Optical anisotropy of the crystals at the nonuniform fields

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Abstract

In this paper a review of the parametrical crystallooptical effects at the influence of the nonhomogeneous fields is presented. It is shown that the parametrical crystallooptics at the static nonhomogeneous fields is the particular branch of the nonlinear optics in which spatial dispersion is taken into account. The coupling spatial dispersion phenomena and gradient crystallooptical effects is shown on the phenomenological level as well as description of the influence of the multicomponent fields on the behavior of the optical indicatrix is realized. It is shown that the mechanical bending and twisting deformations could be described as axial second rank tensor. The experimental investigations of the influence of the nonuniform mechanical strain and temperature field on the refractive and gyrotropy properties of $LiNbO_3$ and $NaBi(MoO_4)_2$ crystals are presented.

Key words: parametrical crystallooptics, nonuniform fields, gyration, nonlinear optics, electrooptics, thermooptics, piezooptics, gradient fields phenomena.

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Contents

1. Introduction

Development of novel physical optics, laser physics and optical material science is connected, on one hand, with expansion of experimental possibilities on the basis of computer engineering, high accuracy polarimetry and laser physics, and on other - with practical needs of optoelectronics and informatics in new optical materials with nontrivial properties. Among these materials - media, in which laser radiation, the external fields and structural phase transitions induce these properties. Finding new optical phenomena, related to these factors, calls for more deep processes understanding, which take place at light interaction with media in nontrivial and extreme conditions (homogeneous and nonhomogeneous fields of different nature, high powers of light radiation, pressure, hard radiation, and others like that). Special interest herewith displays to factors, which takes into account a real media structure. Complex approach to this problem enables to explain nature and mechanisms of already known phenomena, to obtain scientific information, bearing upon interaction of electromagnetic radiation with media of diverse structure (distant from idealized models).

The appearing of nonlinear effects in powerful, harmonically fields, that may be reached by lasers, is connected with the fact, that medium polarization (P_i) on optical frequency (ω) nonlinearly depends on electric field (E_j) of the light wave [1]. Firstly it was believed that the nonlinear optical phenomena arise in media only at sufficiently high field power of the light wave. However, from the point of view of nonlinear electrodynamics, to them one can attribute also effects, which appear in sufficiently weak light fields. This is so called parametrical nonlinear phenomena, which arise in media, the parameters of which change under the action of external influences (electric and magnetic field, mechanical strain, etc.) [2]. Among them can be pointed out so called gradient effects – that is – effects, caused by the presence of spatial dispersion. Today gradient crystallooptics comes forward as partial branch of nonlinear optics.

2. Phenomenological analysis of the influence of the nonhomogeneous actions on optical properties of crystals.

2.1. Symmetry aspects of parametric optical phenomena at nonhomogeneous mechanical, electrical and temperature influence.

Possibility of existence of different effects into first turn follows from symmetry transformations. So, for example, appearance of optical activity (gyration) in crystals, which not possess it, at polar-vector (linear electrogyration) and polar-tensor (piezogyration) actions is connected with the reducing of the point group of the crystal symmetry to the noncentro-symmetrical group. Electro- and piezooptical effects consisting in lowering of crystal symmetry to state, which permits the possibility of existence of new components of polarization constant tensor or changing already existing constants.

For general analysis of nonhomogeneous fields of certain configuration as the influence on physical properties of crystals one can be profited by approach, which provides symmetry similarity of nonhomogeneous action and displacements field symmetry, caused by given action. Consequently, scalar action on crystalline medium (temperature, hydrostatic pressure) does not lower its symmetry and does not induce the arise of new components of polarization constants tensor a_{ij} and gyration tensor g_{ij}. At the same time, if to create in crystal a temperature gradient, which is polar-vector quantity, then under its influence the optical activity ought to arise in sample (here nonsymmetrical tensor g_{ij} is analyzed), polarization constants can also change. These effects from symmetry point of view are analogous to the electrogyration and electrooptics and can be described by correlations $\Delta a_{ij}=r_{ijk}(\mathrm{grad}T)_k$, $\Delta g_{ij}=\nu'_{ijk}(\mathrm{grad}T)_k$, where r_{ijk} and ν'_{ijk} is polar and axial tensors of third rank, respectively. It is interesting to note, that, unlike homogeneous heating and cooling, in the existence of temperature gradient the gyration can arise as in the acentric as well as in the centrosymmetrical crystals. Also under the influence of temperature gradient the constant polarization is ought to arise.

Let's go to the analysis of nonhomogeneous actions, which in homogeneous state are described by polar vectors with ∞mm symmetry. For this we shall model some possible types of nonhomogeneouties, for example, the electric field.

1. Spiral distribution of equal on absolute value vectors of electric field E will possess $\infty 2$ symmetry. Axial tensor of second rank possesses such symmetry. That's why, for example, lowering crystals symmetry at propagating through them of intensive circular polarized wave can be represented as superposition of symmetry elements of axial-tensor action with symmetry elements of crystal. In this case one can consider the changing of tensor of polarization constants and gyration constants as self-induced effect of nonlinear optics, which arises as the result of nonhomogeneouty of spiral distribution of electric field in intensive circular polarized electromagnetic wave.

2. Spiral distribution of polar vectors, which gradually changes on absolute value, possesses symmetry of rotative cone – ∞. The action with symmetry ∞ always can cause the optical activity in medium. Regarding a change of polarization constant, then at homogeneous action, for example, of electric field $E||2$ on crystal with symmetry 422, its symmetry will lower to monoclinic syngony $422\cap\infty mm=2$, that is to optically biaxial state with possible indicatrix rotation around two-fold axis. At the same time under the influence of nonhomogeneous field with symmetry ∞ ($E||2$, $\infty||4$) $422\cap\infty=4$ a crystal will remain optically single-axis, and optical indicatrix can change only along to crystallophysical directions. The difference between influences of fields with symmetry $\infty 2$ and ∞ it can be shown on the following example. At the applying of nonhomogeneous electric field with symmetry $\infty 2$ to crystal its symmetry will lower from $\bar{4}2m$ ($E||2$, $\infty||4$) to 222 group and optical biaxiality ought to arise, but indicatrix can not rotate. At the applying of field with symmetry ∞ ($\infty||4$) $\bar{4}2m\cap\infty=2$ besides that a crystal will become optically biaxial, the turning of the indicatrix around two–fold axis is possible.

3. A nonhomogeneous vector field, that is not changed in direction, but evenly changes in absolute value, will possess symmetry m (E vector and direction of its change is in plane m). At such action optical activity can arise, crystal symmetry will lower to group m or 1. Then the optical indicatrix will have a view of ellipsoid of general type and can rotate around crystalophysical axis, which is perpendicular to plane of symmetry in m group, or around all axes in 1 group. Regarding optical activity at twisting, its appearing admits in crystals of all symmetry groups. Actually axial-tensor action always leads to lowering of crystal symmetry to optically active state. The existence of such effect testifies the fact that induced by twisting polarization ought to cause secondary electrogyration. Here it is suitable to remember symmetry rule, which is consequence of Curie and Neuman principles: among effects, which would appear in crystal under the influence of field of some symmetry always will exist such effects, symmetry of which coincides the symmetry of action or is limiting symmetry group, which includes action symmetry as sub–group. It can be noticed, that at any nonhomogeneous actions the optical activity, that is ellipticity of eigen waves [3] is ought to arise.

2.2. Polarization characteristics of crystalline medium with the account of fields' gradients

In presence of effects of spatial dispersion the correlation for polarization of crystalline medium could be written in the form:

$$P_i=\chi_{ij}E_j+d_{ijk}E_jE_k+\gamma_{ijk}\partial E_j/\partial x_k+R_{ijkl}E_l\partial E_j/\partial x_k+...(1)$$

Two last terms in expression (1) describe the optical activity and electrogyration effect [4, 5]. It can be noticed, that the optical activity may be represented as bilinear effect (that is effect, which describes medium polarization decomposition on electric field of light wave and wave vector), whereas electrogyration is, perhaps, first known effect of gradient nonlinear optics [6]. Taking into account different gradient invariants in correlation (1) one can foresee and define the existence conditions of new gradient effects, connected with spatial dispersion [7, 8].

Medium polarization at linear electrogyration effect can be represented as follows:

$$^{nl}P_i^{\omega}=R_{ijkl}E_l\partial E_j/\partial x_k = ie_{ikm}\gamma_{mjl}E_j^{o}E_k^{\omega}k_l \qquad (2),$$

where γ_{mil} is axial tensor of the third rank, e_{ikm} is Levi-Civita tensor. Then, realizing frequency transposition: $\omega=\omega+0$, we get:

$$^{nl}P_i^{\omega}=R_{ijkl}E_l^{\omega}\partial E_j^{0}/\partial x_k \qquad (3),$$

$$\Delta a_{ij}=R_{ijkl}\partial E_k/\partial x_l \qquad (4),$$

where Δa_{ij} is the change of polarization constants. As one can see from expressions (3, 4), the gradient of the constant electric field caused the change of polarization constants, that is the correlation (4) that describes the change of medium refractive indexes in the nonhomogeneous electric field. It is interesting to mark, that, unlike linear electrooptical effect, which one can observe only in noncentrosymmetrical media, given effect can arise in all media. Moreover, accepting the existence of an electrical field distribution at the frequency Ω in the crystals (e.g. ferroelectric enantiomorphic domain structure, where $\Omega<<\omega$), so called electrogyration light diffraction is possible. This effect may be described by

$$^{nl}P_i^{\omega\pm\Omega}=ie_{ikm}\gamma_{mjl}E_k^{\omega}E_j^{\Omega}k_l. \qquad (5)$$

Such distribution may be created by grating written in photorefractive crystal by two optical waves interference [9].

Let us consider following possible term in expression (1):

$$^{nl}P_i=M_{ijklm}\partial E_j/\partial x_k\partial E_l/\partial x_m. \qquad (6)$$

At the propagation of electromagnetic wave in noncentrosymmetrical medium, to which constant nonhomogeneous electric field is applied, the arising of gradient electrogyration effect is possible. Herewith the correlation (6) will be written down as follows:

$$^{nl}P_i=M_{ijklm}\partial E_j^{\omega}/\partial x_k\partial E_l^{0}/\partial x_m=$$

$$= ie_{ijn}m_{nklm}\partial E_l^{0}/\partial x_mE_j^{\omega}k_k, \qquad (7)$$

and changing of gyration tensor can be represented as $\Delta g_{nk}=m_{nklm}\partial E_l/\partial x_m$. At the propagation of two waves with frequency ω in the noncentrosymmetrical medium, the correlation (7) can describe effects of second harmonic generation and optical detection [1, 10, 11]:

$$^{nl}P_i^{2\omega}=M_{ijklm}\partial E_j^{\omega}/\partial x_k\partial E_l^{\omega}/\partial x_m=M_{nklm}E_l^{\omega}E_j^{\omega}k_mk_k,$$
at $\omega+\omega=2\omega$,

$$^{nl}P_i^{o}=M_{ijklm}\partial E_j^{\omega}/\partial x_k\partial E_l^{\omega}/\partial x_m=M_{nklm}\partial E_l^{\omega}E_j^{\omega}k_mk_k,$$
at $\omega-\omega=0$.

