Influence of Non-Linear Recording of Thick Hologram on the Effective Value of Modulated Refractive Index

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Abstract

Diffraction of a plane electromagnetic wave falling onto a thick hologram at the first-order Bragg angle at the non-linear recording conditions is analyzed using the coupled-wave equations. It is shown that, under the non-linear conditions, one can use double-wave approximation and the effective values for the modulated refractive index. Analytical dependences of the hologram diffraction efficiency on thickness are obtained. A possibility is revealed for using the effective value of the refractive index for the case of slight deviations from the Bragg angle.

Key words: grating, diffraction, coupled waves.

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Introduction

A refractive index change of a finite magnitude takes place while recording purely phase holograms. This leads to a non-linear record of the hologram under the influence of the sine spatial distribution of light intensity . In this case the refractive index along a given direction can be expressed as follows:

$$n(x) = n_0 + n_1 \cos\left(\frac{2\pi}{\Lambda}x\right) + n_2 \cos\left(\frac{4\pi}{\Lambda}x\right),$$

where Λ is the spatial period of the refractive index change, n_0 the constant component of the refractive index and n_1 and n_2, respectively, the modulation amplitudes for the basic and the doubled spatial periods. For the analysis of diffraction properties of such the hologram it is necessary to use a four-wave approximation for the diffraction at the first-order Bragg angle [1] or a three-wave approximation for the diffraction at the second-order Bragg angle as it has been done in [2]. In this work the diffraction efficiency dependence on the hologram thickness is obtained using the analytical solution of the linear system of three differential equations for the case of diffraction at the second-order Bragg angle. This dependence is similar to the Kogelnik's solution but it uses n_{2ef} which depends on both n_2 and n_1. When the optical wave diffraction at the first-order Brag angle is dealt with, the Kogelnik formula is usually used in the calculations of diffraction efficiency, i.e. n_2 is not considered. However, in this case n_2 influences considerably the diffraction efficiency (see the results obtained in [4]). If the maximum change in the refractive index is low (like for photopolymeric materials), the influence of n_2 can be insignificant. But n_2 must be taken into consideration [4], if we wish to calculate n_1 using the angular dependence of the diffraction efficiency. However, some photosensitive materials like a chalcogenide glass [5] show a high change of the refractive index Δn (over 0.1 under exposition). Therefore the chalcogenide glasses would possess a high non-linearity [6], so it is necessary to account for n_2 in the dependences of diffraction efficiency on the hologram thickness and the incidence angle of the beam onto the hologram in the vicinity of the first-order Bragg angle. On the other hand, the higher-order term of the expansion (n_3) of spatial distribution of the refractive index does not practically influence the diffraction efficiency, as shown in [4] with numerical calculations. As a consequence, we shall not consider n_3 in our study, but we shall instead estimate its influence with our method and show that it is negligible. To obtain the diffraction efficiency dependence on n_1 and n_2 at the first-order Bragg angle, it is necessary to solve a system of four linear differential equations, which is quite difficult to be done because of considerable mathematical transformations. Such the solutions, which involve n_1 and n_2, are not known for the authors. At the same time, after the analysis of the results [2] and the coupled-wave equations proposed in [4], we conclude that the diffraction efficiency dependence on n_1 and n_2 can be obtained in a simpler way, without coming to analytical solution of the said differential equation system. The relevant method is proposed in this work. We also provide a comparison of the diffraction efficiency results obtained with the two methods, the analytical dependences derived in this work and the numerical solution of the differential equation system in a four-wave approximation. The influence of n_2 values on the dependences of diffraction efficiency will be studied, too.

Theory of S-polarization optical wave diffraction at a thick hologram with taking a nonlinear record into account

Let us assume that two coherent laser beams with plane wave fronts are falling onto a photosensitive medium and the beams inside the medium propagate at the 2φ angle with respect to each other (Fig. 1). The bisectrix of that angle

