Modeling of phase relief of holographic diffraction gratings: application to self-developing photopolymers

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Received 13.09.2001
After revision 21.01.2002

Abstract

The thin holographic diffraction gratings with periodic surface/internal spatial phase modulation are investigated. The dependences of the efficiency in the various diffraction orders are measured and calculated in supposed profiles of phase relief. The calculated results are compared with experimental data of holographic phase gratings recorded in self-developing photopolymers. The numerical values of the phase shift $\Delta\phi$ of experimental samples are determined. It was found, that the surface relief conforms satisfactory the two-parabolic model, whereas the internal modulation profile in the volume of photopolymer can be well described with two-cylindrical model of the phase relief.

Keywords: thin phase holographic grating, diffraction orders, Wiener spectrum, spatial phase modulation, self-developing holographic photopolymer, surface and volume spatial modulation

PACS: 42.30Kq, 42.70.Ln

1. Introduction

The Damman approach is the most popular method to design the periodic phase relief of diffracted optical elements [1]. The arbitrary phase profile of the elementary cell is resulted in addition to finite number of steps, characterized by the constant phase shift. Such a model corresponds to the conditions of preparing the “multilevel” optical elements by means of photolithography.

However the description of the phase relief by the steps can’t be considered as the satisfactory one in the case of the periodic profile of the optical elements obtained by the holographic recording, in particular, in the case of the self-developing photopolymer materials. In the latter case the modulating function of the phase relief is smooth and continuous. While recording the interference pattern of two light waves one often supposes a typical harmonic phase profile light diffraction which can be well described by means of the expansion into a series with respect to the Bessel functions. Such an expansion can be also used in more complicated inharmonic phase profile [2]. Recently, the phase elements with parabolic [3,4] and cylindrical [5] phase relief have attracted a special attention. They are characterized by a higher diffraction efficiency in comparison with the harmonic phase relief.

In the present paper, basing on the method of signal coordinate-frequency distribution [6,7] we have studied the peculiarities of the Wiener spectra of space frequencies of the thin holographic gratings with the phase profile which is modelled by the curves of the second order. Early [8] such an approach was used in description of the Wiener spectra of quasi-orthogonal and random binary phase elements.

2. Wiener spectra of the periodic phase elements (PPE)

Let us consider the periodic phase element

$$f(x)=\sum_{n=-\infty}^{\infty} f_0(x)\delta(x-nT),$$
$$f_0(x)=rect\left(\frac{x}{T}\right)\exp[i\pi\phi(x)], \qquad (1)$$

where $rect(x/T)$ is the rectangle function [9], which is described by the continuous phase function $\phi(x)$ for $-T/2<x<T/2$, where T is the spatial period. For a given element, which forms the stationary optical signal with the given phase distribution while a plane monochromatic wave illuminates it, let us construct the basic functional in the form of the ambiguity function [6]

$$W_{ff^*}(x_0;\omega_0)=$$
$$=\int_{-\infty}^{\infty} f\left(x+\frac{x_0}{2}\right)f^*\left(x-\frac{x_0}{2}\right)\exp(-i\omega_0 x)dx \qquad (2)$$

where $f^*(x)$ is the complex conjugate optical signal. On the basis of the definition (1) the coordinate-frequency distribution of PPE is given by the expression:

$$W_{ff^*}(x_0;\omega_0)=\omega_{01}\times$$
$$\sum_{n}^{\infty}\sum_{m=-\infty}^{\infty}(-1)^{nm}W_{f_0f_0^*}(x_0-nT;m\omega_{01})\times$$
$$\delta(\omega_0-m\omega_{01}) \qquad (3)$$

where $W_{ff^*}(x_0;\omega_0)$ is the coordinate-frequency distribution of the elementary cell, $\omega_{01}=2\pi/T$ is the spatial frequency of the phase profile.