These phenomena lead to arising small changes in traditional effects of second harmonic generation and optical detection depending on the wave vector. It is necessary to mark, that above mentioned effects can also arise in the presence of constant gradient of electric field:

$$\left.\begin{array}{l} ^{nl}P_i^{2\omega}=Q_{ijklm}\partial E_j^{o}/\partial x_kE_l^{\omega}E_m^{\omega} \\ \text{at } 2\omega=0+\omega+\omega, \\ ^{nl}P_i^{0}=M_{ijklm}\partial E_j^{0}/\partial x_kE_l^{\omega}E_m^{\omega} \\ \text{at } 0=0+\omega-\omega. \end{array}\right| \qquad (8)$$

An interesting circumstance is that at frequency transposition $\omega=\omega+0+0$ the correlation (8) describes quadratic electrogyration effect where the change of gyration tensor will be written down in the next view $\Delta g_{nk}=q_{nklm}E_lE_m$.

Let us consider more in detail the quadratic parametrical effects. For example, an effect of quadratic electrooptics (Kerr effect) is well studied now [12]. Dynamic analogue of this effect is the effect of self-induced changing of refraction indexes by intensive electromagnetic wave [13]. The possibility of such phenomenon existence does not contradict fundamental principles of light interaction with medium. Actually, medium polarization for dynamic electrooptics effect can be written in the view:

$$^{nl}P_i^{2\omega}= R_{ijkl}E_j^{\omega}E_k^{\omega} E_l^{\omega} \qquad (\omega=\omega+\omega-\omega)$$

and $$\Delta a_{ij} = R_{ijkl}E_kE_l. \tag{9}$$

With that, in the case of self-induced quadratic electrogyration [14] in expression (9) electric field with components E_l and E_k possess frequency ω. Then inconsequent looks like the description of dynamic electrogyration on the base of the correlation (9) - incomprehensible protrudes circumstance that at identical frequencies of fields E_j, E_l and E_k, in one case takes into account derivative of coordinate x_k, and in the other – not. Then the effect of dynamic electrogyration can be described by the correlation:

$$^{nl}P_i^{\omega}=Q_{ijklmnt}\partial E_j^{\omega}/\partial x_k\partial E_l^{\omega}/\partial x_m\partial E_n^{\omega}/\partial x_t= =ie_{ijr}q_{rklmnt}E_j^{\omega}E_l^{\omega}E_n^{\omega}k_kk_mk_t \tag{10}$$

and can appear only as effect of spatial dispersion of the third order.

Phenomena of gradient parametrical crystal optics can appear also in fields of another nature. So, for example, well-known piezogyration effect [15-21] is described by following expression:

$$^{nl}P_i^{\omega}=\theta_{ijmkl}\sigma_{kl}^{0}\partial E_j^{\omega}/\partial x_m= =ie_{ijn}\theta_{nmkl}\sigma_{kl}^{0}E_j^{\omega}k_m \tag{11}$$

$$\Delta g_{nm} = \theta_{nmkl}\sigma_{kl}, \tag{12}$$

where σ_{kl} is mechanical strain. With that, at the propagation of acoustic wave with frequency Ω in acentric medium, a light diffraction on the grate, formed on the imaginary part of dielectric permitivity, must arise:

$$^{nl}P_i^{\omega\pm\Omega}=\theta_{ijmkl}\sigma_{kl}^{0}\partial E_j^{\omega}/\partial x_m= =ie_{ijn}\theta_{nmkl}\sigma_{kl}^{\Omega}E_j^{\omega}k_m \tag{13}$$

This effect was for the first time observed in quartz crystals [22] and called acoustogyration light diffraction.

The effect of optical activity appearance in centrosymmetrical $NaBi(MoO_4)_2$ crystals under the influence of twisting deformation was also observed. This effect is connected with arising of nonhomogeneous mechanical strains in crystal because piezogyration can not appear in centrosymmetrical crystals:

$$^{nl}P_i^{\omega}=S_{ijklmn}\partial\sigma_{kl}^{0}/\partial x_m\partial E_j^{\omega}/\partial x_n= =ie_{ijr}k_{nrkml}\partial\sigma_{kl}^{0}/\partial x_mE_j^{\omega}k_n \tag{14}$$

$$\Delta g_{rn} = k_{rnklm}\partial\sigma_{kl}/\partial x_m, \tag{15}$$

where tensor k_{rnklm}, according to its symmetry $\varepsilon[V^2][V^2]V$, differs from zero even in isotropic medium. So, optical activity is a suitable tool for control and investigation of nonuniformly strained state of media.

Nonhomogeneous mechanical strains also lead to the changes of refractive properties of crystals. This phenomenon was called gradient piezooptics effect and as it was shown in [8] it can be described by the expression:

$$\Delta a_{ij} = K_{ijklm}\partial\sigma_{kl}/\partial x_m, \tag{16}$$

where K_{ijklm} is polar tensor of 5-th rank.

2.3. Influence of the multicomponent fields on optical indicatrix.

For the description of effects of parametric crystallooptics (electro- and piezooptics) usually optical indicatrix is used. External influences can cause turnings of optical indicatrix around its axes. As far as we know, cases of inducing by external influence the turning of indicatrix around only one axis were considered up-to-now. This may be explained by the circumstance, that to bring ellipsoid equation to canonical view (to obtain correlation for changed principle refraction indexes) at the turnings of indicatrix around two or three axes in extant view is practically impossible. Therefore the attention does not spare tasks, related to inducing by external influence the turning of characteristic refraction indexes surface around a few axes simultaneously. On the other side, the turning around the third coordinate axis must be observed at turnings present of any geometrical figure around two coordinate axes. In crystallooptics the turning of indicatrix around the third coordinate axis can be unconnected with the existence of a respective field component, which should induced it, or according to tensor components, for example, electrooptics or piezooptics tensor constants. Then such indicatrix turning is reasonable to describe as non-direct turning, so far as it must be described over its turnings around two other axes.

The tensor approach in crystallooptics is based on the description of not one resulting turning of optical indicatrix, but on the description of its turnings around coordinate axes. On the other side, matrix approach is impossible to use to turnings description around a few axes, which occur simultaneously. Then let us assume these turnings as one turning, that one can do with the help of turning conception as axial vector.

In general case equation of optical indicatrix may be represented in the next view:

$$a_1x^2+a_2y^2+a_3z^2+2a_4yz+2a_5xz+2a_6xy=1 \quad (17),$$

where a_i is the component of polarization constants tensor, x, y, z - coordinates. It is known, that the presence of constants a_4, a_5, a_6 testifies about turnings of optical indicatrix around axes x, y, z on angles $tg2\xi_1=2a_4/(a_2-a_3)$, $tg2\xi_2=2a_5/(a_3-a_1)$, $tg2\xi_3=2a_6/(a_1-a_2)$, respectively. For example, let us consider piezooptical effect in crystals of 3m symmetry group, which arises under the influence of shift strains σ_{31} and σ_{12}. These strains are inducing indicatrix turnings around axes z and y on angles ξ_3 and ξ_2, respectively. The equation of optical indicatrix herewith will have the view (17), where $a_4=0$, $a_5=\pi_{44}\sigma_{13}+2\pi_{41}\sigma_{12}$, $a_6=\pi_{14}\sigma_{13}+\pi_{66}\sigma_{12}$. It must be noticed that, originating from the view of piezo-optics constants tensor, the component of tensor of polarization constants $a_4=0$, that is indicatrix turning around axis x should be absent.

As it is already mentioned, the summation of turnings around two diverse axes with the help of multiplication of matrices of direction cosines does not bring unequivocal result. Actually, the matrix expression $C=C^{(2)}C^{(1)}$ (C - matrix of directing cosines of summary turning, $C^{(2)}$ and $C^{(1)}$ - matrices of directing cosines of separate turnings) is non-commutative and then the summary coordinates system turning depends on the sequence of realized turnings [23]. But, so far as these two turnings take place simultaneously (in fact takes place one turning, which consists from the turnings around two axes), is impossible to distinguish anyone of them as such, which takes place first.

Using the conception of turning in the axial vector, one can obtain the resulting turning, which describes the resulting axial vector (this vector is directed along rotation axis, and its value is equal to the turning angle) by summarizing of two turnings around diverse axes as two axial vectors. It is necessary to mark that at turning description as axial vector, we, taking into account the turning periodicity, adopt the value of axial vector equal to the fractal part of division of turning angle on 2π. Knowing the direction cosines of rotation axis and the turning value (in our case $\cos\alpha=0$,

$$\cos\beta=\frac{\{\xi_2/2\pi\}}{\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}},$$

$\cos\gamma = \frac{\{\xi_3/2\pi\}}{\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}}$, and $\theta = 2\pi\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}$), it is easy to obtain the matrix of direction cosines for the resulting turning [24]:

$$
\begin{aligned}
c_{xx} &= \cos\theta = \cos 2\pi\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2};\\
c_{xy} &= \cos\gamma\sin\theta = \frac{\{\xi_3/2\pi\}}{\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}}\sin 2\pi\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2};\\
c_{xz} &= -\cos\beta\sin\theta = -\frac{\{\xi_2/2\pi\}}{\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}}\sin 2\pi\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2};\\
c_{yx} &= -\cos\gamma\sin\theta = -\frac{\{\xi_3/2\pi\}}{\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}}\sin 2\pi\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}; \qquad (18)\\
c_{yy} &= \cos\theta+\cos^2\beta(1-\cos\theta) = \frac{\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\cos 2\pi\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}}{\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2};\\
c_{yz} &= \cos\beta\cos\gamma(1-\cos\theta) = \frac{\{\xi_2/2\pi\}\{\xi_3/2\pi\}}{\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2}\left(1-\cos 2\pi\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}\right);\\
c_{zx} &= \cos\beta\sin\theta = \frac{\{\xi_2/2\pi\}}{\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}}\sin 2\pi\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2};\\
c_{zy} &= \cos\beta\cos\gamma(1-\cos\theta) = \frac{\{\xi_2/2\pi\}\{\xi_3/2\pi\}}{\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2}\left(1-\cos 2\pi\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}\right);\\
c_{zz} &= \cos\theta+\cos^2\gamma(1-\cos\theta) = \frac{\{\xi_3/2\pi\}^2+\{\xi_2/2\pi\}^2\cos 2\pi\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}}{\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2}.
\end{aligned}
$$

From above-mentioned matrix by transformations one can obtain the information, which interests us. So, for example, the axis x′ does not belong to the plane xy, neither to the plane xz (as in the case of turning around one axis), that one can describe with the help of correlation for angles between projection of axis x′ on the plane yz and axis z:

$$\alpha' = \pi - \arccos\frac{\{\xi_2/2\pi\}}{\left(\{\xi_2/2\pi\}^2+\{\xi_3/2\pi\}^2\right)^{1/2}} \qquad (19).$$

Consequently, although strains σ_{31} and σ_{12} directly do not cause the turning of optical indicatrix around axis x (as was already mentioned, the component of tensor of polarization constants a_4=0, that is the turning of indicatrix around the axis x should be absent), this turning nevertheless exists thanks to the presence of turnings around two other axes. The angle α' may be used for quantitative description of non-direct turning of optical indicatrix around the third axis. Considering the example of 3m symmetry crystals a given angle will be related to the piezooptics constants as:

$$\alpha' = \pi - \arccos\frac{\left\{\operatorname{arctg}\frac{2(\pi_{44}\sigma_{13}+2\pi_{41}\sigma_{12})}{a_3-a_1}\Big/2\pi\right\}}{\left(\left\{\operatorname{arctg}\frac{2(\pi_{44}\sigma_{13}+2\pi_{41}\sigma_{12})}{a_3-a_1}\Big/2\pi\right\}^2+\left\{\operatorname{arctg}\frac{2(\pi_{14}\sigma_{13}+\pi_{66}\sigma_{12})}{a_1-a_2}\Big/2\pi\right\}^2\right)^{1/2}} \quad (20).$$

Let us consider some possible cases of correlations between angles ξ_2 and ξ_3 in the formula (19):

1) $\xi_2 = \xi_3$, then $\alpha'_1=3\pi/4$;

2) $\xi_2 << \xi_3$, then $\alpha'_2\approx\pi/2$;

3) $\xi_2 >> \xi_3$, then $\alpha'_3\approx\pi$.