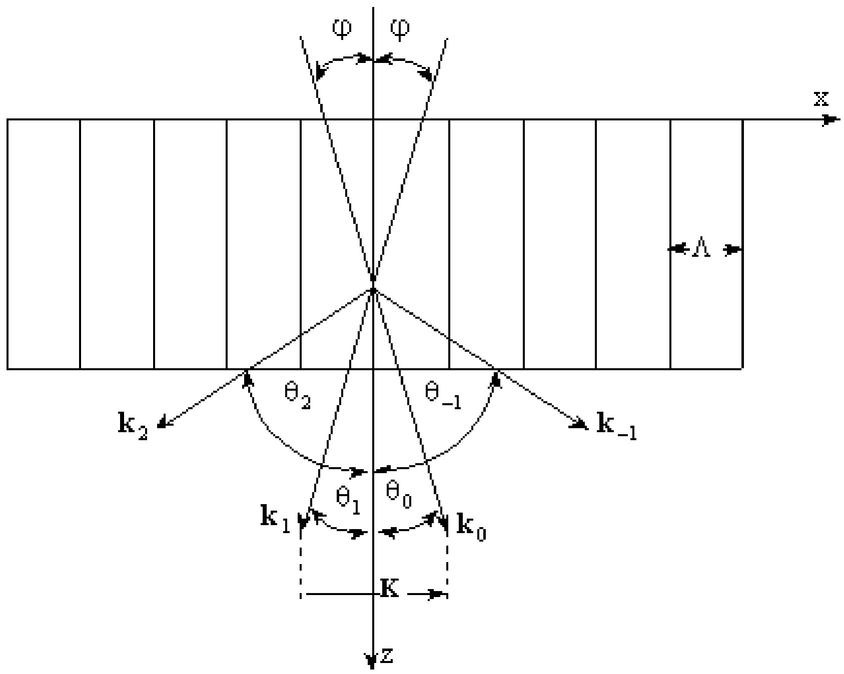

Fig.1. Representation of distribution of the beams at the record of thick hologram and the diffracted beams.

is orthogonal to the two parallel planes, between which there is the photosensitive medium, and the distance between these planes determines the hologram thickness.

If the axis x lays in one of the planes and, at the same time, in the incident plane of recording beams, then the total light intensity $I(x)$ inside the volume of a photosensitive medium with a slight absorption can be determined by the following relation:

$$I(x) = \bar{I}[1 + \cos(2\pi x/\Lambda)],$$

where $\bar{I}$ is the total light intensity, $\Lambda = \dfrac{\lambda_0}{2n_0 \sin\varphi}$ the interference pattern period along x, n_0 the refractive index of the photosensitive medium and λ_0 the light wavelength at the hologram recording. According to this, the refractive index after the hologram record can be expressed as

$$n(x) = n_0 + n_1 \cos\left(\frac{2\pi}{\Lambda}x\right) + n_2 \cos\left(\frac{4\pi}{\Lambda}x\right), \quad (1)$$

A system of linear differential equations describing the propagation of S-polarization waves through a transparent periodic medium has been obtained in [4]. The refractive index of the medium is described by formula (1).

The electric field of optical wave in a periodic medium is presented as a linear combination of plane waves, whose amplitudes are changing while propagating and depend only on the coordinate *z*. The axis *z* is perpendicular to the planes that bound the periodic medium. So the field in the medium equals to [4,7]:

$$\mathbf{E}(x,z) = \vec{m}\sum_i A_i(z)\frac{\exp[-j(k_{i,x}x + k_{i,z}z)]}{\sqrt{\cos\theta_i}}, \quad (2)$$

where $A_i(z)$ denoted the variable amplitude of the plane wave, $\mathbf{k_i}$ the wave vector of the plane wave with the amplitude $A_i(z)$, module of which is equal to $k = 2\pi n_0/\lambda$, θ_i the angle between the axis z and the wave vector $\mathbf{k_i}$, $k_{i,x} = k\sin\theta_i$, $k_{i,z} = k\cos\theta_i$, $\vec{m}$ the ort vector directed perpendicular to the incident and the diffraction planes (i.e. to the plane *xz*).