For the calculation of the Wiener spectrum of PPE let us put $\omega_0=0$ in the distribution (2). Then the Wiener spectrum is calculated as follows:

$$\left[F(\omega)\right]^2=\int_{-\infty}^{\infty} W_{ff^*}(x_0;0)\exp(-i\omega x_0)dx_0 \qquad (4)$$

After substituting (4) the explicit expression of the distribution (3) the Wiener spectrum of the spatial frequencies of the periodic phase elements can be written in the general form

$$\left[F(\omega)\right]^2=\omega_{01}\sum_{n=-\infty}^{\infty}\left[F_0(n\omega_{01})\right]^2\delta(\omega-n\omega_{01}) \qquad (5)$$

where $\left[F_0(n\omega_{01})\right]^2$ is the intensity of the Wiener spectrum of the spatial frequencies of the periodic phase element in the n-th order of the diffraction, which is calculated as a discrete sample of intensities in the equidistance points from the continuous Wiener spectrum $\left[F_0(\omega)\right]^2$ of the elementary cell of the phase relief:

$$\left[F_0(\omega)\right]^2=\left[\int_{-T/2}^{T/2}\exp[i\pi\phi(x)]\exp(-i\omega x)dx\right]^2. \qquad (6)$$

Thus, having the results of the calculation of the Wiener spectrum of the elementary cell one uniquely obtains the intensities of the Wiener spectrum of the phase elements in different orders of diffraction.

Study of some typical profiles, in particular, of the parabolic and harmonic ones was performed in [3,4]. Within the frames of the suggested approach let us describe more complicated profiles of the elementary cell of the periodic phase element.

Let us write the phase profile of the elementary cell in the general form of the joined curves of the second order $\phi_1(x)$ and $\phi_2(x)$ shifted by a half period

$$f_0(x)=rect\left(\frac{x}{\gamma T}\right)\exp[i\pi\phi_1(x)]$$
$$+rect\left(\frac{x-T/2}{(1-\gamma)T}\right)\exp[i\pi\phi_2(x)] \qquad (7)$$

where a join point of two rectangle profiles is given by the parameter $\gamma=2x_c/T \quad (0\ \gamma\ 1)$, where the coordinate of continuous joining of two curves x_c is determined on the conditions:

$$d\phi_1(x)/dx_{x=x_c}=d\phi_2(x)/dx_{x=x_c}; \qquad (8)$$

$$\phi_1(x_c)=\phi_2(x_c). \qquad (9)$$

Two-parabolic relief of the elementary cell. According to Eq. (7) let us write the equation of the parabolic reliefs:

$$\phi_1(x) = \phi_{\max} - p_1\left(\frac{2x}{T}\right)^2;$$

$$\phi_1(x) = \phi_{\min} + p_2\left(\frac{2x}{T} - 1\right)^2. \qquad (10)$$

A construction of two-parabolic relief of the elementary cell at $\alpha = p_1 / p_2 = 2$ is shown in Fig. 1 (downward). Here to simplify, we consider the following unit interval in which the phase changes: $\Delta\phi = |\phi_{\max} - \phi_{\min}| = 1$. From Eq. (8) of two-parabolic relief one gets the following value of the joining parameter γ:

$$\gamma = \frac{p_2}{p_1 + p_2}. \qquad (11)$$

While Eq. (9) is satisfied with the continuous joining of the parabolas and with the coefficients p_1 and p_2 the phase different $\Delta\phi$ of the two-parabolic relief as it is shown in Fig. 1 (upward) can be determined according to

$$\Delta\phi = \frac{p_1 p_2}{p_1 + p_2} = \gamma p_1. \qquad (12)$$

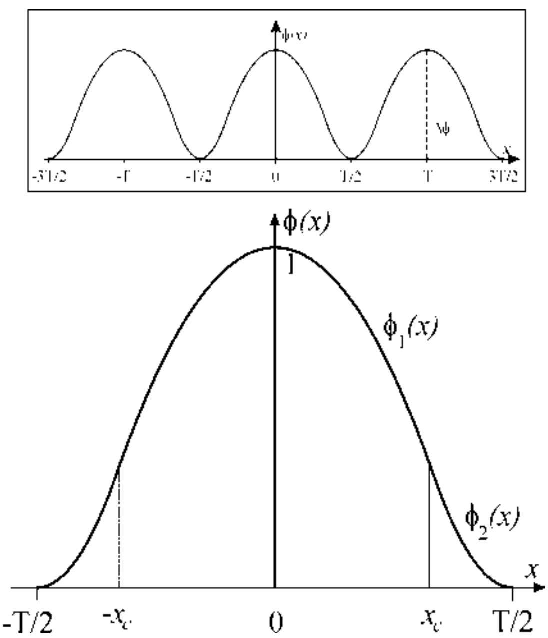

Fig. 1. The construction of the two-parabolic relief.