All of these cases are depicted on figure 1. It is visible, that non-direct turning is most essential at close values of the angles ξ_2 and ξ_3; if one of the angles is far from the other, then non-direct turning is insignificant.

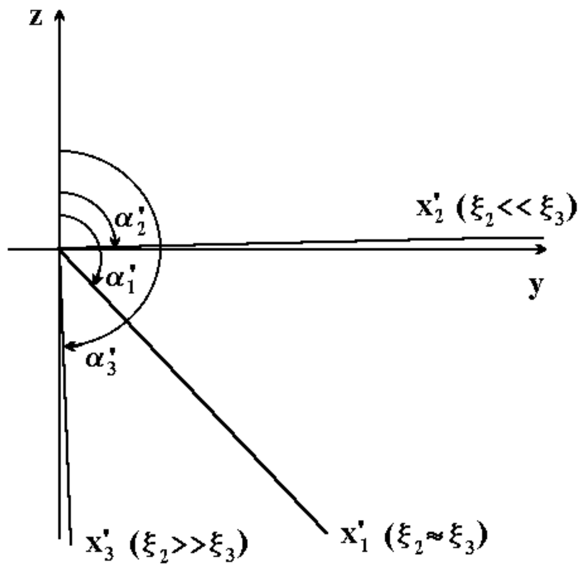


Fig. 1. Non-direct turning for diverse correlations between angles ξ_2 and ξ_3.

The conclusion about the existence of turning around the third axis at turnings present around two other axes one can also do, originating from symmetry consideration. So, turnings of optical indicatrix at external influences, which do not change crystal symmetry (temperature, hydrostatic pressure, etc.), are possible only in monoclinic and triclinic syngonies. In monoclinic syngony the turning of optical indicatrix is permitted only around one axis, and in triclinic - around three axes. Point groups of symmetry, in which at above-mentioned conditions the permitted turnings around two axes do not exist. On the other side, under the influence of external action, which changes the crystal symmetry and induces the turnings of optical indicatrix around two axes, a crystal always will lower symmetry to triclinic syngony, in which the turnings are permitted around three axes.

So, the conception of turning as axial vector allows to obtain the information about the behavior of indicatrix at the presence of the turnings around a few axes [25]. Also one can make a conclusion, that at inducing of the turning of characteristic refraction indexes surface around two axes by external influence always the turning around the third axis appears. It is necessary to mark that the conclusion about non-direct turning of optical indicatrix is general for the turnings of all of characteristic surfaces as of second order, as well as of higher orders.

2.4. Tensor description of gradient piezogyration and piezooptics.

The phenomena of gradient piezooptics and piezogyration consist in changing of polarization (a_{ij}) and gyration (g_{ij}) constants in proportion of gradient of mechanical strain and are described by the correlations [8]:

$$\Delta a_{ij} = K_{ijklm}\partial\sigma_{kl}/\partial x_m$$

$$\Delta g_{ij} = k_{ijklm}\partial\sigma_{kl}/\partial x_m \quad (21)$$

respectively, where K_{ijklm} and k_{ijklm} is polar and axial tensors of the fifth rank. For the simplification of these expressions we shall consider more in detail the strains tensor at twisting and bending. From the formulas of σ_{kl} [23] it is visible, that bend is caused by the distribution of diagonal components σ_{ii} and twisting - by nondiagonal ones σ_{kl} (k≠l), and rotor of strains tensor [23] is equal

$$(Rot\sigma)_{il}=e_{ijk}\partial\sigma_{kl}/\partial x_j \qquad (22),$$

where e_{ijk} is Levi-Civita tensor. Taking into account equations of elastostatics ($\partial\sigma_{ik}/\partial x_k=0$), from (22) we can get

$$Rot\sigma=\begin{pmatrix} \partial\sigma_{31}/\partial x_2-\partial\sigma_{21}/\partial x_3 & -\partial\sigma_{22}/\partial x_3 & \partial\sigma_{33}/\partial x_2 \\ \partial\sigma_{11}/\partial x_3 & \partial\sigma_{12}/\partial x_3-\partial\sigma_{32}/\partial x_1 & -\partial\sigma_{33}/\partial x_1 \\ -\partial\sigma_{11}/\partial x_2 & \partial\sigma_{22}/\partial x_1 & \partial\sigma_{23}/\partial x_1-\partial\sigma_{13}/\partial x_2 \end{pmatrix} \qquad (23).$$

In our case nondiagonal components of strains tensor σ_{kl} (k≠l), which are connected with twisting of crystal, attend only in diagonal components of second rank axial tensor $(Rot\sigma)_{ii}$, but the diagonal components σ_{ii}, which are connected with bending, exist only in nondiagonal components $(Rot\sigma)_{ik}$ (i≠k), so the separation of the components, which cause bending and twisting takes place. It is also necessary to remark next circumstance: at twisting

$$\partial\sigma_{kl}/\partial x_j=\partial\sigma_{jl}/\partial x_k \quad (k\neq l\neq j). \qquad (24)$$

From the correlation (22), elastostatics equations and (24) one can get

$$\partial\sigma_{kl}/\partial x_j=Ae_{kij}(Rot\sigma)_{il}\ , \qquad (25)$$

where $A=(2-\delta_{kl})^{-1}$, δ_{kl} is Cronecker symbol. By installation of (25) in (21), we get

$$\Delta a_{ij} = AK_{ijklm}\delta_{kmn}(Rot\sigma)_{nl}\ . \qquad (26)$$

Obviously, the tensor of gradient piezooptics at twisting and bending with internal symmetry $[V^2]V^2$

$$\eta_{ijln}=AK_{ijklm}\delta_{kmn} \qquad (27)$$

has already the fourth rank. Then we can write down

$$\Delta a_{ij}=\eta_{ijln}(Rot\sigma)_{nl} \qquad (28)$$

(analogous correlations just for Δg_{ij}: $\Delta g_{ij}=\mu_{ijln}(Rot\sigma)_{nl}$, where $\mu_{ijln}=k_{ijklm}\delta_{kmn}$), that is for the description of changes of polarization constants one can already use the tensor of not fifth but of fourth rank.

In general case axial second rank tensor $(Rot\sigma)_{il}$ is nonsymmetrical. Let's formulate the conditions, when it becomes antisymmetrical or symmetrical. From antisymmetrical $(Rot\sigma)_{il}^{*}=-(Rot\sigma)_{li}$ or symmetric conditions $(Rot\sigma)_{il}=(Rot\sigma)_{li}$ we get:

$$\begin{cases} \dfrac{\partial\sigma_{31}}{\partial x_2}-\dfrac{\partial\sigma_{21}}{\partial x_3}=0 \\ \dfrac{\partial\sigma_{12}}{\partial x_3}-\dfrac{\partial\sigma_{32}}{\partial x_1}=0 \\ \dfrac{\partial\sigma_{23}}{\partial x_2}-\dfrac{\partial\sigma_{13}}{\partial x_2}=0 \\ \dfrac{\partial\sigma_{11}}{\partial x_2}=\dfrac{\partial\sigma_{33}}{\partial x_2} \\ \dfrac{\partial\sigma_{22}}{\partial x_3}=\dfrac{\partial\sigma_{11}}{\partial x_3} \\ \dfrac{\partial\sigma_{33}}{\partial x_1}=\dfrac{\partial\sigma_{22}}{\partial x_1} \end{cases} \qquad (29)$$

or

$$\begin{cases} \dfrac{\partial\sigma_{11}}{\partial x_2}=-\dfrac{\partial\sigma_{33}}{\partial x_2} \\ \dfrac{\partial\sigma_{22}}{\partial x_3}=-\dfrac{\partial\sigma_{11}}{\partial x_3} \\ \dfrac{\partial\sigma_{33}}{\partial x_1}=-\dfrac{\partial\sigma_{22}}{\partial x_1} \end{cases} \qquad (30)$$

respectively. Then antisymmetrical or symmetrical tensors will have next view:

$$\begin{pmatrix} 0 & -\partial\sigma_{22}/\partial x_3 & \partial\sigma_{33}/\partial x_2 \\ \partial\sigma_{11}/\partial x_3 & 0 & -\partial\sigma_{33}/\partial x_1 \\ -\partial\sigma_{11}/\partial x_2 & \partial\sigma_{22}/\partial x_1 & 0 \end{pmatrix}, \quad (31)$$

$$\begin{pmatrix} \partial\sigma_{31}/\partial x_2 - \partial\sigma_{21}/\partial x_3 & \partial\sigma_{22}/\partial x_3 & \partial\sigma_{33}/\partial x_2 \\ \partial\sigma_{11}/\partial x_3 & \partial\sigma_{12}/\partial x_3 - \partial\sigma_{32}/\partial x_1 & \partial\sigma_{33}/\partial x_1 \\ -\partial\sigma_{11}/\partial x_2 & \partial\sigma_{22}/\partial x_1 & \partial\sigma_{23}/\partial x_1 - \partial\sigma_{13}/\partial x_2 \end{pmatrix} \quad (32).$$

As it is visible from (31), antisymmetrical tensor contains only the diagonal components of tensor of strains, which are connected with bending and does not contain strains, which are connected with twisting. It means that the diagonal components of the symmetrical part of second rank axial tensor describe the twisting as well as nondiagonal - bending.

It is reasonable to consider in more details the case, when the tensor $(\mathrm{Rot}\sigma)_{nl}$ is antisymmetrical. Let us use the correlation of duality for antisymmetrical axial tensor of second rank and polar vector [23]:

$$\mathrm{Rot}(\sigma)_{nl} = B\delta_{pnl}a_p, \quad (33)$$

where B is constant. This vector (a_p) describes a displacement field in crystal, which arises under the influence of two-coordinate bending strains (figure 2). After installing (33) into (28) we shall obtain:

$$\Delta a_{ij} = B\eta_{ijln}\delta_{pnl}a_p \quad (34)$$

and once more will lower the rank of tensor, which is used for the link between polar vector, which describes the two-coordinate bending, and tensor of polarization constants:

$$\theta_{ijp} = B\eta_{ijln}\delta_{pnl}, \quad (35)$$

$$\Delta a_{ij} = \theta_{ijp}a_p \quad (36)$$

So, twisting and bending deformation can be described by axial tensor of second rank. The diagonal components of this tensor describe crystal twisting around crystallophysical axes, and nondiagonal ones - bending. Antisymmetrical part of axial tensor of second rank describes two-coordinate bending [26].

