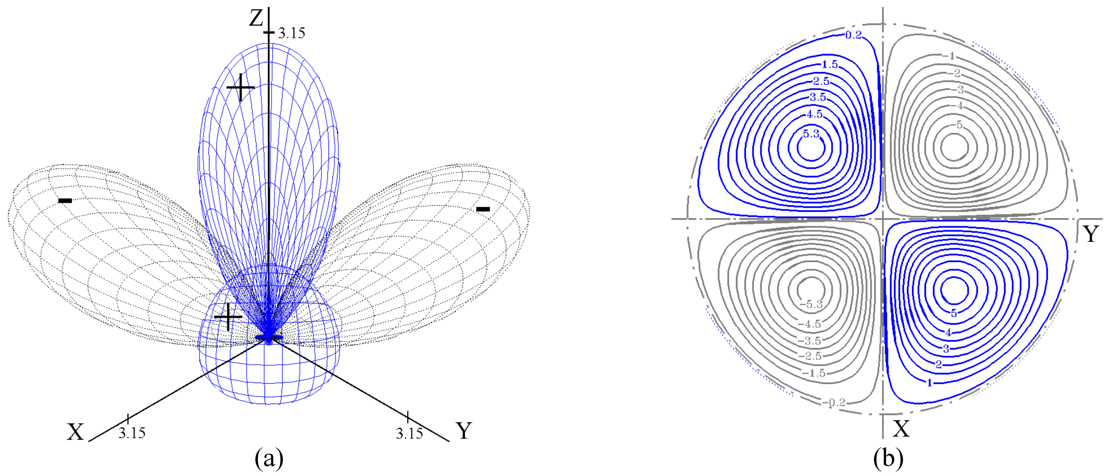


Fig. 5. Half the indicative surface of electrogyration tensor (a) and its stereographic projection (b) for $BiGeO_{20}$ crystals (in the units of 10^{-13}*m/V*).

increment of refractive indices Δn due to the action of mechanical stresses σ_{kl} in different directions. Moreover, it is possible to construct a part of the indicative surface if one takes an interest in this. For example, in the case of PO anisotropy induced by a chosen component of mechanical stress tensor, it is reasonable to take into account only one column that corresponds to a given stress component.

Let us construct the indicative surface of the PO tensor for $LiNbO_3$ and β-BaB_2O_4 crystals. This would be instructive enough for the illustrative purposes, as well as for making necessary corrections to our previous construction procedure [6,7]. Both $LiNbO_3$ and β-BaB_2O_4 crystals belong to the same point symmetry group *3m*. Then the equation of the indicative surface takes the same form for those crystals:

$$\begin{aligned} R(\theta,\varphi) &= \pi_{11}\sin^4\theta + \\ &+(\pi_{13}+\pi_{31}+2\pi_{44})\sin^2\theta\cos^2\theta \\ &+\pi_{33}\cos^4\theta + \\ &+(2\pi_{41}+\pi_{14})\sin^3\theta\cos\theta\sin 3\varphi \end{aligned} . \quad (4)$$

Using the known values of PO coefficients for β-BaB_2O_4 crystals,
$\pi_{11} = (-1.7 \pm 0.15) pm^2/N$,
$\pi_{12} = (-1.35 \pm 0.07) pm^2/N$,
$\pi_{13} = (1.75 \pm 0.23) pm^2/N$,
$\pi_{31} = (-1.6 \pm 0.15) pm^2/N$,
$\pi_{33} = (3.7 \pm 0.37) pm^2/N$,
$\pi_{14} = (-2.0 \pm 0.8) pm^2/N$,
$\pi_{41} = (-2.03 \pm 0.07) pm^2/N$,
$\pi_{44} = (-26.3 \pm 0.9) pm^2/N$, one can represent Eq. (4) as

$$\begin{aligned} R(\theta,\varphi) &= (-1.7\sin^4\theta - 52.45\sin^2\theta\cos^2\theta + \\ &+3.7\cos^4\theta - 6.06\sin^3\theta\cos\theta\sin 3\varphi)\times 10^{-12} \end{aligned} . \quad (5)$$

In case of $LiNbO_3$ crystals we have
$\pi_{11} = -47.7 pm^2/N$, $\pi_{12} = 0.11 pm^2/N$,
$\pi_{13} = 2 pm^2/N$, $\pi_{31} = 0.47 pm^2/N$,
$\pi_{33} = 1.6 pm^2/N$, $\pi_{14} = 0.7 pm^2/N$,
$\pi_{41} = -1.9 pm^2/N$, $\pi_{44} = 0.21 pm^2/N$ and so we obtain

$$\begin{aligned} R(\theta,\varphi) &= (-0.47\sin^4\theta + 2.89\sin^2\theta\cos^2\theta + \\ &+1.6\cos^4\theta - 3.1\sin^3\theta\cos\theta\sin 3\varphi)\times 10^{-12} \end{aligned} . \quad (6)$$

We have also constructed the indicative surfaces of that part of the PO tensor which is responsible for the effect occurring under the application of mechanical stresses σ_{33} and σ_{11}. They are described respectively by the equations

(i) for σ_{33}:

$$R(\Theta,\varphi) = \pi_{13}\sin^2\Theta\cos^2\Theta + \pi_{33}\cos^4\Theta \,; \quad (7)$$

(ii) for σ_{11}:

$$\begin{aligned} R(\Theta,\varphi) &= (\pi_{11}+\pi_{12})\sin^4\Theta\cos^2\varphi + \\ &+\pi_{31}\sin^2\Theta\cos^2\Theta\cos^2\varphi + \\ &+\pi_{41}\sin 3\Theta\cos^2\varphi\sin\varphi\cos\Theta \end{aligned} . \quad (8)$$

These surfaces are displayed in Fig. 3 and 4.

It is seen from Fig. 3 and 4 that the indicative surfaces belong to a lowered point symmetry group, which is actual under application of the chosen mechanical stress component. Since the point symmetry group of crystals remains to be *3m* when the component σ_{33} is applied, the indicative surface also possesses all of the symmetry elements of the group *3m*. When the mechanical stress σ_{11} is applied, the point symmetry group of crystal becomes *m* and so the symmetry of the indicative surfaces is also characterized by this group.

Applicability of Eq. (2) for the construction of indicative surfaces of different higher-rank tensors could be demonstrated on the example of indicative surface of electrogyration tensor for $BiGeO_{20}$ crystals, which belong to the point symmetry group *23*. The components of the electrogyration tensor for these crystals are equal to $\gamma_{41} = \gamma_{52} = \gamma_{63} = 0.95 pm/V$ [10]. The indicative surface of the electrogyration tensor for $BiGeO_{20}$ crystals is shown in Fig. 5.

Conclusion

We have reconstructed the surfaces of the PO tensor for β-BaB_2O_4 and $LiNbO_3$ crystals, which have been incorrectly presented in our previous papers. We thus demonstrate a correct approach for constructing the indicative surfaces of higher-rank tensors. The indicative surfaces of the PO effect published earlier (see [1-7]) are in fact the surfaces built in frame of the definition [8], though in a certain error. Moreover, characterization of piezoelectric effect by means of representation surfaces (but not indicative ones) in the work [8] has a well-defined practical meaning, in contrast to the case of PO effect.

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