Taking different values of the coefficients p_1 and p_2 and consequently changing the value of γ, one gets different forms of the parabolic phase relief.

Two-cylindric relief of the elementary cell. Let us write the equations of cylindrical profiles in the form

$$\phi_1(x) = \phi_{\max} - \left[\frac{2r_1}{T} - \sqrt{\left(\frac{2r_1}{T}\right)^2 - \left(\frac{2x}{T}\right)^2}\right];$$

$$\phi_2(x) = \phi_{\min} + \left[\frac{2r_2}{T} - \sqrt{\left(\frac{2r_2}{T}\right)^2 - \left(\frac{2x}{T} - 1\right)^2}\right]. \quad (13)$$

As it is shown in Fig. 2 we consider the general case when the centers of the cylinders lie on the line which has a slope φ with respect to the axis *0X*. Since the point of touching of both cylinders is in that line the condition (8) is fulfilled automatically. In this case for the cylinder radii one can write $(r_1 + r_2)\cos\varphi = T/2$. Therefore, the coordinate of touching the cylinders x_c and the joining parameter γ are calculated according to the formula

$$x_c = r_1 \cos\varphi; \quad \gamma = \frac{r_1}{r_1 + r_2}. \quad (14)$$

For two-cylindrical relief the phase difference $\Delta\Phi$ of the periodic phase element as is shown in Fig.2 (upward) is calculated according to the formula:

$$\Delta\phi = |\phi_{\max} - \phi_{\min}| = (r_1 + r_2)(1 - \sin\varphi). \qquad (15)$$

In this case we have two independent parameters: the parameter γ and the angle φ. This circumstance provides additional possibilities to design different forms of two-cylindrical phase profile. The suggested method to design the elementary cell may be used in the description of the more complicated phase profile. However, for modeling the phase relief which is formed in a real self-developing photopolymer material it is sufficiently to use only the curves of the second order.

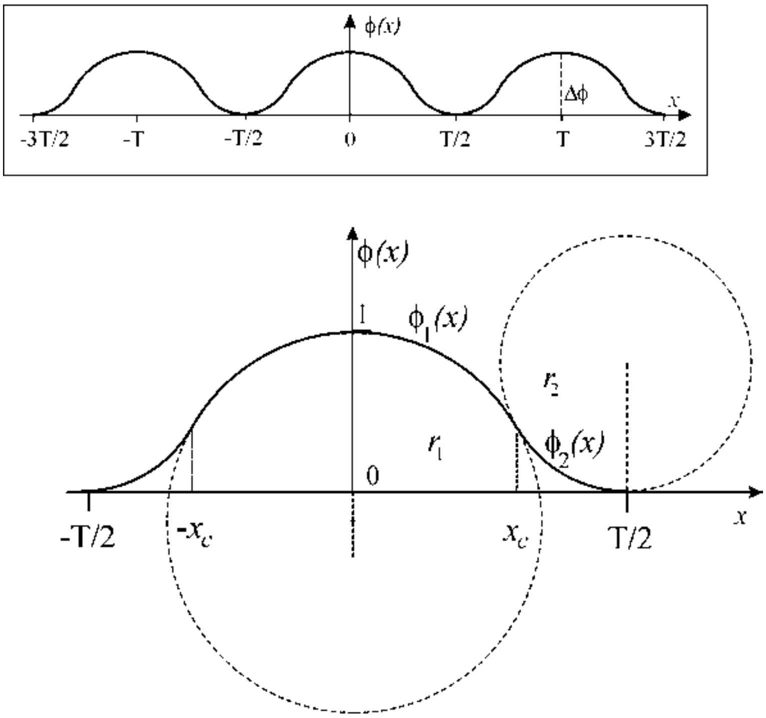

Fig.2. The construction of two-cylindrical phase relief.