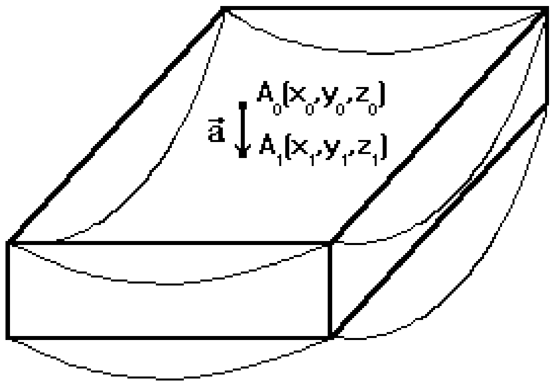


Fig. 2. The view of crystalline plate before (thick line) and after (thin line) applying of two-coordinate bending strains. Vector **a** shows displacement of point A_0 under the influence of bending strains.

3. Light interaction in crystals at static nonuniform influences.

3.1. Influence of nonuniform mechanical actions on optical properties of crystals.

3.1.1. Investigations of optical properties of CdS and CdSe crystals and their CdS_xSe_{1-x} solutions under twisting deformation influence in isotropic point vicinity

Recently a few interesting papers appeared on research of optical properties of CdS and CdSe crystals and their CdS_xSe_{1-x} solutions under twisting deformation influence [27-30]. These hexagonal crystals belong to the point symmetry group 6 mm and are weakly gyrotropic and possess isotropic point. In above-mentioned measurements authors applied the twisting, which symmetry is ∞2, around the optical axis, that lowers symmetry of crystal. As well as linear birefringence, as stronger factor, could mask the effects of gyrotropy, so the measuring was carried out not far from the isotropic point, where an optical anisotropy disappears.

The transmittance of system polarizator-crystal-analyser (PCA) at the application to crystal twisting deformation and magnetic field [27], the change of polarization parameters of light at passing over such crystal and also absorption spectrum of circularly polarized light and circular dichroism was investigated.

Some peculiarities were found out at the investigation of PCA system spectrums of transmittance. A deep minimum in isotropic point is surrounded by two sharp ("principle") and a little weak maximums (figure 3).

A value of "principle" maximums essentially depends on the value of twisting moment (figure 4,a), although at this does not bring any spectral displacements (figure 4,b).

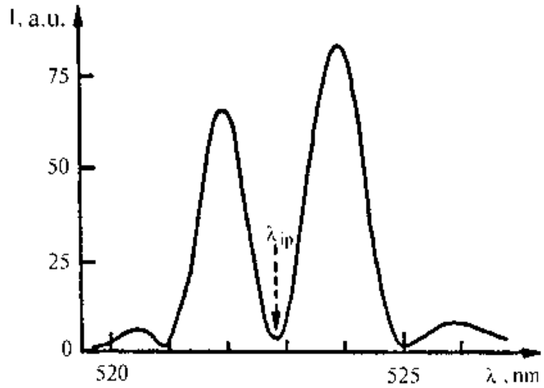


Fig. 3. The spectrum of transmittance of PCA system with twisted CdS crystal and with electrical vector of incident light, parallel to the optical axis [27].

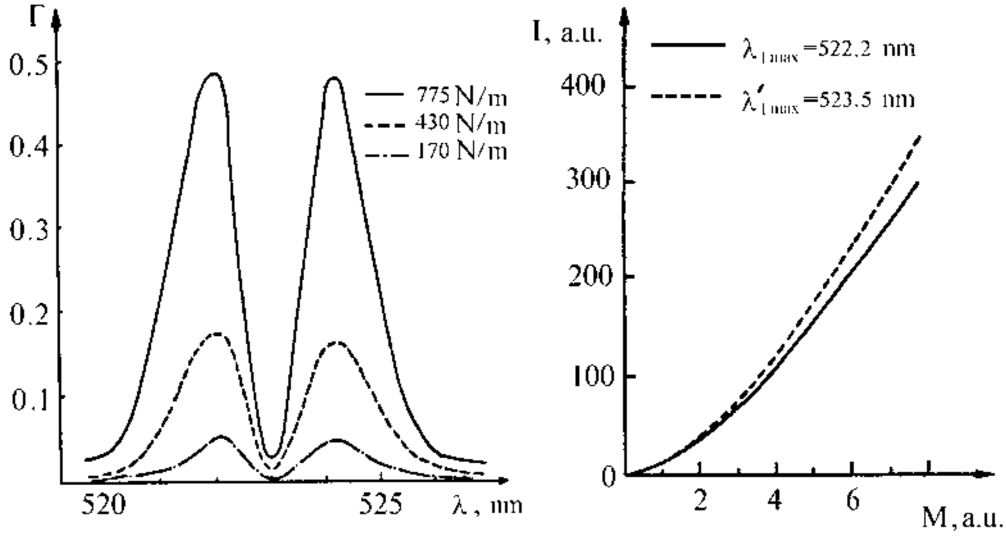
A spectral position of "principle" maximums depends on crystal thickness and can be described as:

$$d=\text{const}/(\lambda_m-\lambda_{ip}), \tag{37}$$

where λ_m is a spectral position of the "principle" maximum, λ_{ip} is a spectral position of the isotropic point (figure 5). Spectral dependence of transmittance on CdS_xSe_{1-x} samples with commensurate S and Se content have only one "principle" maximum, and its position does not depend on sample thickness.

Light on outlet of all samples is elliptically polarized in all spectral diapason. Spectral dependence of ellipticity is similar to spectral dependence of transmittance - a deep minimum in isotropic point is surrounded by two sharp ("principle") and a little weak maximums (figure 6,a), and their spectral positions are exactly equal to a founded transmission investigation ones.

Azimuth of polarization ellipse oscillates around zero, changing its sign in points, which corresponds to the "principle" maximums and minimum between them, irrespective to value of twisting moment (figure 6,b). A polarization ellipse orientation and its azimuth changes its sign in all spectral diapason when twisting moment changes its direction.



a) b)

Fig. 4. The spectrum of transmittance of PCA system for different twisting moments (a) and the dependence of transmittance on the value of twisting moment (b) [27].

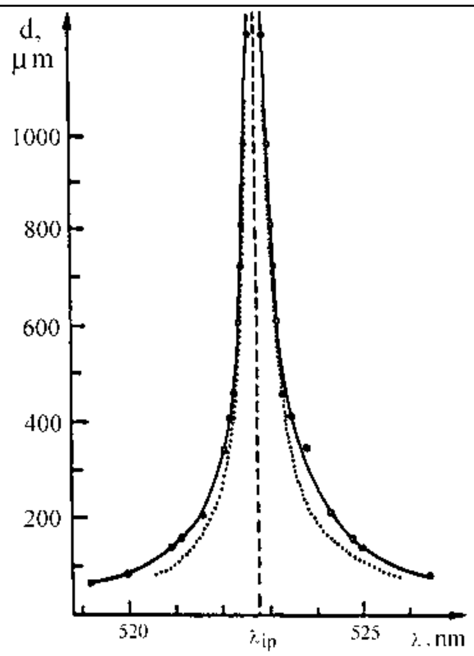


Fig. 5. Dependence of spectral position of “principle” maximum on the crystal thickness [27].

The application of magnetic field, collinear to optical axis, leads to insignificant spectral displacement of absorption spectrums as well as increasing one of the “principle” maximums and decreasing another one. The direction of spectral displacement depends on the direction of magnetic field, and increasing or decreasing of maximums determines as by the direction of magnetic field as well as by the direction of twisting moment (figure 7). The above-mentioned effects are observed for samples with insignificant of S or Se concentration (<5%). At commensurate S and Se concentration only one “principle” maximum is observed, which is a little bit displaced respectively to its position dependently on magnetic field, and it increases or diminishes in dependence of direction of magnetic field (figure 8). On spectral dependence of ellipticity only one maximum at the wavelength of intensity maximum on absorption spectrum is observed, and a polarization ellipse azimuth at this wavelength changes its sign (figure 9).

Also the changes in spectral dependencies of absorption of linearly and circular polarized light and linear and circular dichroism not far from isotropic point, under the influence of twisting moment were observed [29].

All experimental data were theoretically explained. On the base of obtained results it was proposed to use these crystals as a narrow-band optical filter.

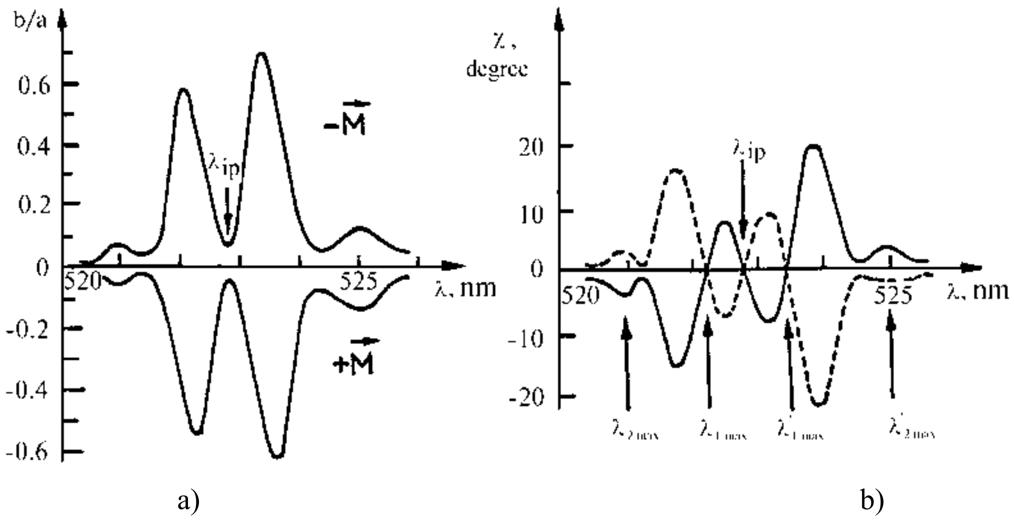


Fig. 6. Spectral dependence of ellipticity (a) and azimuth (b) of ellipse polarization of the CdS crystals with insignificant (<5%) Se concentration [27].

From the above-mentioned results one can make a conclusion that under the twisting deformation the optical transmittance of the PCA system and the ellipticity of light changes in the CdS_xSe_{1-x} crystals at the wavelength of the isotropic point. It means that under the torsion deformation in the CdS_xSe_{1-x} crystals at the wavelength of the isotropic point as optical birefringence as well as gyrotropy could appear. On the other hand the torsion deformation possesses spatial distribution in the sample and optical birefringence due to the elastooptical effect also should be non-homogeneous. In such a case eigen waves should be elliptically polarized and optical activity that appear in the result of gradient piezogyration is masked by the linear birefringence.