After inserting (2) into the scalar wave equation for the S-polarization [8] we would obtain the following system of linear differential equations [4]:

$$\frac{d^2A_i}{dz^2} - 2jk_{i,z}\frac{dA_i}{dz} + \\ + \sum_{r=1}^{2} a_r\left[\sqrt{\frac{\cos\theta_i}{\cos\theta_{i-r}}}A_{i-r}\exp(j\Delta_{i-r}z) + , \right. \quad (3) \\ \left. + \sqrt{\frac{\cos\theta_i}{\cos\theta_{i+r}}}A_{i+r}\exp(-j\Delta_{i+r}z)\right] = 0$$

where

$$a_r = k^2\frac{n_r}{n_0},\ \Delta_{i-r} = k_{i,z} - k_{i-r,z},\ \Delta_{i+r} = k_{i+r,z} - k_{i,z}$$

The system of the coupled-wave equations can exist and be written as (3) only under the following conditions [4]:

$$rK + k_{i,x} - k_{i-r,x} = 0;\ rK - k_{i,x} + k_{i+r,x} = 0, \quad (4)$$

where $K = 2\pi/\Lambda$ is the ort module of the grating vector $\mathbf{K}$ directed along the axis *x*.

The angles θ_i are determined from the condition (4). Knowing $k_{0,x}$ (given by a direction of the incident wave) and using formulae (4), one can determine $k_{\pm1,x}$, then $k_{\pm2,x}$, and so on. As a consequence, $k_{i,z}$ is expressed as $k_{i,z} = \sqrt{k^2 - k_{i,x}^2}$. So all the quantities can be determined in the systems of equations (3). Let us note that the experimental examination [4] proves that the condition (4) is indeed true.

It is worth to mention that the systems of equations (3) are infinite in general ($-\infty < i < \infty$). In practice those systems are cut off, provided that i is bounded. For example, in the Kogelnik's theory we have $i = 0, 1$. To consider the second harmonics of the refractive index modulation, n_2, it is necessary to use the four-wave approximation, i.e. $i = -1, 0, 1, 2$. At the same time, the condition $-1 < \sin(\theta_i) < 1$ should be true.

If $n_1, n_2 << 1$, one can use a parabolic approximation [7] for the calculations of diffraction at thick holograms, thus neglecting the second-order derivative in the system (3). The simplified system of equations would be as follows:

$$\frac{dA_i}{dz} + \sum_{r=1}^{2} jb_r \times \left[\frac{A_{i-r}\exp(j\Delta_{i-r}z)}{\sqrt{\cos\theta_i \cos\theta_{i-r}}} + \frac{A_{i+r}\exp(-j\Delta_{i+r}z)}{\sqrt{\cos\theta_i \cos\theta_{i+r}}} \right], \quad (5)$$

with $b_r = \pi n_r / \lambda$.

This system of differential equations is linear, with variable coefficients. It is convenient for theoretical investigations, which will be performed below. For the four-wave approximation under the Bragg conditions we obtain

$$\begin{cases} \dfrac{dA_{-1}}{dz} = -jb_1 \dfrac{A_0 \exp(-j\Delta z)}{\sqrt{\cos\theta_{B1}\cos\theta_{-1}}} - \\ -jb_2 \dfrac{A_1 \exp(-j\Delta z)}{\sqrt{\cos\theta_{B1}\cos\theta_{-1}}}, \\ \dfrac{dA_0}{dz} = -jb_1 \dfrac{A_{-1}\exp(j\Delta z)}{\sqrt{\cos\theta_{B1}\cos\theta_{-1}}} - \\ -jb_1 \dfrac{A_1}{\cos\theta_{B1}} - jb_2 \dfrac{A_2 \exp(j\Delta z)}{\sqrt{\cos\theta_{B1}\cos\theta_{-1}}}, \\ \dfrac{dA_1}{dz} = -jb_2 \dfrac{A_{-1}\exp(j\Delta z)}{\sqrt{\cos\theta_{B1}\cos\theta_{-1}}} - \\ -jb_1 \dfrac{A_0}{\cos\theta_{B1}} - jb_1 \dfrac{A_2\exp(j\Delta z)}{\sqrt{\cos\theta_{B1}\cos\theta_2}}, \\ \dfrac{dA_2}{dz} = -jb_2 \dfrac{A_0\exp(-j\Delta z)}{\sqrt{\cos\theta_{B1}\cos\theta_2}} - \\ -jb_1 \dfrac{A_1\exp(-j\Delta z)}{\sqrt{\cos\theta_{B1}\cos\theta_2}}, \end{cases} \quad (6)$$

where $\theta_{-1} = -\theta_2 = \theta$, $\Delta = k(\cos\theta_{B1} - \cos\theta)$, $\sin\theta = 3\sin\theta_{B1}$, according to (4).