3. Diffraction efficiency of PPE with the parabolic and cylindrical profiles

In the frame of the work it was important to see the effect of the form of the phase relief on the diffraction efficiency of the periodic phase elements. In Figs. 3 and 4 the results of numerical computation of the diffraction efficiency in the 1 st order of diffraction depending on rations of p_1 and p_2 and r_1 and r_2 are shown.

For two types of the studied phase elements one observes a general tendency of an increasing of the maximum of the diffraction efficiency while passing from the one-element (α=0, σ=0) to the symmetric two-element (α=1, σ=1) phase profile. Thus, the formation of the phase profile, which is close to the symmetric two-element one is the definite way to increase the diffraction efficiency of the thin gratings.

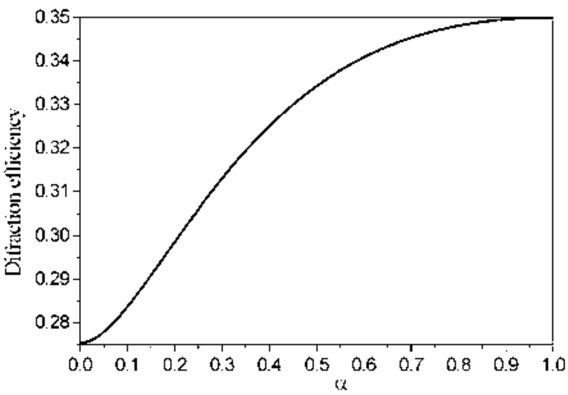

Fig.3. The dependence of the maximum of the diffraction efficiency in the $\pm 1^{st}$ order of diffraction by two-parabolic phase profile on the parameter $\alpha = p_2 / p_1$.

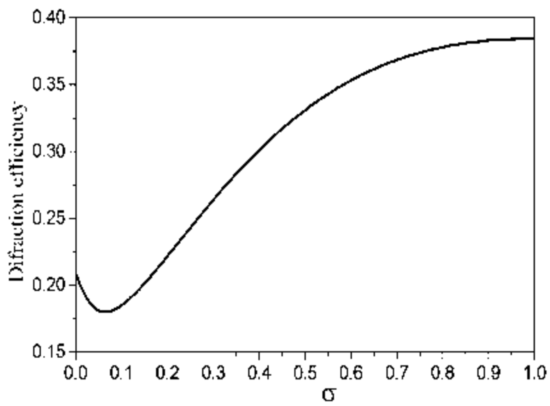

Fig.4. The dependence of the maximum of the diffraction efficiency in rhe $\pm 1^{st}$ order of diffraction by two-cylindric phase profile on the parameter $\sigma = r_2 / r_1$.

The quantities $DE_{\max}$ essentially depends on the form of the basic phase profile. The parabolic profile of the minimal value [3,4] $DE_{\max}$ = 0.2753 smoothly increases to the value $DE_{\max}$ = 0.35. the cylindrical interval 0 ≤ σ ≤ 0.07 the quantity DE decreases somewhat. However, with further increases of σ the diffraction efficiency of two-cylindrical phase profile rapidly grows up and becomes $DE_{\max} \simeq$ 0.37 which overcomes the limiting value $DE_{\max}$ for the symmetric two-parabolic phase profile.

In Fig. 5 and 6 the numerical calculation of the phase difference $\Delta\phi$ of the optical element, in the case the value $DE_{\max}$ is max, are shown different values of parameters α and σ. In both studied types of PPE the position of $DE_{\max}$ in

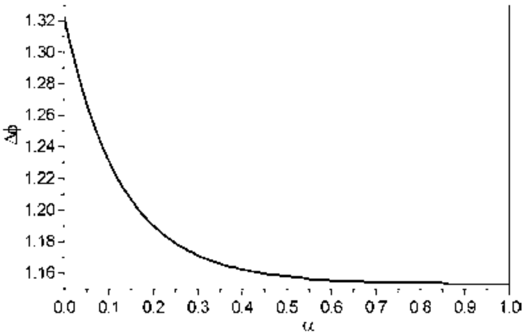

Fig.5. The change of the phase shift $\Delta\phi$ in the maximum of diffraction efficiency (Fig.3) as a function of the relation $\alpha = p_2 / p_1$.