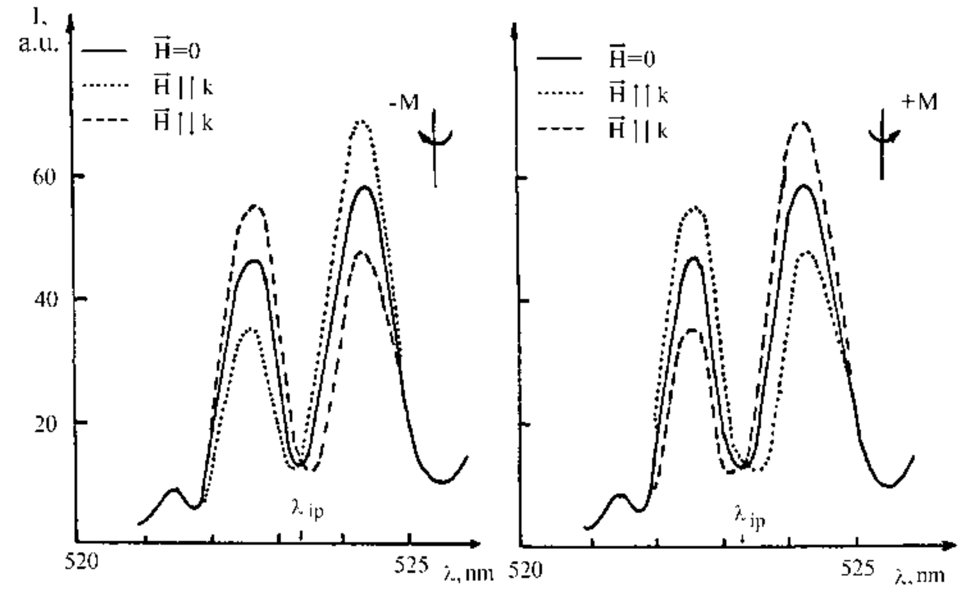


Fig. 7. Influence of magnetic field on transmittance of PCA system for CdS crystals with insignificant (<5%) Se concentration [28].

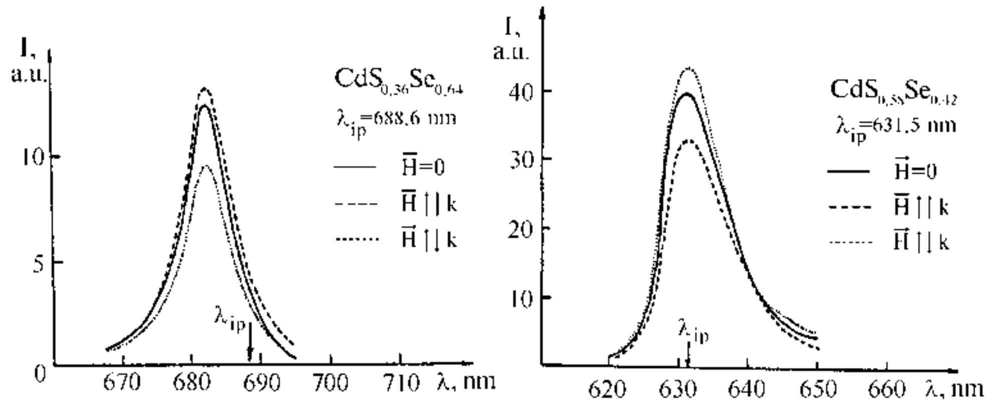


Fig. 8. Influence of magnetic field on transmittance of PCA system for crystals with commensurate S and Se concentration [28].

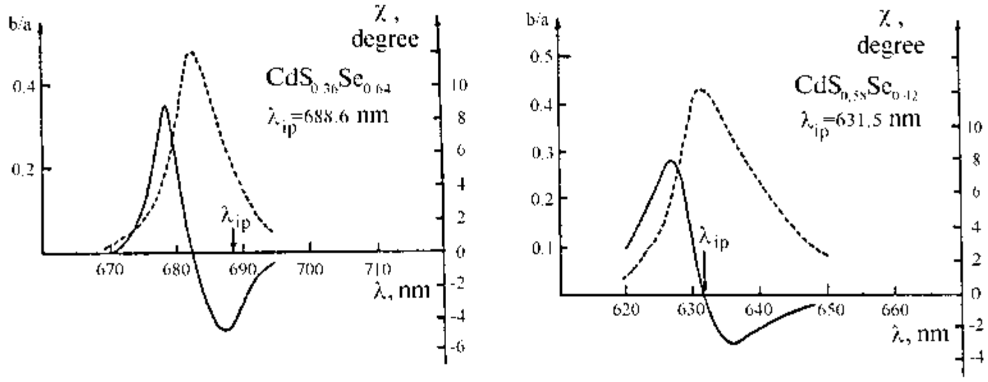


Fig. 9. Spectral dependencies of ellipticity and polarization ellipse azimuth for crystals with commensurate S and Se concentration [28].

3.1.2. Distribution of optical indicatrix turning in $LiNbO_3$ crystals at twisting

The turning of optical indicatrix at scanning by laser ray in xy-crossing in the $LiNbO_3$ crystal was investigated [25] under the application of the torsion moment around z axis. As it was found out, the distribution of optical indicatrix (figure 10) possesses special directions, which coincides with crystallophysical axes x and y. So, at scanning by light ray in xz plane and propagation of light along z axis the turning of indicatrix is not observed, and at scanning by ray in yz plane and propagation of light along z axis the optical indicatrix is turned to the angle ±45°. Really, the equation of optical indicatrix for crystals with symmetry 3m at the presence of shift strains σ_{31} and σ_{32} has next view:

$$\begin{aligned}
&(a_{11}+\pi_{14}\sigma_{32})x^2+(a_{11}-\pi_{14}\sigma_{32})y^2+a_{33}z^2+2\pi_{44}\sigma_{32}zy-2\pi_{44}\sigma_{31}zx-2\pi_{14}\sigma_{31}xy=1 && \text{in the quadrant xy,}\\
&(a_{11}-\pi_{14}\sigma_{32})x^2+(a_{11}+\pi_{14}\sigma_{32})y^2+a_{33}z^2-2\pi_{44}\sigma_{32}zy-2\pi_{44}\sigma_{31}zx-2\pi_{14}\sigma_{31}xy=1 && \text{in the quadrant -xy,}\\
&(a_{11}-\pi_{14}\sigma_{32})x^2+(a_{11}+\pi_{14}\sigma_{32})y^2+a_{33}z^2-2\pi_{44}\sigma_{32}zy+2\pi_{44}\sigma_{31}zx+2\pi_{14}\sigma_{31}xy=1 && \text{in the quadrant -x-y,}\\
&(a_{11}+\pi_{14}\sigma_{32})x^2+(a_{11}-\pi_{14}\sigma_{32})y^2+a_{33}z^2+2\pi_{44}\sigma_{32}zy+2\pi_{44}\sigma_{31}zx+2\pi_{14}\sigma_{31}xy=1 && \text{in the quadrant -yx,}
\end{aligned} \tag{38}$$

where a_{ij} -polarization constants; its turning around z axis will be written down in the view:

$$\begin{aligned}
&tg2\xi=-\sigma_{31}/\sigma_{32} && \text{in the quadrant xy,}\\
&tg2\xi=\sigma_{31}/\sigma_{32} && \text{in the quadrant -xy,}\\
&tg2\xi=-\sigma_{31}/\sigma_{32} && \text{in the quadrant -x-y,}\\
&tg2\xi=\sigma_{31}/\sigma_{32} && \text{in the quadrant -yx.}
\end{aligned} \tag{39}$$

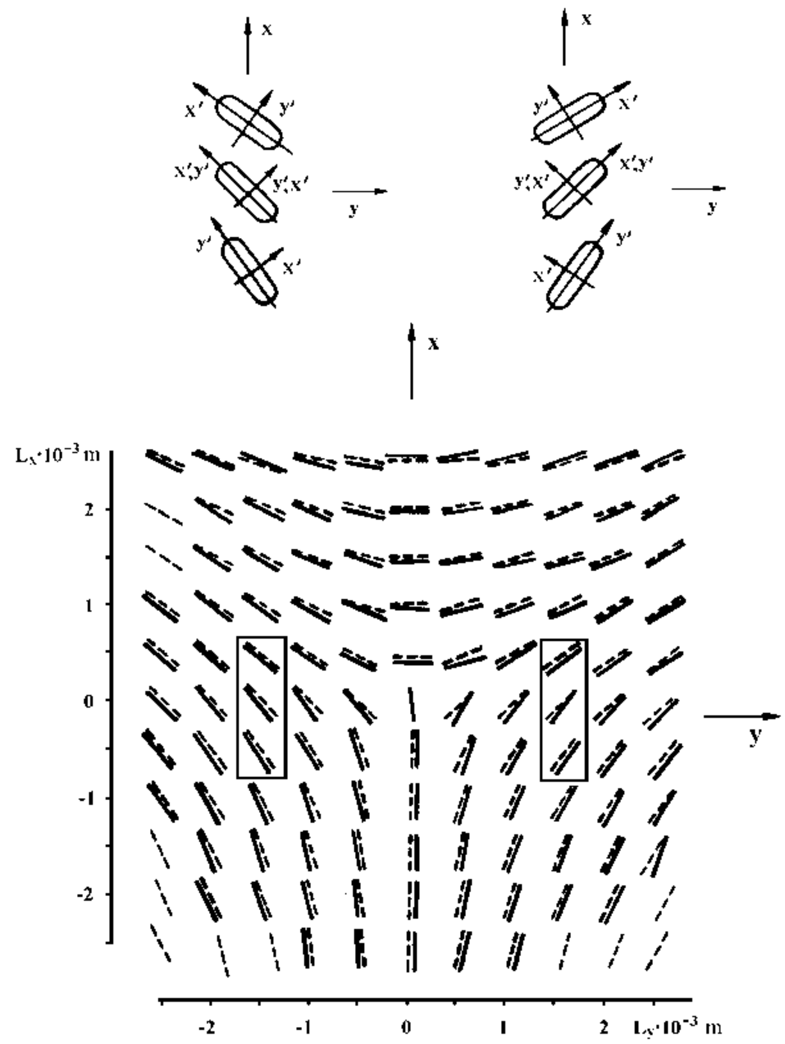


Fig. 10. Distribution of optical indicatrix turnings at constant twisting moment on xy - crossing of $LiNbO_3$ crystal. Theoretically calculated orientations of optical indicatrix are marked by strokes. On the insert - the turn of optical indicatrix to angle 90° with the saving of orientation of long axis at transition from quadrants xy and x-y to quadrants -xy and -x-y, respectively. x and y axes orientation of undeformed crystal is marked by arrows.

These equations describe obtained figure of turnings distribution of optical indicatrix. So, elementary volumes, those belong to yz "plane", which pass over the center of cylindrical sample, are under the action of the strain σ_{32}, which changes a sign at transition through crystal center. All other elementary volumes are under the action of the strains σ_{31} and σ_{32} at twisting, and the direction and value of the turning of the optical indicatrix is determined by formulas given above and they depend on sign and correlation of mentioned strains. At the same time, in a given case there exists an interesting peculiarity: at transition from the xy quadrants and x-y to -xy and -x-y, respectively, optical indicatrix turns to angle 90° with the saving of the orientation of long axis (figure 10, on insert). This situation corresponds to the crystallo-physical x axis interchanging with y axis, because in the unity system of coordinates the turning angle of optical indicatrix can not be more, than 45°.