This system of equations will be further studied in order to determine the dependence of diffraction efficiency on the hologram thickness at a non-linear record for the case of diffraction at the first-order Bragg angle.

Effective value of the modulated refractive index of hologram material at a non-linear record

In the case of diffraction at the first-order Bragg angle, the boundary conditions are following: $A_{-1}(0) = 0,\ A_0(0) = 1,\ A_1(0) = 0,\ A_2(0) = 0$. Let us assume that n_1 and n_2 are so small that the thickness $A_0(z) \approx 1$ at the borders of hologram. Neglecting the first and the fourth equations of the system (6), we obtain:

$$A_{-1}(z) \approx b_1 \frac{\exp(-j\Delta z) - 1}{\Delta\sqrt{\cos\theta_{B1}\cos\theta}},\ A_2(z) \approx \approx b_2 \frac{\exp(-j\Delta z) - 1}{\Delta\sqrt{\cos\theta_{B1}\cos\theta}}. \quad (7)$$

Let us insert (7) into the third equation of the system (6) and integrate it. This would result in

$$A_1(z) \approx -\frac{jb_1 z}{\cos\theta_{B1}} - \frac{j2b_1b_2 z}{\Delta\cos\theta_{B1}\cos\theta}. \quad (8)$$

The relation (8) is obtained when considering that $z >> \left|\dfrac{\exp(-j\Delta) - 1}{\Delta}\right|$ for thick holograms. After inserting $b_1 = \pi n_1 / \lambda$, $b_2 = \pi n_2 / \lambda$, the expression (8) becomes

$$A_1(z) \approx -j\frac{\pi z}{\lambda\cos\theta_{B1}} \times \left[n_1 + \frac{n_1 n_2}{n_0 \cos\theta_1 (\cos\theta_{B1} - \cos\theta_1)} \right]. \quad (9)$$

Let us consider the first term in the r. h. s. of expression (9) as the first term of Macloren series of the function

$$A_1(z) = -j\sin\left(\frac{\pi z}{\lambda\cos\theta_{B1}}\times\right.$$
$$\left.\times\left[n_1 + \frac{n_1 n_2}{n_0\cos\theta_1(\cos\theta_{B1}-\cos\theta_1)}\right]\right). \tag{10}$$

It is known that the diffraction efficiency of hologram in the parabolic approximation equals to

$$\eta = |A_1(z)|^2 = \sin^2\left\{\frac{\pi z}{\lambda\cos\theta_{B1}}\times\right.$$
$$\left.\times\left[n_1 + \frac{n_1 n_2}{n_0\cos\theta_1(\cos\theta_{B1}-\cos\theta_1)}\right]\right\}. \tag{11}$$

The calculated points in the four-wave approximation are obtained when solving the system (5) with the fourth-order Runge-Kutte method. To apply the double-wave approximation, n_1 should also satisfy the next condition [9]:

$$n_1 < 0.4 n_0 \sin^2\theta_{B1}.$$

For $\theta_{B1} = 0.325$, n_1 can equal to $n_1 = 0.061$. That is why it is possible to apply double-wave approximation at $n_1 = 0.061$. The latter is proved in Fig.2, where the calculated points fit well the full curves.

Fig. 3a shows the dependences of the diffraction efficiency on the hologram thickness (3a) at $n_1 = 0.008$ and $n_2 = 0$ (thin line) and $n_1 = 0.008$ and $n_2 = -0.008$ (thick line), whereas Fig. 3b shows the angular dependence of the diffraction efficiency at the same conditions.

It is seen from Fig. 3 that, at the small values of n_2 and n_1, the influence is notable for high thickness holograms. This behaviour changes appreciably if n_1 is high, for example, 0.05. In this case it becomes possible to use the double-wave approximation [9] and replace n_1 with n_{1ef}, in accordance with the formula (11). Fig. 4 illustrates the dependences of the diffraction efficiency on the hologram thickness and the incident angle of the beam. The numbers near the curves specify the values of n_2.