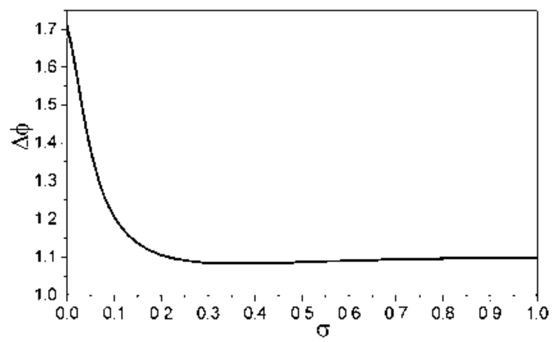

Fig.6. The change of the phase shift $\Delta\phi$ in the maximum of diffraction efficiency (Fig.4) as a function of the relation $\sigma = r_2 / r_1$.

the ± 1st diffraction order moves towards smaller values $\Delta\phi$ while passing the symmetric two-element form of phase profile.

As can be seen, the parabolic phase relief the value $\Delta\phi$ right along decreases in the interval $1.32\pi \doteq 1.15\pi$. As to the cylindrical phase profile one observes somewhat different behavior. At first the phase shift $\Delta\phi$ in the maximum of the diffraction efficiency decreases fast from its maximal value $\Delta\phi \simeq 1.7\pi$ as σ increases. Then as to $\sigma > 0.25$ the position of $DE_{\max}$ remains constant and corresponds to the value $\Delta\phi \simeq 1.1\pi$.

The obtained result permits to state, that forming the two-cylindrical profile of the elementary cell gives the possibility to achieve the value $DE_{\max}$ at smaller value of the phase shift $\Delta\phi$

4. Experimental results

The important part of the conducted investigation consists of restoring the phase profile of the thin holographic gratings that were recorded on the self-developing photopolymers PPC-488 [11]. The set task is solved by consequent analysis of the measured and calculated the Wiener spectra of some assumed profiles that can be checked independently of the experiment. The examination of the relief formation under holographic recording of the thin photopolymer gratings has been previously undertaken in our work [11].

Testing the above presented theoretical model three basic types of phase holographic thin grating have been done. The sample #1 was recorded under conditions, when the phase modulation had extremely surfaced the origination and there was no volume modulation of an refraction index. The shape of the surface relief was measured by micro-interferometer and is shown in a Fig. 7a.

The sample #2 in opposite was recorded under conditions when the phase modulation of

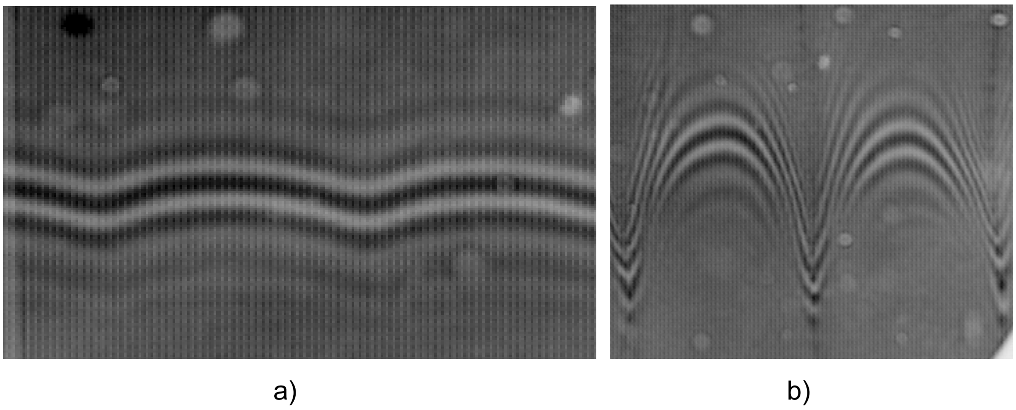

Fig. 7. The surface relief resulted from holographic recording a)-sample #1; b)-sample #3. The thin gratings with thickness 20 μm and spatial period 50 μm are recorder Ar-laser in photopolymer PPC-488.

the photopolymer film was extremely to score the volume variation of an refraction index.

The sample #3 was recorded in most broadly realised conditions, when the surface relief as well as the volume modulation of the refraction index in photopolymer medium took place. We have proved earlier that maximums of the refraction index in polymer film volume coincide with minimums of the relief on its surface.