3.1.3. Neutral birefringence point and neutral birefringence line in $LiNbO_3$ crystals at twisting

The twisting of the crystals was realized by application of forces pair to the opposite faces of the samples. He-Ne laser (λ=632.8 nm) was used as light source. The measuring of birefringence was carried out by Senarmont method. For the investigations samples of the parallelepiped shape were prepared. Twisting axis was directed along the long side of parallelepiped, the center of coordinates was placed in the center of crystal, and the coordinates axes were directed along crystallophysical axes.

As it was mentioned above, the equation of optical indicatrix for $LiNbO_3$ crystals, which belong to symmetry group 3m, at presence of shift strains σ_{31} and σ_{32} can be presented as (38). Let us investigate the crossing of optical indicatrix by the plane z=0. Then (38) it can be written down in the view:

$$\left.\begin{array}{l} (a_{11}+\pi_{14}\sigma_{32})x^2+(a_{11}-\pi_{14}\sigma_{32})y^2-2\pi_{14}\sigma_{31}xy=1 \\ \text{in the quadrant } xy, \\ (a_{11}-\pi_{14}\sigma_{32})x^2+(a_{11}+\pi_{14}\sigma_{32})y^2-2\pi_{14}\sigma_{31}xy=1 \\ \text{in the quadrant } -xy, \\ (a_{11}-\pi_{14}\sigma_{32})x^2+(a_{11}+\pi_{14}\sigma_{32})y^2+2\pi_{14}\sigma_{31}xy=1 \\ \text{in the quadrant } -x-y, \\ (a_{11}+\pi_{14}\sigma_{32})x^2+(a_{11}-\pi_{14}\sigma_{32})y^2+2\pi_{14}\sigma_{31}xy=1 \\ \text{in the quadrant } x-y. \end{array}\right| \quad (40)$$

A solution of respective characteristic equations gives:

$$\lambda_{1,2}=a_{11}\pm\pi_{14}(\sigma^2_{32}+\sigma^2_{31})^{1/2} \quad (41)$$

As well as $\Delta n=n_1-n_2=1/(\lambda_1)^{1/2}-1/(\lambda_2)^{1/2}= =((\lambda_2)^{1/2}-(\lambda_1)^{1/2})/((\lambda_1)(\lambda_1))^{1/2}$ then

$$\Delta n=\frac{\sqrt{a_{11}+p_{14}\sqrt{e_{32}^2+e_{31}^2}}-\sqrt{a_{11}-p_{14}\sqrt{e_{32}^2+e_{31}^2}}}{\sqrt{a_{11}^2-p_{14}^2(e_{32}^2+e_{31}^2)}} \quad (42)$$

While $a_{11}>>\pi_{14}(\sigma^2_{32}+\sigma^2_{31})^{1/2}$, (42) it can be presented as

$$\Delta n\approx\frac{p_{14}\sqrt{e_{32}^2+e_{31}^2}}{a_{11}\sqrt{a_{11}}}=p_{14}\sqrt{e_{32}^2+e_{31}^2}\,n_1^3 \quad (43)$$

Obviously, the dependence of induced birefringence on coordinates is determined by dependences of strains σ_{32} and σ_{31} on coordinates, which are proportional to x and y, respectively. Because $\sigma_{32}=\sigma_{31}=0$ only in the point x=0, y=0, then $\Delta n=0$ only in this point, it is seen on the experiment (figures 11, 12).

Turnings of the optical indicatrix ξ_1, ξ_2, ξ_3 around axes x, y, z one can write down in the view $tg2\xi_1=2a_4/(a_2-a_3)$, $tg2\xi_2=2a_5/(a_3-a_1)$, $tg2\xi_3=2a_6/(a_2-a_1)$. The turning of the indicatrix around z axis will be written down:

$$\begin{array}{l} tg2\xi_3=-\sigma_{31}/\sigma_{32} \text{ in the quadrants } xy \text{ and } -x-y, \\ tg2\xi_3=\sigma_{31}/\sigma_{32} \text{ in the quadrants } -xy \text{ and } x-y. \end{array} \quad (44)$$

These equations describe the distribution of optical indicatrix given on figure 10.

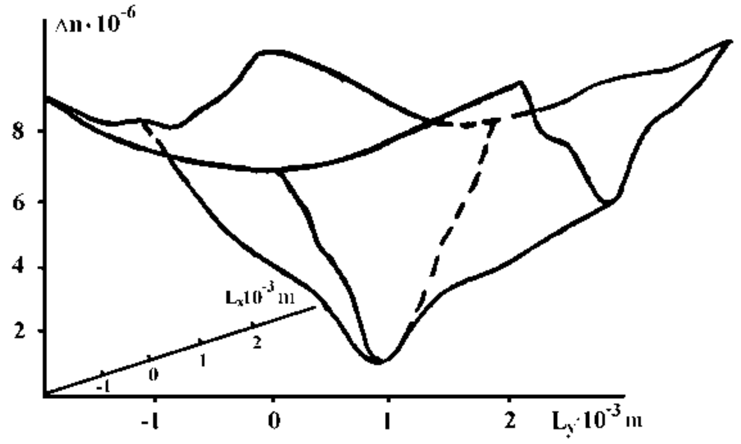


Fig.11. Distribution of the birefringence induced by constant twisting moment M=1.68×10^3 N/m on xy-crossing of $LiNbO_3$ crystal (λ=632.8 nm), (k││z, M││z).

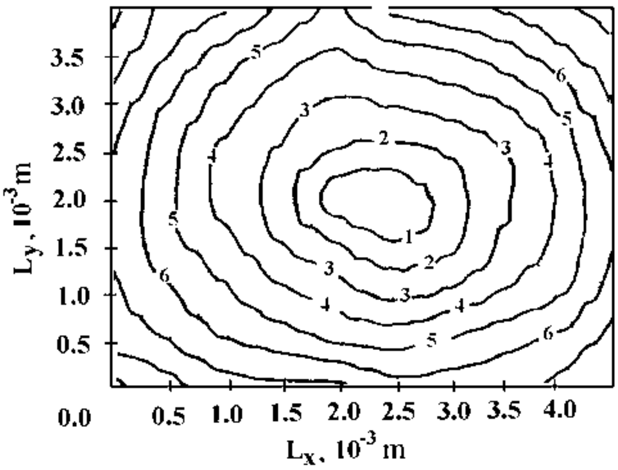


Fig. 12. xy-crossings of the birefringence induced by constant twisting moment in crystal of lithium niobate by planes, perpendicular to optical axis (the distance between planes is $\Delta n=10^{-6}$).

Let's write down the equation (17) for $LiNbO_3$ crystals at presence of shift strains σ_{12} and σ_{31} (it corresponds to the configuration of experiment k||Z, M||x):

in the quadrant yz:

$$a_{11}x^2+a_{11}y^2+a_{33}z^2+2(\pi_{44}\sigma_{31}-\pi_{65}\sigma_{12})zx+2(\pi_{56}\sigma_{31}-(\pi_{11}-\pi_{12})\sigma_{12})xy=1,$$

in the quadrant -zy:

$$a_{11}x^2+a_{11}y^2+a_{33}z^2+2(\pi_{44}\sigma_{31}+\pi_{65}\sigma_{12})zx+2(\pi_{56}\sigma_{31}+(\pi_{11}-\pi_{12})\sigma_{12})xy=1,$$

in the quadrant -yz: (45)

$$a_{11}x^2+a_{11}y^2+a_{33}z^2+2(-\pi_{44}\sigma_{31}-\pi_{65}\sigma_{12})zx+2(-\pi_{56}\sigma_{31}-(\pi_{11}-\pi_{12})\sigma_{12})xy=1,$$

in the quadrant -y-z

$$a_{11}x^2+a_{11}y^2+a_{33}z^2+2(-\pi_{44}\sigma_{31}+\pi_{65}\sigma_{12})zx+2(-\pi_{56}\sigma_{31}+(\pi_{11}-\pi_{12})\sigma_{12})xy=1.$$

So, for optical ray, which propagates above (y>0) and below (y<0) plane y=0, we can obtain after summarizing the respective equations:

$$a_{11}x^2+a_{11}y^2+a_{33}z^2+2\pi_{44}\sigma_{31}zx+2\pi_{56}\sigma_{31}xy=1 \quad (y>0)$$

and

$$a_{11}x^2+a_{11}y^2+a_{33}z^2-2\pi_{44}\sigma_{31}zx-2\pi_{56}\sigma_{31}xy=1 \quad (y<0). \tag{46}$$

The indicatrix turning around z axis is equal to 45°, because $a_{11}=a_{22}$. Let's investigate the crossing of the optical indicatrix by plane z=0. Then (46) one can write down in the view:

$$a_{11}x^2+a_{11}y^2+a_{33}z^2+2\pi_{56}\sigma_{31}xy=1 \quad (y>0)$$

and

$$a_{11}x^2+a_{11}y^2+a_{33}z^2-2\pi_{56}\sigma_{31}xy=1 \quad (y<0). \tag{47}$$

Solution of the respectively characteristic equations gives:

$$\lambda_{1+}=a_{11}-2\pi_{56}\sigma_{31},\ \lambda_{2+}=a_{11}+2\pi_{56}\sigma_{31};$$
$$\lambda_{1-}=a_{11}+2\pi_{56}\sigma_{31},\ \lambda_{1-}=a_{11}-2\pi_{56}\sigma_{31} \tag{48}$$

Induced birefringence could be written as

$$\Delta n_+ = \frac{\sqrt{a_{11}+2p_{56}e_{31}}-\sqrt{a_{11}-2p_{56}e_{31}}}{\sqrt{(a_{11}-2p_{56}e_{31})(a_{11}+2p_{56}e_{31})}} = \frac{\sqrt{a_{11}+2p_{56}e_{31}}-\sqrt{a_{11}-2p_{56}e_{31}}}{\sqrt{a_{11}^2-(2p_{56}e_{31})^2}}$$

and (49)

$$\Delta n_- = \frac{\sqrt{a_{11}-2p_{56}e_{31}}-\sqrt{a_{11}+2p_{56}e_{31}}}{\sqrt{(a_{11}-2p_{56}e_{31})(a_{11}+2p_{56}e_{31})}} = \frac{\sqrt{a_{11}-2p_{56}e_{31}}-\sqrt{a_{11}+2p_{56}e_{31}}}{\sqrt{a_{11}^2-(2p_{56}e_{31})^2}}$$

Note, that $\Delta n_+ = -\Delta n_-$, (figure 13). Taking into account, that $a_{11} >> 2\pi_{56}\sigma_{31}$, one can write down

$$\Delta n_+ = 2n_1^3\pi_{56}\sigma_{31} \text{ and } \Delta n_- = 2n_1^3\pi_{56}\sigma_{31}. \qquad (50)$$

It is visible, that the dependence of induced birefringence on coordinate is determined by the dependence of strain σ_{31} on coordinate. Because at y=0 $\sigma_{31}=0$, then $\Delta n=0$, that is observed on the experiment (figure 13).