Fig. 4 demonstrates that the non-linearity influences appreciably the diffraction efficiency at high n_1. The diffraction efficiency can differ by 10%, even if $n_2 = -0.01$ (i.e., n_2 amounts 20% of $n_1 = 0.05$, that being typical for the holographic recording) . The absolute values of n_2 and n_1 are comparable in case of a pronounced non-linearity of record and the diffraction efficiency could change from 25% ($n_2 = 0$) to 70% ($n_2 = -0.05$). Hence, a cardinal influence on the holographic descriptions is then observed.

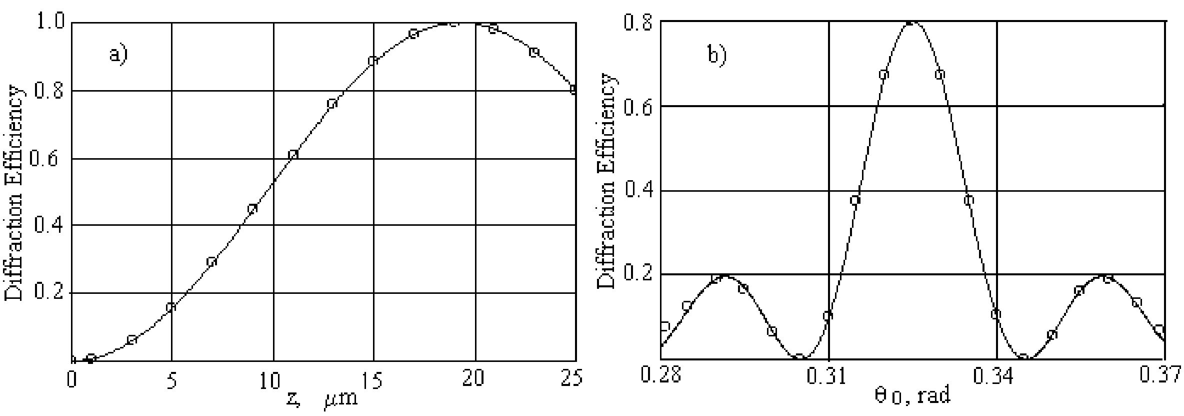

Fig. 2. Dependence of the diffraction efficiency on the thickness of hologram and the beam distribution angle of the zero-order diffraction. Here $n_0 = 1.5, \lambda = 0.633$ μm; (a) $\theta_0 = 0.325$ rad; (b) $z = 25$ μm.

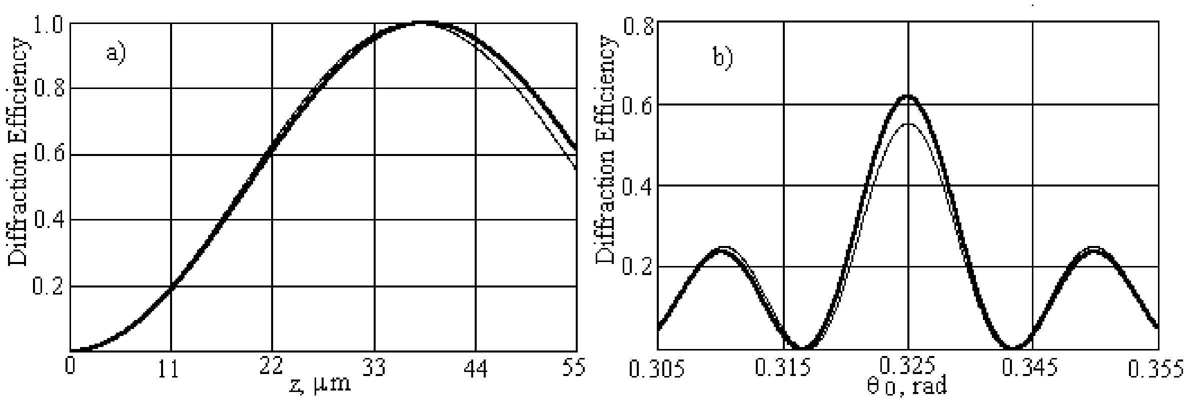

Fig.3. Dependence of the diffraction efficiency on the hologram thickness (a) and the incidence angle of beam (b) at $n_1 = 0.008$.