The used procedure of the phase profile parameters calculation lay in the following. As the first step of the calculation the relative intensities I_n / I_0 are determined, where n=1,2,3… are the diffraction order of the Wiener spectrum. Calculated values depend on the phase shifts that in turn is related to arbitrary selected parameters of a parabolic or cylindrical type of the phase profiles. The family of the calculated dependences I_n / I_0 and the experimental datum as horizontal lines are superimposed. The intersection points of the experimental lines with relevant curves of the diffraction efficiency show the value of the relative intensity of the Wiener spectrum. Since there are some intersection points in experimental lines and a theoretical curve therefore for one order of the diffraction the problem has no singular solution. However with results of measured intensities in several (more than 3 diffraction orders) it is possible to find out the singular spatial profile where all intersection points are located near the same phase shift. For searching the best value $\Delta\phi_{med}$ the special program has been designed.

In a Fig.8 the results of the calculation of relative intensities of the Wiener spectrum of the parabolic relief in the range $0.1 \le \Delta\phi \le 1.0$ are given, for the first six orders of the diffraction. On this diagram the experimental (normalized to zero order intensity) the diffraction intensities of sample 1 are traced by horizontal lines. As it is also seen from Fig.7a, the surface relief of sample #1 is the closest to the parabolic shape. It is found, by the method of computer modelling that for the relation $p_2 / p_1 = 6$ of parabolas the intersection points direct in 2-nd up to 5-th diffraction orders are close bunched near to a mean $\Delta\phi_{med} \approx 0.510\pi$. Taking into account that intersection points biased in 1-st and 6-th diffraction orders in the opposite parts from the mean, we have received for sample #1 the mean phase shift $\Delta\phi_{med} \approx 0.502\pi$, which only in the third sign differs from the previous value. It gives hint to propose that model of the parabolic phase relief most conforms to the shape of surface phase relief of sample #1. As $\Delta\phi = (2\pi h / \lambda)\Delta n$, where h- height of a phase relief, λ-wave length of a light, Δn-odds of indexes of refraction, for a given sample we gain $h\Delta n \approx 0.251\lambda$

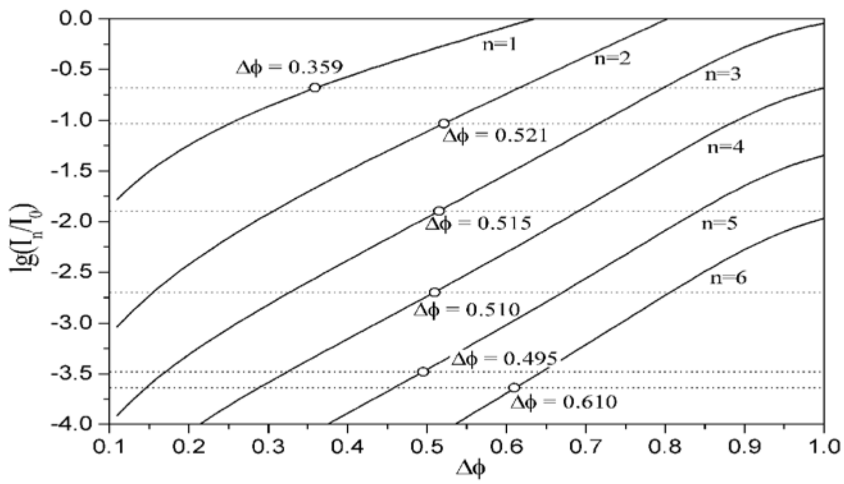

Fig.8. Relative intensities of Wiener spectrum of a parabolic phase relief at $p_2 / p_1 = 6$ from the phase shift $\Delta\phi$ in the first six diffraction orders. (The dashed lines trace experimental dates for a sample #1).

