Light beams focusing in periodically non-uniform crystalline medium

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Abstract

In the present paper the peculiarities of propagation and diffraction of optical waves in the crystalline periodically non-uniform media are considered. The dependence of light beam divergence in the crystal on curvatures of wave vector surface is obtained. It is shown, that presence of the periodic non-uniformity of the crystal and natural anisotropy lead to essential deformation of wave vector surface, namely, to the appearance of local regions with negative curvature and, hence, to the effect of light focusing in corresponding directions. The application of uniform electric field may intensify the light focusing. The crystalline diffraction lenses are proposed which are controlled by electric field and variation of incident light polarization.

Keywords: divergence of laser beam, tensor derivative of group velocity, crystalline periodically non-uniform medium

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Introduction

Much attention has been recently focused on crystalline periodically non-uniform medium (CPNUM) [1-5]. This interest is stipulated by broad variety of application, including the control of spontaneous emission, vertical- cavity surface emitting semiconductor lasers, Bragg reflecting structures, low-threshold optical limiting and switching [1-3]. Such crystals may be widely used for control of the parameters of laser radiation. For example, in the articles [6,7] the possibility of high- effective compression short and ultra-short pulses in periodically non-uniform medium have been grounded.

As a rule, the investigations of laser radiation propagation in the crystals with periodic non-uniformity were limited by the case of structures formed by isotropic media. But anisotropic materials are more interesting. In the present report the peculiarities of transformation of laser beams in CPNUM created on the base of anisotropic media are considered.

Propagation of laser beams in crystalline periodically non-uniform medium

In this section we consider the mathematical model of laser beam propagating in arbitrary medium. We propose matrix approach permitting to obtain analytical expressions describing the peculiarities of diffraction of laser radiation in the crystal with periodic non-uniformity.

2.1. Model of laser beam

In general, the field of light radiation in the crystal is presented as superposition of monochromatic waves by integral

$$\underline{E}(\underline{r},t) = (2\pi)^{-3} \times \int d\underline{k} A(\underline{k},\omega)\underline{a}(\underline{k},\omega)\exp i[\underline{k}\underline{r} - \omega t] , \qquad (1)$$

where $\underline{E}$ is the electric vector, $\underline{a}$ are the polarization vectors of partial waves. The deviation of frequency ω from mean value may be expressed through the deviation of wave vector $\underline{k}$ from its mean value $\underline{k}_0 = (\omega_0 / v_0)\underline{n}_0$ in the following form:

$$\omega(\underline{k}_0 + \underline{q}) \approx \omega_0 + \underline{U}\underline{q} + \frac{1}{2}\underline{q}W\underline{q}, \qquad (2)$$

where ω_0, v_0 are the frequency and phase velocity of central wave of the beam for which wave normal is $\underline{n}_0$, respectively, $\underline{q} = \underline{k} - \underline{k}_0, \underline{U} = \partial\omega / \partial\underline{k}$ is vector of group velocity, $W = \partial\underline{U} / \partial\underline{k}$ is tensor of the gradient of group velocity. To monochromatic beams we have $\omega = \omega_0$, and from (Eq. 2) one may obtain the expression for longitudinal component of the wave vector as a function of its transversal components $\underline{q}_\perp = [\underline{q}\underline{n}_0]$:

$$k_z = k_{0z} - \underline{U}\underline{q}_\perp / v - \underline{q}_\perp W \underline{q}_\perp /(2v). \qquad (3)$$

Then, assuming that the beam is weakly divergent (and, hence, we may neglect the changes of the polarization of partial waves), polarized as the central wave, and accounting that for non- dispersion medium tensor W is plane, one can find from Eq.1 the field of light beam:

$$\underline{E}(\underline{r},t) = (2\pi)^{-2} \times \int d\underline{q}_\perp A(\underline{q}_\perp)\underline{a}\exp i \times \times\left[(\underline{r} - \underline{U}z / v_0)\underline{q}_\perp - \underline{q}_\perp W \underline{q}_\perp z/(2v_0)\right] \times \times \exp i[\underline{k}_0\underline{r} - \omega_0 t)]. \qquad (4)$$

It is possible to show that for arbitrary beam propagating in single crystal the eigenvalues W_i (i=1,2) characterize the divergence of light radiation: by $W_i < 0$ the beam in the crystal is focusing; by $W_i > 0$ it is defocusing , and if $W_i = 0$ the non-diffractional propagation takes place.