It is possible to consider the contribution of n_3 to n_{1ef}, using the same technique for determining the influence of n_2 on n_{1ef}. For this purpose, the summation has to be carried out in the system (5) for r changing from one to three. As a result, the system (6) would consist of six equations. In this case, the additional term proportional to n_3 appears in n_{1ef}:

$$n_{1ef} = n_1 + \frac{n_1 n_2}{n_0 \cos\theta(\cos\theta_{B1} - \cos\theta)} + \frac{n_2 n_3}{n_0 \cos\theta_2(\cos\theta_{B1} - \cos\theta_2)}, \qquad (12)$$

where θ_2 is the propagation angle for the second-order diffraction beam at the Bragg condition (i.e., $\sin\theta_2 = 5\sin\theta_B$).

Upon taking into account that n_3 is in practice much less than n_1 and less than n_2, while the maximal value n_1 for the photosensitive materials does not exceed 0.1 (except for the chalcogenide glasses [5] where n_1 could be higher than 0.1), one can estimate the third term in the equation (12) to be significantly less than the second one. This is just the reason why the numerical calculations performed in the work [4] did not reveal the influence of n_3 on the dependence of diffraction efficiency on the hologram thickness and the incidence angle of the beam. However, this case requires more detailed studies. In particular, one should compare the results for numerical solution of the differential equation system of the sixth order with the Kogelnik's formula, using the effective refraction index taking n_2 and n_3 into account, in accordance with the equation (12).

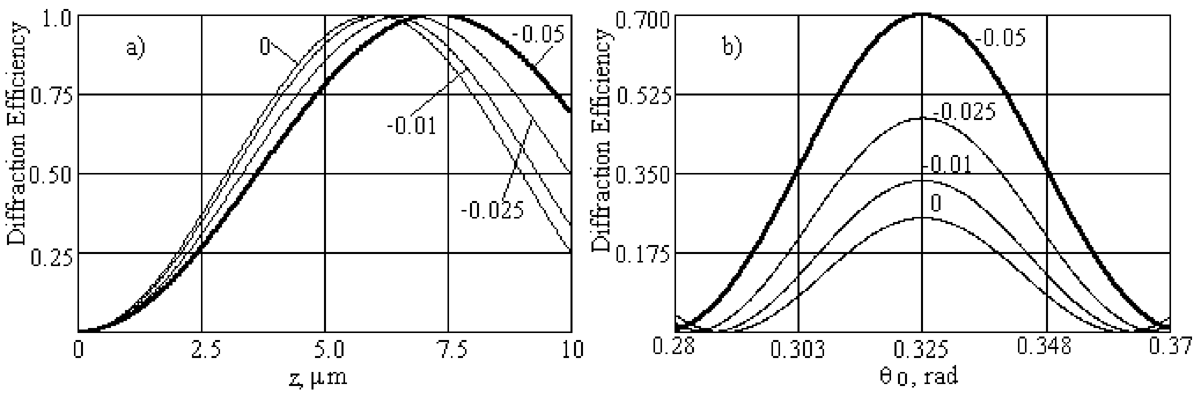

Fig. 4. Dependence of the diffraction efficiency on the hologram thickness (a) and the incidence angle of beam (b) at $n_1 = 0.05$.

Conclusions

Through the analysis of differential equation system (6), describing a distribution of optical waves in a thick diffraction grating generated during the non-linear holographic record, the analytical dependence of the diffraction efficiency of the hologram on its thickness is obtained under the Bragg conditions. The efficiency for the non-linear holograms can be determined with the known Kogelnik's formulae (both under the Bragg condition and at a small deviation from the latter), using, instead of n_1, the effective value n_{1ef} defined by the relation

$$n_{1ef} = n_1 + \frac{n_1 n_2}{n_0 \cos\theta(\cos\theta_{B1} - \cos\theta)} + \frac{n_2 n_3}{n_0 \cos\theta_2(\cos\theta_{B1} - \cos\theta_2)}.$$

The results for the diffraction efficiency derived on the basis of numerical solution of the equation system (6) and the Kogelnik's formulae using n_{1ef} practically coincide. Using the obtained relation, together with the results [2] on the experimental dependences of diffraction efficiency on the angle of incidence of the beam on the hologram in the vicinities of the first and second Bragg angles, it is possible to define the true values of n_1 and n_2, as well as the thickness of the hologram.

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