In the result of carried out investigations [25, 31] we obtain dependencies of birefringence on the twisting moment (figures 14 and 15), and distributions of induced birefringence on samples crossing (figures 11 and 13). It was established that birefringence does not arise only in the case, when an optical ray propagates through the geometrical center of square crossing - neutral birefringence point (figure 11) or in plane y=0 - neutral birefringence line (figure 13) in dependence of configuration of experiment, in all other cases the birefringence induced by twisting increases at displacement of optical ray from crystal crossing center.

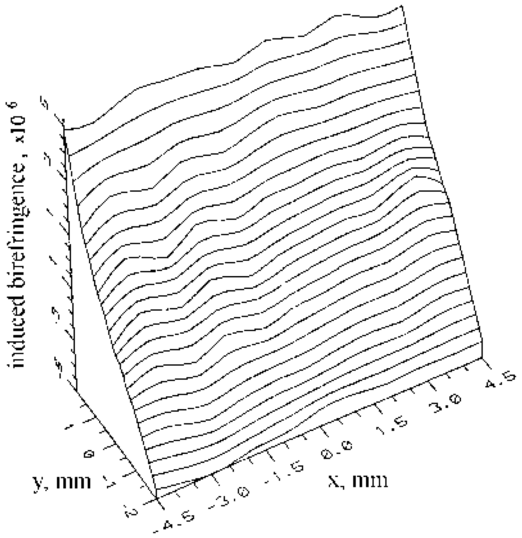


Fig. 13. Distribution of induced by constant twisting moment M=3.008×10^3 N/m birefringence on xy-crossing of the $LiNbO_3$ crystal (λ=632.8 nm), (k||z, M||x).

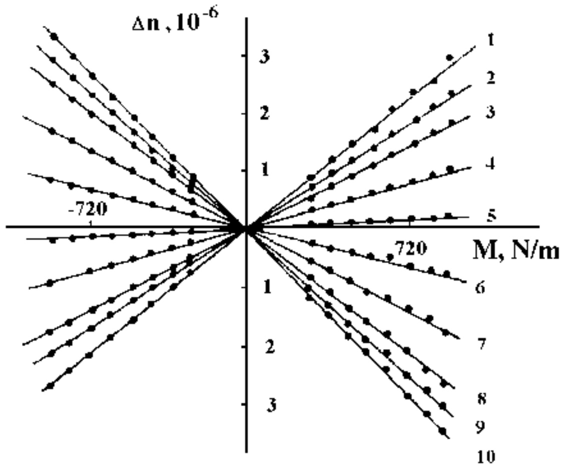


Fig. 14. Dependence of the birefringence on twisting moment at different distances L from center of xy-crossing of $LiNbO_3$ crystal (λ=632.8 nm). L= -130 μm (1), -100 μm (2), -70 μm (3), -40 μm (4), -10 μm (5), 20 μm (6), 50 μm (7), 80 μm (8), 110 μm (9), 140 μm (10), (k||z, M||z).

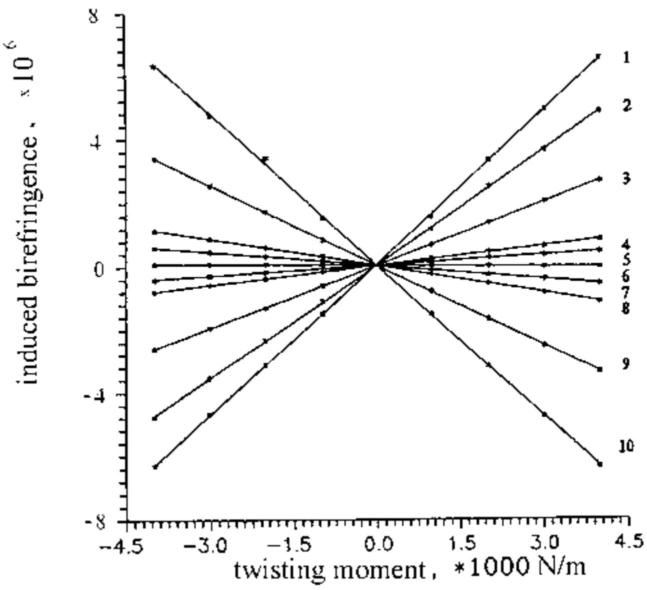


Fig. 15. Dependence of the birefringence on twisting moment at different distances L from line y=0 $LiNbO_3$ crystals xy-crossing (λ=632.8 nm). Ly= -1998 μm (1), -1332 μm (2), -666 μm (3), 0 μm (4), 166.5 μm (5), 333 μm (6), 499.5 μm (7), 666 μm (8), 1332 μm (9), 1998 μm (10), (k| |z, M| |x).

3.1.4. Distribution of birefringence in $LiNbO_3$ crystals at bending.

Bending deformation is as well as twisting deformation one of the most simple non-homogeneous deformations. It can be presented as simultaneous pressing and expansion of different parts of sample (figure 16). For the investigation of the influence of bend deformation on the refractive properties of the $LiNbO_3$ the samples were prepared in parallelepiped form. The center of coordinates was placed in the center of crystal and the coordinates axes were directed along crystallophysical axes. Light was propagated along z axis. Bending moment was applied to the samples by a three-point method.

Bending moment caused the appearance of spatially nonhomogeneous strains $\partial\sigma_{11}/\partial y$. Then equation of optical indicatrix could be written as:

$$(a_{11}+\pi_{11}\sigma_{11})x^2+(a_{11}+\pi_{12}\sigma_{11})y^2+ \\ +(a_{33}+\pi_{31}\sigma_{11})z^2+2\pi_{41}\sigma_{11}zy=1 \qquad (51).$$

It is visible, that turning of optical indicatrix around z axis should be absent. Let us analyze its crossing by the plane z=0. Then (51) one can write down in the view:

$$(a_{11}+\pi_{11}\sigma_{11})x^2+(a_{11}+\pi_{12}\sigma_{11})y^2=1 \qquad (52).$$

The birefringence induced by bending will be equal:

$$\Delta n=(\pi_{12}-\pi_{11})\sigma_{11}/2a_{11}(a_{11})^{1/2} \qquad (53).$$

As it is followed from the investigations (figure 17), nevertheless the turning of the indicatrix around z axis exists, and its value depends on x and y coordinates. This disconjugation can be explained by the disparity of bending moment, applied by a three-point method, to model bending presentation. While using this method additional shift strains are induced in sample, which in our case leads to the turning of optical indicatrix.

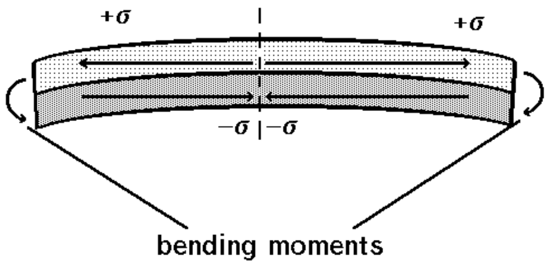


Fig. 16. Schematic view of the bending deformation applied to sample and induced strains.

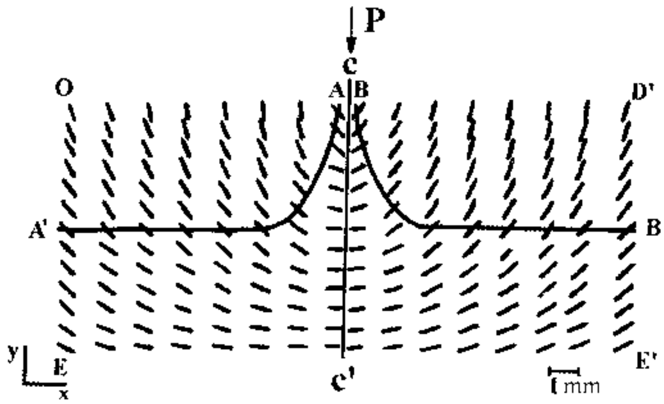


Fig. 17. Distribution of turnings of optical indicatrix in $LiNbO_3$ crystal at bending applied by a three-point method.

We have also investigated the distribution of the birefringence induced by bending deformation in xy plane in $LiNbO_3$ crystal (figure 18). It is visible, that obtained results can not be described by (53). In the points, which belong to CC' curve, the turning of optical indicatrix is absent, that is - additional strains are absent. So, the equations (51-53) are only just for the points that belong to this curve, and that's why one can define the strain σ_{11}:

$$\sigma_{11}=2\Delta n/n_o^3(\pi_{12}-\pi_{11}) \quad (54).$$

Similar investigations were carried out on $NaBi(MoO_4)_2$ crystals, too. Obtained results, as for $LiNbO_3$ crystals, considerably deviated from calculated on the base of optical indicatrix equation. Evidently, a three-point method of bending moment application leads to the appearance of the additional shift strains. May be, it can be used for samples with considerable length exceeding over thickness; then regions with nonideal strains distribution will not fill up all sample.

In connection with this we used a so-called single-arm method of the bending application and thinner sample. The turning of optical indicatrix was not observed; obtained distribution of induced birefringence (figure 19) corresponds to the calculation on the base of the formula (53), especially a so-called neutral birefringence line (line of absence of induced birefringence) was observed. It is necessary to note, that the essential deviations from theoretically predicted dependencies were observed on sample edges - in places of pasting up and loading, and thus in these regions additional strains exist.

In the result of carried out study the distributions of induced birefringence on samples crossing (figures 18 and 19) were obtained. Birefringence does not arise in that case, when an optical ray propagates in the plane y=0 - neutral birefringence line (figure 19), as well as at displacement of optical ray from plane y=0 induced by bending birefringence increases.

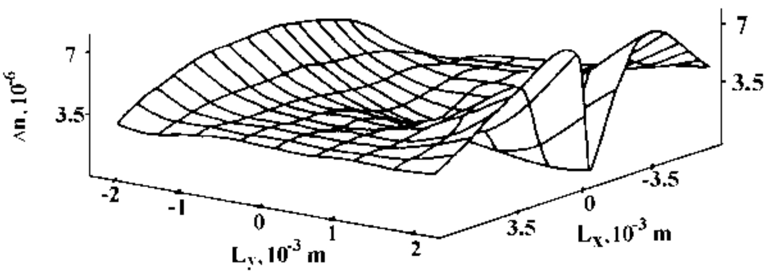


Fig. 18. Distribution of induced birefringence on xy-crossing of $LiNbO_3$ crystal at mechanical bending (case of a three-point bending).