Such a model of laser beam may be used in the case of crystals with periodic non-uniformity. With the help of this, the main task is the determination of eigenvalues W_i. It will be considered in the next part of the report.

2.2. Diffraction of laser beam in the crystal with periodic non-uniformity

Let us consider now the influence of periodical non-uniformity of medium on its diffraction properties. Let us assume that the light beam propagates in the crystal where we'll use the model of periodic two-layered structure, formed by anisotropic media possessing the symmetry center (see fig.1).

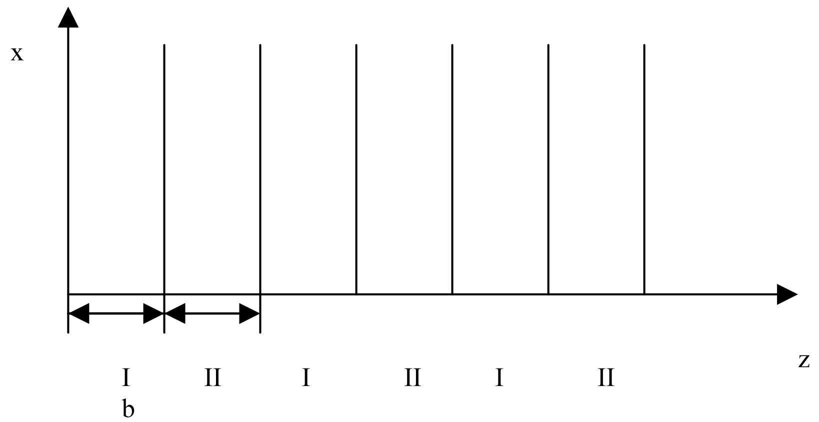

Fig. 1. Periodic layered structure as a model of crystalline periodically non-uniform medium.

The electric field in such structure is described by wave equation:

$$-\nabla^2 \underline{E}(\underline{r}) - \frac{\omega^2}{c^2}\varepsilon'(\underline{r})\underline{E}(\underline{r}) = \frac{\omega^2}{c^2}\varepsilon^0 \underline{E}(\underline{r}) . \qquad (5)$$

Here $\varepsilon^0_{ij} = (\varepsilon^{(1)}_{ij} b + \varepsilon^{(2)}_{ij}(D-b))/D$ is invariable (on the distance of period D of the structure) part of the tensor of dielectric permeability. The variable part of the latter has the following form:

$$\varepsilon'_{ij}(\underline{r}) \equiv \varepsilon'_{ij}(z)$$

$$\varepsilon'_{ij}(z) = \begin{cases} \varepsilon^{(1)}_{ij} - \varepsilon^0_{ij}, if\ mD < z < mD + b \\ \varepsilon^{(2)}_{ij} - \varepsilon^0_{ij}, if\ mD + b < z < (m+1)D \end{cases} \qquad (6)$$

Here i=1,2 is the number of layer, b is the thickness of the first layer, *m* is the number of cell. Note that in general case

$$\varepsilon^{(i)}_{ij} = \tilde{\varepsilon}^{(i)}_{ij} - \tilde{\varepsilon}^{(i)}_{ki}\tilde{\varepsilon}^{(i)}_{lj} R_{klmn} E^0_m E^0_n , \qquad (7)$$

where $\tilde{\varepsilon}^{(i)}_{ij}$ is the tensor of linear dielectric permeability of i –th layer in the absence of electric disturbance, R_{klmn} is the tensor of quadratic electrooptical coefficients, $\underline{E}^0$ is the quasi-stationary external electric field. In the absence of periodical non-uniformity wave the equation (5) in general case determines two light waves propagating in one direction, but having different wave vectors and mutually orthogonal polarization vectors. Let us analyze the influence of induced anisotropy in the propagation of the mode with given wave vector $\underline{k}_i$ and polarization vector $\underline{a}$.