In the table 1 we cite the normalized intensities (I_0 is the zero-order diffraction intensity)

Sample #2 presents the particular interest because there was no information about the shape of the phase profile in volume medium of the photopolymer material. The conducted examinations have shown that for the sample #2 parabolic models of the phase relief are not suitable. In Fig. 9 the results of calculation of relative intensities of the Wiener spectrum in the first four diffraction orders of the sample #2 are presented in the case of the cylindrical shape of the phase relief. The cylindrical model at value of parameters $\sigma = 2.85, \phi = 0.13\pi$, conforms to a phase profile, quite well which was shaped in the photopolymer film volume medium of the photopolymer compound. The mean of the phase shift equals $\Delta\phi_{med} \approx 0.227\pi$.

Let us analyze this quantity. Let us assume that harmonic shape of the phase profile in this case is fit and the diffraction intensity in all orders is described by the Bessel functions: $I_n / I_0 = J^2{}_n(\pi\Delta\phi/2)/J^2{}_0(\pi\Delta\phi/2)$. Using numerical values of the Bessel functions for all orders [10], for the first diffraction order one has: $J^2{}_n(\pi\Delta\phi/2)/J^2{}_0(\pi\Delta\phi/2) = 0.056$, that corresponds $\pi\Delta\phi/2 \cong 0.4606$. From here we gain $\Delta\phi \cong 0.293$. Thus, we have gained $\Delta\phi$ quite close to one in our procedure. For other orders the pattern essentially differs. Data of experimental measurements that were treated with the harmonic model give: 2 order diffraction - $\Delta\phi \cong 0.464\pi$; 3-rd order diffraction - $\Delta\phi \cong 0.487\pi$; 4-th order diffraction - $\Delta\phi \cong 0.785\pi$. Thus for sample #2 the harmonic model gives poor description of higher diffraction orders, in spite of the first order is described well.

Results of examination thin gratings #1 and #2

Table 1.

Sample	I_1/I_0	I_2/I_0	I_3/I_0	I_4/I_0	I_5/I_0	I_6/I_0
1	2.1×10^{-1}	9.3×10^{-2}	1.3×10^{-2}	2.0×10^{-3}	3.3×10^{-4}	2.3×10^{-4}
2	5.6×10^{-2}	5.3×10^{-3}	1.1×10^{-4}	7.3×10^{-5}		
3	8.8×10^{-1}	6.6×10^{-1}	3.4×10^{-1}	3.9×10^{-1}	3.4×10^{-1}	

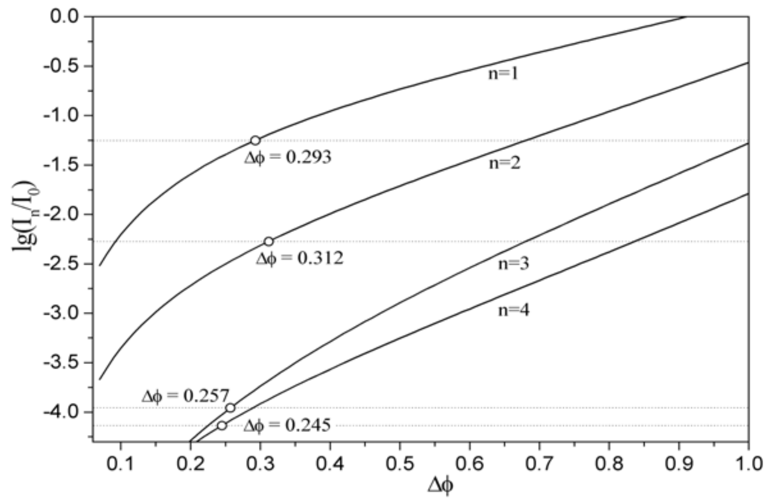

Fig.9. Relative intensities of Wiener spectra of a cylindrical phase profile at parameters $\sigma = 2.85, \phi = 0.13\pi$, as function of the phase shift $\Delta\phi$ in the first four diffraction. (The dashed lines trace experimental data for a sample #2)

allow to put forward the guess, that for the sample of grating #3, which was formed by the surface and volume phase modulation, is possible to describe by model of "two-parabolic+two-cylindrical" of the phase profile. For the similar model the calculation program of the Wiener spectrum in different diffraction orders depending on both profiles is composed simultaneously. Fig. 10 presents the calculation results for first five orders of the Wiener spectra. The experimental lines are seen to intercross the relevant theoretical curves in points, which are distributed in much more greater interval of values $\Delta\varphi$, than for samples #1 and #3. It is the hint that the offered model features make the case worse and the next improvement of the model is necessary.