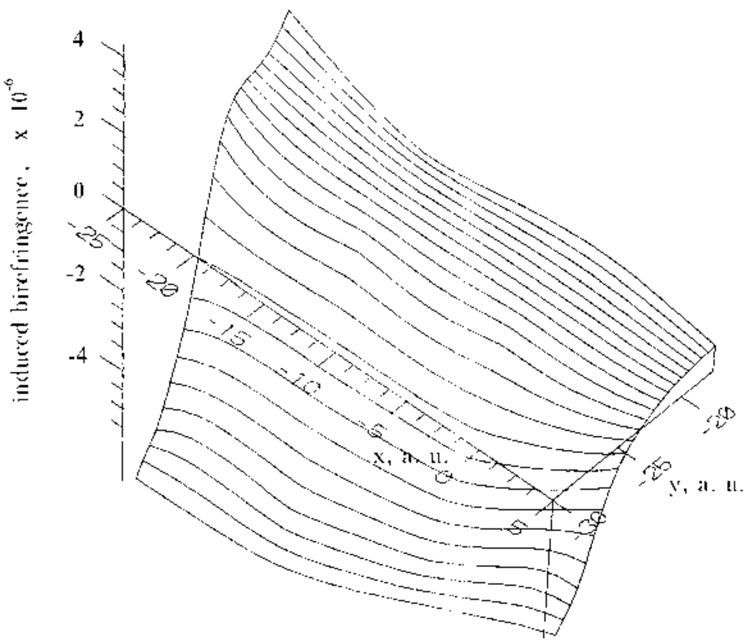


Fig. 19. Distribution of induced birefringence on xy-crossing of $LiNbO_3$ crystal at mechanical bending (case of the single-arm bending).

3.1.5. Torsion-gyrational effect (gradient piezogyration) in $NaBi(MoO_4)_2$ crystals

From the symmetry point of view the lowering of crystal symmetry to the noncentrosymmetrical group causes the appearing of gyration in the centrosymmetrical crystals. That's why gyration, induced by homogeneous mechanical strain (piezogyration), is impossible in centrosymmetrical crystals. At the same time, the change or appearing of the gyration at crystals twisting (torsion-gyrational effect) can be described by the correlation

$$\Delta g_{ij}=\xi_{ijkl}M_{kl}, \qquad (55)$$

where ξ_{ijkl} is a polar tensor of 4-th rank, M_{kl} is a symmetrical axial tensor of 2-nd rank. Most comfortable from the point of view of founding of the torsion-gyrational effect is an experiment with using single-axis centrosymmetrical crystals at light propagation along optical axis and twisting crystal around it, because under such twisting neutral birefringence point should exist and linear birefringence should not mask an investigated effect.

Torsion-gyrational effect was observed in $NaBi(MoO_4)_2$ crystals, which belong to the group of the symmetry 4/m, at propagation of linearly-polarized light along optical axis [32]. As it was found out, the dependence of component of gyration tensor g_{33} on the value of applied moment is linear and its sign changes at the change of twisting direction (figure 20). On the base of obtained experimental results the component of torsion-gyration tensor was calculated – $\xi_{3333}=3.7\times10^{-11}$ m/N.

It is necessary to note, that the change of optical activity under the twisting could be induced by a secondary electrogyration, because at twisting of centrosymmetrical crystals electric polarization can arise [33]:

$$\Delta g_{ij}=(\xi^{D}_{ijkl}+\gamma^{*}_{ijn}B_{nkl})M_{kl}, \qquad (56)$$

where $\gamma^{*}_{ijn}=\gamma_{ijn}/\varepsilon_o(\varepsilon-1)$ are the coefficients, which link electric polarization with optical activity, B_{nkl} are the components of polarization tensor at twisting, ξ^{D}_{ijkl} are torsion-gyrational coefficients for a short-circuit sample. As well as data about B_{333} coefficient is absent, it is impossible to estimate the role of a secondary electrogyration in torsion-gyrational effect.

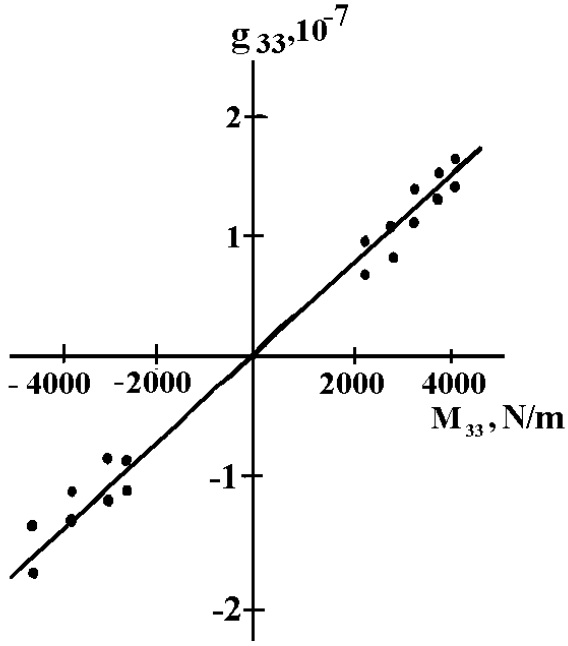


Fig. 20. Dependence of gyration tensor component g_{33} on value of twisting moment M_{ij} for $NaBi(MoO_4)_2$ crystals for configurations of experiment k||z, M||z.

3.2. Influence of the temperature gradient on the refractive properties of the $LiNbO_3$ crystals

The study of the influence of nonhomogeneous temperature distribution on the crystallooptical properties is actual from the point of view of the stability of operating elements of different optoelectronical devices in the natural environment. Among some literature date one can note the investigations of the temperature gradient influence and a special temperature treatment of domain structure [34-36] and biaxiality arising at fast heating (over 10°C/min) in cylindrical furnaces [35]. Biaxiality arising in lithium niobate at the presence of temperature gradient along optical axis was observed [36].

The change of the conoscopic pictures and birefringence were investigated [36] in $LiNbO_3$ crystals with $1\times10\times10$ mm^3 dimensions, cut out perpendicularly to optical axis, along which with the help of specially constructed furnace a temperature gradient up to 40°C/mm was created. The observation of conoscopic pictures suggests that the sample becomes optically biaxial. The behavior of the conoscopic pictures was investigated with the help of He-Ne laser (λ=632.8 nm) or halogen lamp. From a measured distance d between two isogyres branches the angle between optical axis $\tan\theta = d/2L$ (figure 21) was calculated. On figure 21 the angle's dependence on the temperature gradient is given. At all measuring time a value of gradient was supported constantly. The angle θ remained practically invariable on a sample crossing. After smoothing the temperature, a crystal again became a single axis one.

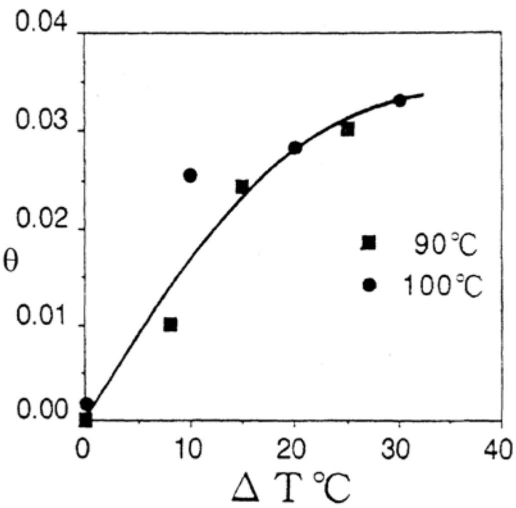


Fig. 21. Dependence of isogyres splitting angle on the value of temperature gradient [37].

The measuring of the birefringence conducted by the Senarmont method, confirmed the arising of optical anisotropy at the presence of the temperature gradient and show its increasing proportional to the temperature gradient (figure 22). However, it was quite difficult to describe the obtained results only on the base of the temperature gradient action as a polar vector influence. Really, the application of the vector action along a three-fold axis of the lithium niobate crystals can not low down the symmetry to the optical biaxial state. So, perhaps, it is necessary to take into account, for example, polarization and deformation induced by the thermal gradient as well as possible nonhomogeneouty of the temperature gradient.

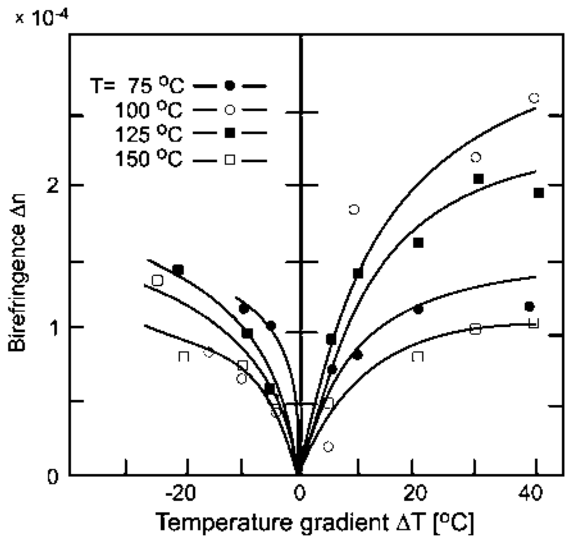


Fig. 22. Dependence of birefringence Δn_z of the $LiNbO_3$ crystals on the temperature gradient [37].

We have also investigated the influence of the $\partial T/\partial x$ and $\partial T/\partial y$ on the refractive properties of the lithium niobate crystals. The optical beam was propagated along z axis. The temperature gradient with maximum value 15 K/mm was induced by the Peltier semiconductor elements. As it comes out from the observation of the conoscopic pictures the temperature gradient induced the birefringence at light propagation in z direction of $LiNbO_3$ crystals. Under the influence of $\partial T/\partial x$ optical indicatrix is turned to 45^o around z axis, but $\partial T/\partial y$ didn't induce the turn of optical indicatrix. Such behavior of the optical indicatrix under the temperature gradient influence corresponds to the gradient thermooptical effect described by the relation $\Delta a_{ij}=s_{ijk}\partial T/\partial x_k$, where s_{ijk} is the third rank polar tensor.

4. Conclusion

Phenomenological analysis of nonlinear optical phenomena caused by nonhomogeneous fields (so called gradient effects) was made. On the base of relation between crystal medium polarization on the optical frequency and gradient of the field with different nature as well as on the base of the symmetry approach it was shown that the electrogyration effect was the first founded phenomena of the gradient nonlinear optics. The existence condition of the new different crystallooptical effects induced by the gradients of fields was analyzed.

It was shown that bending and torsion strains could be described by the axial tensor of the second rank. The presentation of the turning of the optical indicatrix at the multicomponent field influence by the axial vector permits to show, that at the presence of turning of the indicatrix around two crystallophysical axes there always should exist the turning around the third axis. The relation that describes the angle of this turning was derivated and analyzed.

The experimental investigations of mechanical twisting and bending influence on optical properties of crystals were made and the distributions of optical indicatrix, conoscopic pictures and birefringence in $LiNbO_3$ crystals were analyzed. The experimental study permits to find out and to investigate predicted by theory gradient piezogyration effects on the example of $NaBi(MoO_4)_2$ crystals. These investigations testify that the optical topography method of investigation of the distribution of the gyrotropy induced by the gradient strains allows to determine peculiarities of nonuniform strained state. The gradient crystallooptical effects could be applied in the operation of optical radiation [37,38] and in the optical method of the orientation of crystals [39]. Unfortunately only last time the attention was paid to the investigations of nonuniform temperature distribution influence on optical properties of crystals that is very actual from the point of view of characteristics stability of optical elements of different devices.

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