We search for the solution of the equation (5) as superposition of two waves, namely, refracted $\underline{E}_i$ and diffracted $\underline{E}_d$ waves:

$$\begin{aligned} \underline{E} &= \underline{E}_i(z)\exp i(k_i z - \omega t) + \\ & \underline{E}_d(z)\exp i(k_d z - \omega t). \end{aligned} \qquad (8)$$

Here k_i, k_d are wave numbers of waves determining from equation (5) by $\varepsilon'(r) \equiv 0$. By this, the presence of periodical non-uniformity of medium causes the appearance of dependence of wave amplitudes on propagation coordinate (z).

Periodical part of $\varepsilon(z)$ tensor may be presented in the form:

$$\varepsilon'(z) = \varepsilon_1 \exp ik_G z + \varepsilon_{-1}\exp - ik_G z, \qquad (9)$$

where $k_G = 2\pi/D$, for medium without absorption $\varepsilon_1 = \varepsilon^*_{-1}$ and by $b = D/2$, for example, $\varepsilon_1 = -2ia/\pi$; $a = [\varepsilon^{(1)} - \varepsilon^{(2)}]/2$. Substituting (8), (9) into (5), we obtain within approximation slowly varying amplitudes the set of equations:

$$\begin{aligned} \frac{d\underline{E}'_i}{dz} &= P\underline{E}'_i + M\underline{E}'_d; \\ \frac{d\underline{E}_d}{dz} &= Q\underline{E}'_i \end{aligned} \qquad (10)$$

Here $P = i\Delta k; M = i\chi_1; Q = -i\chi_2$; $\underline{E}'_i = \underline{E}_i \exp i\Delta kz$, $\Delta k = k_i + |k_d| - k_G$ is wave mismatch.

$$\chi_1 = \frac{\omega^2}{2c^2 k_i}\varepsilon_1; \chi_2 = \frac{\omega^2 \varepsilon^*_1}{2c^2 |k_d|}; \qquad (10a)$$

The Eq. 10 may be reduced to differential equation of the second order:

$$\frac{d^2 \underline{E}_d}{dz^2} + R\frac{d\underline{E}_d}{dz} + S\underline{E}_d = 0, \qquad (11)$$

where $R = -QPQ^{-1}; S = -QM$. The solution of Eq.11 is:

$$\begin{aligned} &\underline{E}_d = \exp(-Rz/2) \\ &\{\cos(zT^{1/2})C_1 + T^{-1/2}\sin(zT^{1/2})C_2\} \end{aligned} . \qquad (12)$$

As it follows from Eq.12 and Eq.10, the electric vector of the refracted wave is

$$\begin{aligned} &\underline{E}_i = \exp(-i\Delta kz)Q^{-1}\{-(R/2)\exp(-Rz/2) \\ &\{\cos(zT^{1/2})C_1 + T^{-1/2}\sin(zT^{1/2})C_2\} + \\ &+ \exp(-Rz/2)\{-T^{1/2}\sin(zT^{1/2})C_1 + \\ &\cos(zT^{1/2})C_2\}\} \end{aligned} \qquad (13)$$

Here

$$T = \exp(Rz/2)(S - R^2/4)\exp(-Rz/2).$$

$$R = -i\Delta k;\ S = -\chi_2\chi_1;\ T = \frac{(\Delta k)^2}{4} - \chi_2\chi_1.$$

Constants C_1, C_2 in Eq.12 and Eq.13 may be obtained with boundary conditions taken into account:

$$\underline{E}_d(L) = 0; \qquad \underline{E}_i(0) = \underline{E}^{00}, \tag{14}$$

where L is the length of periodic structure. So,

$$C_1 = -A^{-1}T^{-1/2}\sin(LT^{1/2})Q\underline{E}^{00},$$
$$C_2 = [1-(R/2)A^{-1}T^{-1/2}\sin(LT^{1/2})]Q\underline{E}^{00}, \tag{15}$$
$$A = \cos(LT^{1/2}) + T^{-1/2}\sin(LT^{1/2})(R/2).$$

Obtained expressions Eq.13–Eq.15 permit to analyze the peculiarities of light diffraction of the periodic structure of arbitrary symmetry of components. In the case of diagonal tensors in Eq.13, the expression Eq.13 may be simplified. This takes place, for example, in layered structure formed by crystals of **mmm** symmetry class with X_3 along crystallographic axis, which is to orthogonal the layer boundaries.