In spite of the proposed theoretical model of the phase profile is approximate, the

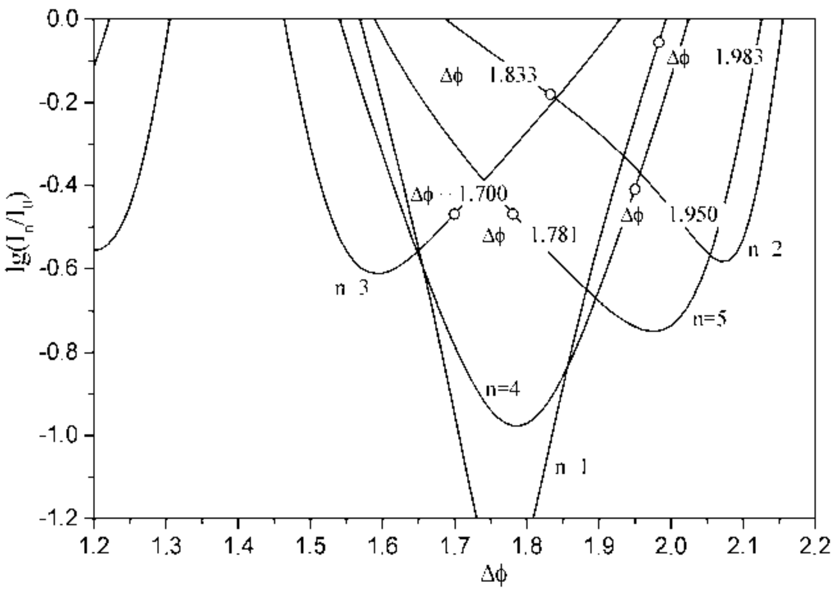

Fig.10 Relative intensities of a Wiener spectrum cylindrical + parabolic phase profile at parameters $\gamma = 0.75, \phi = 0$ as function of $\Delta\phi$. (The dashed lines trace experimental dates for a sample #3).

intersection points of the experimental lines correspond to major values $\Delta\phi$, that agree with experimental data on Fig. 7b. On the basis of the conducted examinations for sample #3 we have gained the mean phase shift $\Delta\phi \approx 1.849\pi$. The medial quadratic dispersion of values in different diffraction orders equals value 0.105π. Therefore we hope that the offered model can be used as the first approximation in the description of similar gratings.

The conducted investigation is concluded in the following: the comparative examination of the experimental Wiener spectra and the theoretical calculation of the Wiener spectra of the periodic phase structures presents the basic to restore parameters of the phase profile. The offered procedure, which is founded on prime experimental examinations, can be used for manufacturing the holographic optical elements with the given phase profile.

References

1. H. Damman. Optics **43(5)** (1979) 409-417.
2. K. Patorski, P.Szwaykowski. Opt.Aplicata XI **(4)** (981) 627-631.
3. G. Neto, L. B. Roberto, P. Verdonck, R. D. Mansano, G.A. Cirino, M. A. Stefani. Proc. SPIE **3749**, (1999) 487-488.
4. Langois and R. Beaulieu., Appl.Opt. **29(23)** (1990) 3434-3439.
5. Magnusson and D. Shin. J.Opt.Soc.Amer. A **6(3)** (1989) 412-414.
6. Shovgenyuk. Preprint ICMP-93-22U (1994) 32 p. (in Ukrainian)
7. Shovgenyuk, P. A. Hlushak. Proc. SPIE **2647**, (1995) 468-474.
8. M.V. Shovgenyuk, T.Ye. Krokhmalskii, M.P.Kozlovskii. Proc. SPIE **3749** (1999) 689-690.
9. D.Gaskill. Linear systems Fourier transforms and Optics., John Wiley & Sons, New York / Chichester / Brisbane / Toronto (1978).
10. Handbbok of Mathematical Function. Edited by M. Abramowitz and I. A. Stegun. National Bureau of Standards Appl. Math. **(55)** (1964) 830 p
11. V. Sakhno, T. N. Smirnova, E. A. Tikhonov. Optica & spectroscopia, **76(5)** (1994) 805-812.