Let us consider the case when $\Delta k^2 >> \chi_2\chi_1$. As it follows from (13), (15),

$$\underline{E}_i = \exp iz\{-iz\chi^2/\Delta k\}\underline{E}^{00}. \tag{16}$$

Here $\chi^2 = \chi_2\chi_1$. The equation 16 describes different types of scattering in anisotropic layered structures: $o \to e, e \to o, o \to o, e \to e$. As it follows from Eq.16, Eq.10a the refracted waves for these types of interaction are different, too.

One can see, that the presence of periodic non-uniformity of medium affects the polarization of light beam and has essential influence on its divergence.

Taking into account that Δk in Eq.16 is presented through a group velocity and its derivative Eq.3, we obtain the beam divergence in periodically non-uniform medium that is determined by a tensor

$$W' = W[1 + \frac{2\chi^2}{\Delta k k_0}]. \tag{17}$$

Eigenvalues of the tensor W' in (17) may be negative by $\Delta k < 0$ and

$$|\Delta k| < 2\chi^2/k_0. \tag{18}$$

That corresponds to light focusing. By this, if Gaussian beam falls on the structure, the latter is analogous to focusing lens with focal length

$$F = -Lp/n_0, \tag{19}$$

where $p = W'\omega_0/v_0$, n_0 is a refractive index for central wave of the beam. In wave polarized along X_1 axis in Eq.17 we have:

$$\frac{v^2}{c^2} = \varepsilon_{22}^{-1}. \quad W_1 = \frac{c^2}{v}\varepsilon_{22}^{-1}, W_2 = \frac{c^2}{v}\varepsilon_{33}^{-1}. \tag{20a}$$

While in wave polarized along Y_1 axis in expression Eq.17 it becomes:

$$\frac{v^2}{c^2} = \varepsilon_{11}^{-1}, \quad W_1 = \frac{c^2}{v}\varepsilon_{33}^{-1}, W_2 = \frac{c^2}{v}\varepsilon_{11}^{-1}. \tag{20b}$$

Note that owing to difference of eigenvalues of tensor W', the beam is defocusing differently in two orthogonal directions of cross-section. This leads to the transformation of the beam structure. Such a transformation will increase the natural anisotropy of layers and induced anisotropy stimulated by periodicity of optical properties.

The existence of different types of scattering in anisotropic layered structure lead to the opportunity of focusing either all four refracted waves or some of them.

The application of external electric field changes the wave numbers of interacting waves and their divergence (see Eq.20a and Eq.20b). By changing the value of controlling electric field, as it follows from Eq.18, we may realize the condition when defocusing without field wave becomes focusing one. This phenomenon takes place on conditions:

$$\Delta k < 0, |\Delta k| - 2\chi^2/k_0 < |\Delta k^E|, \tag{21}$$

where Δk^E is the change of wave mismatch Δk owing to the application of external electric field.

Therefore, crystalline periodically non-uniform medium may be used as diffraction lenses, controlled by polarization of incident light, in which the focus length is determined by natural and induced anisotropy (see Eq.19). The essential property of proposed lenses is their large angular aperture. The application of external electric field permits to change parameters of such lenses.

Conclusion

The analysis is presented in laser radiation diffraction in periodically non-uniform anisotropic crystals. The laser beam divergence is determined by common influence of two physical mechanisms: natural anisotropy and periodical non-uniformity. The latter may lead to the focusing by means of wave mismatch. Owing to difference in eigenvalues of tensor of derivative of the group velocity, that affects the beam divergence, the beam is focusing differently in two orthogonal directions of cross-section. This leads to beam transformation in the structure. Such transformation increases the growth of natural anisotropy of layers and induces the anisotropy stimulated by periodicity of optical properties. The application of external electric field gives opportunities to control the divergence of laser radiation.

The results presented may be used for the explanation of experimental facts by investigation of non-uniform crystals; in creating and optimizing parameters of focusing (focusing-defocusing) laser systems controlled by external electric field.

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